ZZZZ293 – Validation of the position of the under-points of the multifibre beams

Summary:
This test validates the calculation of the position of the under-points of integration in the total reference mark for modeling POU_D_EM and POU_D_TGM. An elementary mechanical calculation is carried out in order to allow the creation of a table with CREA_TABLE starting from the result. Only the coordinates of some under-points are tested in the table.
1 Problem of reference

1.1 Geometry

![Figure 1.1-a: orientation of the element.](image)

The beam is directed in space as indicated on the Figure 1.1-a. Total coordinates of the points $P_1$ and $P_2$:

- $X_{P_1} = 0.0; Y_{P_1} = 0.0; Z_{P_1} = 0.0$
- $X_{P_2} = 2.0; Y_{P_2} = 2.0; Z_{P_2} = 2.0$

Length: $L = 2\sqrt{3}\ m$

1.2 Properties of materials

Concrete:

- Young modulus $E = 3.7272 \times 10^7\ Pa$
- Poisson's ratio $\nu = 0.0$

1.3 Boundary conditions and loadings

On the point $P_1$ one blocks displacements according to $X, Y, Z$ and rotation around the axes $X, Y, Z$:

- $D_{X}^{P_1} = 0.0; D_{Y}^{P_1} = 0.0; D_{Z}^{P_1} = 0.0; DR_{X}^{P_1} = 0.0; DR_{Y}^{P_1} = 0.0; DR_{Z}^{P_1} = 0.0$

On the point $P_2$ one applies a loading according to $X, Y, Z$:

- $F_{X} = 100.0\ N; F_{Y} = 100.0\ N; F_{Z} = -100.0\ N$
2 Reference solution

2.1 Method of calculating

One calculates the position of the nodes, points of integration and under-points of integration from their cordonnées in the local axes of the beam and the matrices of passage between the local axes and the total axes.

By defaults, the local axes and the total axes coincide (Figure 2.1-a).

One applies two rotations (see Figure 1.1-a) to direct the axis of the beam, and the third rotation to position the cross-section:

- \( \alpha = 45^\circ \) around \( Z \)
- \( \beta = -35.26^\circ \) around the new axis \( Y_1 \)
- \( y = 0^\circ \) or \( 90^\circ \) around the new axis \( X_2 \)

**Note:**

one uses conventions of the nautical angles of Code_Aster (see the keyword ORIENTATION of AFFE_CARA_ELEM)

Rotation around the axis \( Z (\alpha) \) is made starting from the following matrix:

\[
T_z(\alpha) = \begin{bmatrix}
\cos(\alpha) & -\sin(\alpha) & 0 \\
\sin(\alpha) & \cos(\alpha) & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Rotation around the new axis \( Y_1 (\beta) \) is made starting from the following matrix:

\[
T_{y_1}(\beta) = \begin{bmatrix}
\cos(\beta) & 0 & \sin(\beta) \\
0 & 1 & 0 \\
-\sin(\beta) & 0 & \cos(\beta)
\end{bmatrix}
\]
Rotation around the new axis $X_2$ is made starting from the following matrix:

$$T_{x2}(y) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(y) & -\sin(y) \\ 0 & \sin(y) & \cos(y) \end{bmatrix}$$

Therefore, for any point of coordinates $(X, Y, Z)$ before rotations, one can calculate his coordinates $(X', Y', Z')$ after rotations with the following transformation:

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \left[ T_{x2}(y) \right] \cdot \left[ T_{z}(\beta) \right] \cdot \left[ T_{y}(\alpha) \right] \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

### 2.2 Sizes and results of reference

One calculates the position of some under-points of integration in the total reference mark knowing their position in the local axes.

With the angles chosen, the digital application gives:

$$T_{y1}(\beta) = \begin{bmatrix} 0.8165 & 0 & -0.5774 \\ 0 & 1 & 0 \\ 0.5774 & 0 & 0.8165 \end{bmatrix} \quad T_{z}(\alpha) = \begin{bmatrix} 0.7071 & -0.7071 & 0 \\ 0.7071 & 0.7071 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and

$$T_{x2}(y) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{or} \quad T_{x2}(y) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For an element length $L = 2 \cdot \sqrt{3} \, m$, the distance from the second point of Gauss compared to the first node is:

For the elements POU_D_EM (modelings A and B) which have two points of Gauss:

$$L = 1 + \sqrt{3} = 2.7320508075688772 \, m$$

For the elements POU_D_TGM (modelings C and D) which have three points of Gauss:

$$L = \frac{1}{2} = 1.7320508075688772 \, m$$
The section of the beam \((0.2\,m \times 0.1\,m)\) in 4 quadrilaterals (Figure is discretized 2.2-a).

![Diagram of a beam section](image)

**Figure 2.2-a**: position of the under-point in the section

The position of the under-points chosen in the initial reference mark for modelings A and B (POU_D_EM) is thus:

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<tr>
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<th>Under-point</th>
<th>x</th>
<th>y</th>
<th>z</th>
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<td>-0.025</td>
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<td>2.732050807568877</td>
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<td>-0.05</td>
<td>0.025</td>
</tr>
</tbody>
</table>

And the position of the under-points chosen in the initial reference mark for modelings C and D (POU_D_TGM) is:

<table>
<thead>
<tr>
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<th>Under-point</th>
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<th>y</th>
<th>z</th>
</tr>
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### 2.3 Uncertainties on the solution

No, exact solution.
3 Modeling A

3.1 Characteristics of the grid

The grid of the beam is composed of an element SEG2. The grid of the section is composed of 4 quadrilateral elements. The group of nodes ENC is composed of the node P1. The group of nodes CHA is composed of the node P2.

3.2 Characteristics of modeling

It is about a modeling POU_D_EM (elements at 2 points of Gauss). The angle of gimlet is equal to $0^\circ$.

Modeling:

```plaintext
MOPOU=AFFE_MODELE (MAILLAGE=MAPOU,
    AFFE_F (TOUT=' OUI', PHENOMENE=' MECANIQUE',
       MODELISATION=' POU_D_EM'),)
```

Boundary conditions:

```plaintext
BLOCAGE=AFFE_CHAR_MECA (MODELE=MO,
    DDL_IMPO=_F (GROUP_NO=' ENC',
        DX=0.0, DY=0.0, DZ=0.0, DRX=0.0, DRY=0.0, DRZ=0.0),)
```

Mechanical loading:

```plaintext
FORCE_NODALE=_F (GROUP_NO=' CHA', FX = 100, FY = 100, FZ = 100.),
```

Assignment of the characteristics of the elements:

```plaintext
POUCA_0=AFFE_CARA_ELEM (MODELE=MOPOU, GEOM_FIBRE=GF,
    POUTRE=_F (GROUP_MA= ('BEAM'),
       SECTION=' RECTANGLE', CARA= ('HY', 'HZ'), VALE= (0.2, 0.1),
       PREC_AIRE=5., PREC_INERTIE=10.,),
    ORIENTATION=_F (GROUP_MA= ('BEAM'), CARA=' ANGL_VRIL', VALE=0.0,)
    MULTIFIBRE=_F (GROUP_MA= ('BEAM'), GROUP_FIBRE= ('SBET',),)
)
```

3.3 Values tested and results

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<th>Under-point</th>
<th>Reference</th>
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</tbody>
</table>

All the tolerance are those by default: $1.0E-3$
4 Modeling B

4.1 Characteristics of the grid

The grid of the beam is composed of an element SEG2.
The grid of the section is composed of 4 quadrilateral elements.
The group of nodes ENC is composed of the node P1.
The group of nodes CHA is composed of the node P2.

4.2 Characteristics of modeling

It is about a modeling POU_D_EM (elements at 2 points of Gauss). The angle of gimlet is of 90°.

Modeling:

```plaintext
MOPOU=AFFE_MODELE (MAILLAGE=MAPOU,
  AFFE_F (TOUT=' OUI', PHENOMENE=' MECANIQUE',
  MODELISATION=' POU_D_EM',)),
```

Boundary conditions:

```plaintext
BLOCAGE=AFFE_CHAR_MECA (MODELE=MO,
  DDL_IMPO_F (GROUP_NO=' ENC',
  DX=0.0, DY=0.0, DZ=0.0, DRX=0.0, DRY=0.0, DRZ=0.0),
)
```

Mechanical loading:

```plaintext
CHARGE=AFFE_CHAR_MECA (MODELE=MO,
  FORCE_NODALE_F (GROUP_NO=' CHA', FX = 100, FY = 100, FZ = 100),
)
```

Assignment of the characteristics of the elements:

```plaintext
POUCA_0=AFFE_CARA_ELEM (MODELE=MOPOU, GEOM_FIBRE=GF,
  POUTRE_F (GROUP_MA= {'BEAM'}, SECTION=' RECTANGLE',
  CARA=('HY', 'HZ'), VALE=(0.2, 0.1),
  PREC_AIRE=5., PREC_INERTIE=10.),
  ORIENTATION_F (GROUP_MA= {'BEAM'}, CARA=' ANGL_VRIL', VALE=90.0),
  MULTIFIBRE=F (GROUP_MA= {'BEAM'}, GROUP_FIBRE= {'SBET'},),
)
```

4.3 Values tested and results

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<th>Coordinate mesh</th>
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<th>under-point</th>
<th>Reference</th>
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</table>

All the tolerance are those by default: 1.0E−3.
## 5 Modeling C

### 5.1 Characteristics of the grid

The grid of the beam is composed of an element `SEG2`. The grid of the section is composed of 4 quadrilateral elements. The group of nodes `ENC` is composed of the node `P1`. The group of nodes `CHA` is composed of the node `P2`.

### 5.2 Characteristics of modeling

It is about a modeling `POU_D_TGM` (elements at 3 points of Gauss). The angle of gimlet is of $0^\circ$.

**Modeling:**

```plaintext
MOPOU=AFFE_MODELE (MAILLAGE=MAPOU,
               AFFE='F' (TOUT='OUI', PHENOMENE='MECANIQUE',
                             MODELISATION='POU_D_TGM'),)
```

**Boundary conditions:**

```plaintext
BLOCAGE=AFFE_CHAR_MECA (MODELE=MO,
                       DDL_IMPO='F' (GROUP_NO='ENC',
                                  DX=0.0, DY=0.0, DZ=0.0, DRX=0.0, DRY=0.0, DRZ=0.0),)
```

**Mechanical loading:**

```plaintext
CHARGE=AFFE_CHAR_MECA (MODELE=MO,
                         FORCE_NODALE='F' (GROUP_NO='CHA', FX=100, FY=100, FZ=100),)
```

**Assignment of the characteristics of the elements:**

```plaintext
POUCA_0=AFFE_CARAELEM (MODELE=MOPOU, GEOM_FIBRE=GF,
                       POUTRE='F' (GROUP_MA=('BEAM'),
                                  SECTION='RECTANGLE', CARA=('HY', 'HZ'), VALE= (0.2, 0.1),
                                  PREC_AIRE=5., PREC_INERTIE=10.),
                       ORIENTATION='F' (GROUP_MA=('BEAM'), CARA='ANGL_VRIL', VALE=0.0),
                       MULTIFIBRE='F' (GROUP_MA=('BEAM'), GROUP_FIBRE=('SBET'),)
```

### 5.3 Values tested and results

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<tr>
<th>Coordinate</th>
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<th>Under-point</th>
<th>Reference</th>
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All the tolerance are those by default: $1.0E^{-3}$

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6 Modeling D

6.1 Characteristics of the grid

The grid of the beam is composed of an element SEG2.
The grid of the section is composed of 4 quadrilateral elements.
The group of nodes ENC is composed of the node P1.
The group of nodes CHA is composed of the node P2.

6.2 Characteristics of modeling

It is about a modeling POU_D_TGM (elements at 3 points of Gauss). The angle of gimlet is of 90°.

Modeling:
```
MOPOU=AFFE_MODELE (MAILLAGE=MAPOU, 
    AFFE=F (TOUT=' OUI', PHENOMENE=' MECANIQUE', 
    MODELISATION=' POU_D_TGM'),)
```

Boundary conditions:
```
BLOCAGE=AFFE_CHAR_MECA (MODELE=MO, 
    DDL_IMPO=F (GROUP_NO=' ENC', 
    DX=0.0, DY=0.0, DZ=0.0, DRX=0.0, DRY=0.0, DRZ=0.0)),
```

Mechanical loading:
```
CHARGE=AFFE_CHAR_MECA (MODELE=MO, 
    FORCE_NODALE=F (GROUP_NO=' CHA', FX = 100, FY = 100, FZ = 100.),
```

Assignment of the characteristics of the elements:
```
POUCA_0=AFFE_CARA_ELEM (MODELE=MOPOU, GEOM_FIBRE=GF 
    POUTRE=F (GROUP_MA= ('BEAM'), 
    SECTION=' RECTANGLE', 
    CARA= ('HY', 'HZ'), VALE= (0.2, 0.1), 
    PREC_AIRE=5., PREC_INERTIE=10.), 
    ORIENTATION=F (GROUP_MA= ('BEAM'), CARA=' ANGL_VRIL', VALE=0.0), 
    MULTIFIBRE=F (GROUP_MA= ('BEAM'), GROUP_FIBRE= ('SBET'),),
```

6.3 Values tested and results

<table>
<thead>
<tr>
<th>Coordinate mesh SG01</th>
<th>Not of integration</th>
<th>Under-point</th>
<th>Reference</th>
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All the tolerance are those by default: 1.0E−3.
# Summary of the results

The purpose of this test is principal to check if the positions of under points of integration of modelings POU_D_EM and POU_D_TGM is well calculated.

For these modelings, the maximum error found is of $1.3 \times 10^{-7}$ %.