Summary:

The problem consists in calculating the spectral response of a system 2 masses – 4 springs multimedia, subjected to a multiple seismic excitation, by regarding the 3 supports as 2 groups of décorrélés supports.

One tests the discrete element in traction, the calculation of the clean modes, the static modes and the spectral response by modal superposition via the operator COMB_SISM_MODAL. This CAS-test makes it possible to validate the order of the combinations to be considered in this case, namely the office plurality intra-group then the office plurality joint committee.

The got results are in very good agreement with the analytical results of reference.
1 Problem of reference

1.1 Geometry

The system is composed of a set of 4 springs, 2 specific masses, is supported by 3 supports.

1.2 Material properties

Stiffness of connection: \( k = k_1 = k_2 = 10^3 \text{ N/m} \quad k_3 = k_4 = 2.10^3 \text{ N/m} \);
specific mass: \( m = m_1 = m_2 = 10 \text{ kg} \).

1.3 Boundary conditions and loadings

**Boundary conditions:**
Only authorized displacements are the translations according to the axis \( x \).
Points \( NO1 \), \( NO3 \) and \( NO5 \) are embedded: \( dx = dy = dz = drx = dry = drz = 0 \).
The other points are free in translation according to the direction \( x \) : \( dy = dz = drx = dry = drz = 0 \).

**Loading:**
The structure is subjected to a multiple spectral seismic excitation and differential displacements.
The spectra of answers of oscillator in pseudo-acceleration are simplified. Only the values corresponding to the 2 Eigen frequencies of the system are mentioned. They do not depend on damping:

with the node \( NO1 \):
\[
SRO_{NO1}(f_1) = A_{11} = 7 \text{ m/s}^2 \\
SRO_{NO1}(f_2) = A_{21} = 5 \text{ m/s}^2 \\
DDS_{NO1} = D_1 = -0.04 \text{ m}
\]

with the node \( NO3 \):
\[
SRO_{NO3}(f_1) = A_{11} = 7.7 \text{ m/s}^2 \\
SRO_{NO3}(f_2) = A_{21} = 5.5 \text{ m/s}^2 \\
DDS_{NO3} = D_2 = -0.044 \text{ m}
\]

with the node \( NO5 \):
\[
SRO_{NO5}(f_1) = A_{21} = 12 \text{ m/s}^2 \\
SRO_{NO5}(f_2) = A_{22} = 6 \text{ m/s}^2 \\
DDS_{NO5} = D_3 = 0.06 \text{ m}
\]
Excitations with the nodes $NO1$ and $NO3$ are correlated. One sets up 2 groups of décroîtés supports: group 1 is composed of the nodes $NO1$ and $NO3$; group 2 is made up by the only node $NO5$.

1.4 **Initial conditions**

The system is at rest.
2 Reference solution

2.1 Method of calculating used for the reference solution

One calculates the spectral response by modal superposition of a system 2 masses – 4 springs subjected to three distinct excitations. One determines the displacement of the masses to the nodes NO2 and NO4 along the axis x.

One calculates analytically:
- Eigen frequencies \( f_i \),
- associated clean vectors \( \phi_i \), standardized compared to the modal mass,
- static modes of supports \( \Psi_i \) system,
- factors of modal participation \( P_{ij} \) relating to the supports,
- \( Rm_{ij} \) the maximum of answer of each mode starting from the spectra of excitation,
- \( Re_{ij} \) the contribution of the movement of training of each support starting from differential displacements.

2.2 Results of reference

2.2.1 characteristic Matrices and vectors

- matrix of rigidity \( K \)

\[
K = \begin{bmatrix}
  k_1 & -k_1 & 0 & 0 & 0 \\
  -k_1 & k_1+k_2 & -k_2 & 0 & 0 \\
  0 & -k_2 & k_2+k_3 & -k_3 & 0 \\
  0 & 0 & k_3 & k_3+k_4 & -k_4 \\
  0 & 0 & 0 & -k_4 & k_4
\end{bmatrix}
\]

matrix relating to the degrees of freedom 1,2,3,4,5

\[
K_{p} = \begin{bmatrix}
  k_1+k_2 & 0 & -k_2 & 0 & 0 \\
  0 & k_3+k_4 & 0 & -k_3 & -k_4 \\
  -k_1 & 0 & k_1 & 0 & 0 \\
  -k_2 & -k_3 & 0 & k_2+k_3 & 0 \\
  0 & -k_4 & 0 & 0 & k_4
\end{bmatrix}
\]

partitionnée matrix degrees of freedom of structure 2.4, degrees of freedom of support 1,3,5

\[
K_{p} = \begin{bmatrix}
  k & k_{xs} \\
  k_{xs} & k_{ss}
\end{bmatrix} \quad k = \begin{bmatrix}
  k_1+k_2 & 0 \\
  0 & k_3+k_4
\end{bmatrix} \quad k_{ss} = \begin{bmatrix}
  -k_1 & 0 & -k_2 & 0 \\
  0 & -k_3 & 0 & -k_4
\end{bmatrix}
\]

- matrix of mass \( M \)
\[
M = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & m_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & m_2 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
matrix relating to the degrees of freedom 1,2,3,4,5

\[
M^p = \begin{bmatrix}
m_1 & 0 & 0 & 0 & 0 \\
0 & m_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
partitionnée matrix degrees of freedom of structure 2.4, degrees of freedom of support 1,3,5

- modal calculation in embedded base

\[
(K - \lambda_i M) \phi_i = 0 \quad \lambda_i = \omega_i^2
\]

\[
det(K - \lambda_i M) = 0 \implies \lambda_i^2 - \left( \frac{k_1 + k_2}{m_1} + \frac{k_3 + k_4}{m_2} \right) \lambda_i + \frac{(k_1 + k_2)(k_3 + k_4)}{m_1 m_2} = 0
\]

• Eigen frequencies:

\[
\Rightarrow f_1 = \frac{\omega_1}{2\pi}, \quad f_2 = \frac{\omega_2}{2\pi}
\]

• not normalized clean modes:

\[
\phi_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \phi_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\]

• generalized modal masses \( \mu_i = \phi_i^T M \phi_i \):

- \( \mu_1 = \mu_2 = m \)

• own standards modes with the unit generalized modal mass \( \phi_{Nk} \):

\[
\Rightarrow \phi_{N1} = \frac{\phi_1}{\sqrt{\mu_1}}, \quad \phi_{N2} = \frac{\phi_2}{\sqrt{\mu_2}}
\]
• **static modes of supports** $\psi_{Sj}$

Matrix of the static modes reduced to the ddls of structure $\varphi_s = -k^{-1}k_{xs}$

$$\varphi_s = -\frac{1}{4k} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -k & -k \\ 0 & -2k \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix}$$

• **static solution with a unit displacement of the node NO1:**

displacements: $\psi_{S1} = \begin{bmatrix} 1 \\ \frac{1}{2} \\ 0 \\ 0 \end{bmatrix}$

• **static solution with a unit displacement of the node NO3:**

displacements: $\psi_{S2} = \begin{bmatrix} 0 \\ \frac{1}{2} \\ 0 \\ 0 \end{bmatrix}$

• **static solution with a unit displacement of the node NO5:**

displacements: $\psi_{S3} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{2} \end{bmatrix}$

• **rigid mode of body** $\psi_{R1}$

Matrix of the rigid modes reduced to the ddls of structure: $\varphi_r = \varphi_s S_R$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Rigid mode of body $\psi_{R1} = \psi_{S1} + \psi_{S2} + \psi_{S3}$

It is checked well that: $\psi_{R1} = \psi_{S1} + \psi_{S2} + \psi_{S3}$
2.2.2 Loading 1 multi-supports

- factors of modal participation: \( P_{kj} = \phi_{Nk}^T M \psi_j \)

- contribution of the dynamic mode 1 to the movement imposed on the node NO1:
  \[ P_{11} = \phi_{N1}^T M \psi_1 = \frac{\sqrt{m}}{2} \]

- contribution of the dynamic mode 1 to the movement imposed on the node NO3:
  \[ P_{12} = \phi_{N1}^T M \psi_2 = \frac{\sqrt{m}}{2} \]

- contribution of the dynamic mode 1 to the movement imposed on the node NO5:
  \[ P_{13} = \phi_{N1}^T M \psi_3 = 0 \]

- contribution of the dynamic mode 2 to the movement imposed on the node NO1:
  \[ P_{21} = \phi_{N2}^T M \psi_1 = \frac{\sqrt{m}}{2} \]

- contribution of the dynamic mode 2 to the movement imposed on the node NO3:
  \[ P_{22} = \phi_{N2}^T M \psi_2 = \frac{\sqrt{m}}{2} \]

- contribution of the dynamic mode 2 to the movement imposed on the node NO5:
  \[ P_{23} = \phi_{N2}^T M \psi_3 = \frac{\sqrt{m}}{2} \]

- answer of the mode \( i \) with the movement of the support \( j \)

\[ Rm_{kj} = \phi_{Nk} P_{kj} A_{\omega_j} \]

Combined answers of the modal oscillators

Response of mode 1 to the movement of support 1:

\[ Rm_{11} = \phi_{N1} P_{11} \frac{A_{11}}{\omega_1} = \frac{A_{11}}{2 \omega_1}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \]

Response of mode 1 to the movement of support 2:

\[ Rm_{12} = \phi_{N1} P_{12} \frac{A_{12}}{\omega_2} = \frac{A_{12}}{2 \omega_2}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \]

Response of mode 1 to the movement of support 3:

\[ Rm_{13} = \phi_{N1} P_{13} \frac{A_{13}}{\omega_1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]

Response of mode 2 to the movement of support 1:

\[ Rm_{21} = \phi_{N2} P_{21} \frac{A_{21}}{\omega_2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]

Response of mode 2 to the movement of support 2:

\[ Rm_{22} = \phi_{N2} P_{22} \frac{A_{22}}{\omega_2} = \frac{A_{22}}{2 \omega_2}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \]

Response of mode 2 to the movement of support 3:

\[ Rm_{23} = \phi_{N2} P_{23} \frac{A_{23}}{\omega_2} = \frac{A_{23}}{2 \omega_2}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \]

- office plurality intra-group (algebraic sum)
mode 1:
\[ R_{m_{groupe1}} = R_{m_{11}} + R_{m_{12}} = \frac{A_{11} + A_{12}}{2 \omega_1^2} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \]

mode 2:
\[ R_{m_{groupe1}} = R_{m_{21}} + R_{m_{22}} = \frac{A_{22}}{2 \omega_2^2} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \]

- contribution of the support \( j \) with the movement of training

\[ R_{e_j} = \psi_j D_j \]

contributions of groups 1 and 2 to the movement of training

\[ R_{e_{groupe1}} = R_{e_1} + R_{e_2} = \frac{1}{2} \begin{pmatrix} D_1 + D_2 \\ D_2 \end{pmatrix} \quad R_{e_{groupe2}} = R_{e_3} = \frac{1}{2} \begin{pmatrix} 0 \\ D_3 \end{pmatrix} \]

- office plurality on the modes (quadratic)

\[ R_{m_{groupe1}} = \sqrt{(R_{m_{11}} + R_{m_{12}})^2 + (R_{m_{21}} + R_{m_{22}})^2} = \begin{pmatrix} \frac{A_{11} + A_{12}}{2 \omega_1^2} \\ \frac{A_{22}}{2 \omega_2^2} \end{pmatrix} \]

\[ R_{m_{groupe2}} = \sqrt{R_{m_{13}}^2 + R_{m_{23}}^2} = \frac{A_{23}}{2 \omega_3^2} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \]

- answer of the groups of supports 1 and 2

\[ R_1 = \sqrt{R_{m_{groupe1}}^2 + R_{e_{groupe1}}^2} \]
\[ R_1^2 = \frac{1}{4} \begin{pmatrix} \frac{(A_{11} + A_{12})^2}{\omega_1^2} + (D_1 + D_2)^2 \\ \frac{A_{22}^2}{\omega_2^2} + D_2^2 \end{pmatrix} \]

\[ R_2 = \sqrt{R_{m_{groupe2}}^2 + R_{e_{groupe2}}^2} \]
\[ R_2^2 = \frac{1}{4} \begin{pmatrix} 0 \\ \frac{A_{23}^2}{\omega_3^2} + D_3^2 \end{pmatrix} \]

- office plurality joint committee (quadratic)

\[ R = \sqrt{R_1^2 + R_2^2} \]
\[ R^2 = \frac{1}{4} \begin{pmatrix} \frac{(A_{11} + A_{12})^2}{\omega_1^2} + (D_1 + D_2)^2 \\ \frac{A_{22}^2 + A_{23}^2}{\omega_2^2 + \omega_3^2} + D_2^2 + D_3^2 \end{pmatrix} \]

2.3 Uncertainty on the solution

No (analytical solution)
3 Modeling A

3.1 Characteristics of modeling

The system is modelled by:

- 4 discrete elements $K_{T \_ D \_ L}$,
- 2 discrete elements $M_{T \_ D \_ N}$.

3.2 Characteristics of the grid

The grid consists of 4 meshes $SEG2$.

3.3 Parameters of modeling

Answer on the first 2 modes without static correction (combination of modal answers SRSS)
4 Results of modeling A

4.1 Eigen frequencies

<table>
<thead>
<tr>
<th>NODE</th>
<th>Reference</th>
<th>Code_Aster</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.2507900E+00</td>
<td>2.25079079E+00</td>
<td>3.5E-05</td>
</tr>
<tr>
<td>2</td>
<td>3.1830000E+00</td>
<td>3.18309886E+00</td>
<td>0.003</td>
</tr>
</tbody>
</table>

4.2 Total answer on complete modal basis

Modes 1 and 2 are taken into account. The components inertial (primary education) and statics (secondary) of the answer are directly cumulated on the level of the supports.

\[
\text{COMB\_MODE=' SRSS'}
\]

- answer of the support \( j = 1 \) (node \( NO1 \)):
  \[
  R_1 = \sqrt{Rm_1^2 + Re_1^2}
  \]
  with \( Rm_1 = \sqrt{Rm_{11}^2 + Rm_{21}^2} \)

- answer of the support \( j = 2 \) (node \( NO4 \)):
  \[
  R_2 = \sqrt{Rm_2^2 + Re_2^2}
  \]
  with \( Rm_2 = \sqrt{Rm_{12}^2 + Rm_{22}^2} \)

- total answer:
  \[
  R = \sqrt{R_1^2 + R_2^2}
  \]

- absolute displacements: DEPL

<table>
<thead>
<tr>
<th>NODE</th>
<th>Reference</th>
<th>Code_Aster</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NO1</td>
<td>4.00000E-02</td>
<td>4.00000E-02</td>
<td>0.000</td>
</tr>
<tr>
<td>NO2</td>
<td>5.58000E-02</td>
<td>5.58083E-02</td>
<td>0.015</td>
</tr>
<tr>
<td>NO3</td>
<td>4.40000E-02</td>
<td>4.40000E-02</td>
<td>0.000</td>
</tr>
<tr>
<td>NO4</td>
<td>3.85600E-02</td>
<td>3.85683E-02</td>
<td>0.022</td>
</tr>
<tr>
<td>NO5</td>
<td>6.00000E-02</td>
<td>6.00000E-02</td>
<td>0.000</td>
</tr>
</tbody>
</table>

- nodal reactions: REAC\_NODA

<table>
<thead>
<tr>
<th>NODE</th>
<th>Reference</th>
<th>Code_Aster</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NO1</td>
<td>3.68000E+01</td>
<td>3.68044E+01</td>
<td>0.012</td>
</tr>
<tr>
<td>NO2</td>
<td>0.00000E+00</td>
<td>2.23756E-14</td>
<td>2.2E-14</td>
</tr>
<tr>
<td>NO3</td>
<td>8.64900E+01</td>
<td>8.64906E+01</td>
<td>7.0E-04</td>
</tr>
<tr>
<td>NO4</td>
<td>0.00000E+00</td>
<td>3.071843E-14</td>
<td>3.1E-14</td>
</tr>
<tr>
<td>NO5</td>
<td>7.71400E+01</td>
<td>7.71367E+01</td>
<td>-0.004</td>
</tr>
</tbody>
</table>

Warning: The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
5 Summary of the results

Perfect agreement of the results Aster with the analytical values of reference.