PERF018 - Contact between a sphere included in a sphere

Summary:

The objective of this test is to validate method of Mortar type for problems of contact. The grids are incompatible so to show the robustness of the method.
1 Problem of reference

1.1 Geometry

One is considered a sphere digs external ray 30 mm and of internal ray 20 mm, included in a hollow sphere of external ray 40 mm and of internal ray equal to the external ray of the sphere included.

1.2 Properties of material

The material is elastic isotropic whose properties are:

- \( E = 2000 \text{ MPa} \)
- \( \nu = 0.3 \)
1.3 Boundary conditions and loadings

Conditions of symmetry $DX=0$ on surface $ADEF$, $DY=0$ on surface $BCEF$ and $DZ=0$ on surface $ABCD$ are forced in order to model only one eighth of the model.

A pressure $P=25 \text{ MPa}$ is imposed on surface $ABE$, inside the first sphere. The two spheres are maintained in contact with the interface.

2 Reference solution

2.1 Method of calculating

There exists a known analytical solution has this problem, if one considers $R_1$ the interior ray and $R_2$ the external ray, then the radial constraint expressed in polar coordinates is written:

$$\sigma_{rr}(r) = \frac{R_1^3}{R_2^3 - R_1^3} \frac{R_2^3 - r^3}{r^3} P.$$ 

Thus, one finds well $\sigma_{rr}(R_1) = P$ and $\sigma_{rr}(R_2) = 0$.

2.2 Sizes and results of reference

One tests the contact pressure on the interfaces, in $r=30 \text{ mm}$, on both sides of discontinuity. With $R_1=20 \text{ mm}$ and $R_2=40 \text{ mm}$, one has then:

$$\sigma_{rr}(r) = 4.894179894 \text{ MPa}.$$
3 Modeling A

3.1 Characteristics of modeling

A modeling is used 3D.

3.2 Characteristics of the grid

The grid contains 33807 elements of the type TETRA10. Surfaces of contact are not in conformity.

3.3 Sizes tested and results

It is tested the contact pressure on the entirety of surfaces of main contact and slave.

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<th>Identification</th>
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<td>LAGSC</td>
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<td>4.894179894</td>
<td>4%</td>
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4 Modeling B

4.1 Characteristics of modeling

A modeling is used 3D.

4.2 Characteristics of the grid

The grid contains 4080 elements of the type \texttt{HEXA20}. Surfaces of contact are not in conformity.

4.3 Sizes tested and results

It is tested the contact pressure on the entirety of surfaces of main contact and slave.

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5 Modeling C

5.1 Characteristics of modeling

A modeling is used 3D.

5.2 Characteristics of the grid

The grid contains 4080 elements of the type **HEXA27**. Surfaces of contact are not in conformity.

5.3 Sizes tested and results

It is tested the contact pressure on the entirety of surfaces of main contact and slave.

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6 Summary of the results

One seeks on this example very simple to test a novel method of integration of the terms of contact based on the subdivision by subelements. This method is activated when pairing is chosen MORTAR for a zone of contact in formulation CONTINUOUS. One validates in 3D the method for the quadratic elements, in the case of surface curves.