SDLL401 - Tilted right beam with 20°, subjected with sinusoidal efforts

Summary:

This test is resulting from the validation independent of version 4 of the models of beams. It makes it possible to check the internal efforts on an inclined beam, for sinusoidal loadings according to time (a modeling with elements POU_D_T, right beam of Timoshenko).
1 Problem of reference

1.1 Geometry

Right beam length 1 m.

slope 20° compared to x (trigonometrical direction).

Characteristics of the section:

\[ S = \pi \times 0.01^2 \, m^2 \]

1.2 Properties of materials

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young modulus</td>
<td>( E = 2 \times 10^{11} , Pa )</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>( \nu = 0.3 )</td>
</tr>
<tr>
<td>Density</td>
<td>( \rho = 7800 , kg/m^3 )</td>
</tr>
</tbody>
</table>
1.3 Boundary conditions and loading

Boundary condition:

- For the loading distributed [Figure -1.1-a]
  Nodes A and B embedded: DX, DY, DZ, DRX, DRY, DRZ blocked
- For the specific loading [Figure -1.1-b]
  Node A embedded: DX, DY, DZ, DRX, DRY, DRZ blocked

Loadings:

- \( f(t) = 1000 \times \cos(t) \) according to the direction AB either distributed or applied at the end B
- \( M_T(t) = 1000 \times \cos(t) \) applied at the end B
2 Reference solutions

2.1 Method of calculating used for the reference solutions

2.1.1 Loading distributed of traction and compression

A right beam length $L$ working only in traction and compression is subjected to a loading distributed constant according to $x$ but varying in a sinusoidal way according to time. It is embedded at its two ends.

$$\begin{align*}
\rho S \frac{\partial^2 u}{\partial t^2} - ES \frac{\partial^2 u}{\partial x^2} &= f(t) \\
u(0) &= 0, \quad u(L) = 0
\end{align*}$$

To solve, one applies to the equation the transform of Fourier in time:

$$\frac{\partial^2 \hat{u}}{\partial x^2} = -\frac{\rho}{E} 4\pi^2 \omega^2 \hat{u} + \frac{1}{ES} \hat{f}(\omega)$$

$\hat{u}$ : transform of Fourier of $u$,
$\hat{f}$ : transform of Fourier of $f$.

Thus, we have for $f(t)=F \cos(2\pi \omega_0 t)$ :

$$u(x, t) = \frac{a^2 F}{ES 4\pi^2 \omega_0^2} \left[ \cos \left( \frac{2\pi \omega_0}{a} L \right) - 1 \right] \sin \frac{2\pi \omega_0}{a} x - \frac{a^2 F}{ES 4\pi^2 \omega_0^2} \left[ \cos \left( \frac{2\pi \omega_0}{a} L \right) - 1 \right] \sin \frac{2\pi \omega_0}{a} x$$

with: $a^2 = \frac{E}{\rho}$.

The use of the law of behavior gives us the tractive effort compression:

$$N(x, t) = \frac{a F}{2\pi \omega_0} \left[ 1 - \cos \left( \frac{2\pi \omega_0}{a} L \right) \cos \left( \frac{2\pi \omega_0}{a} x \right) \sin \left( \frac{2\pi \omega_0}{a} L \right) - \sin \left( \frac{2\pi \omega_0}{a} x \right) \cos \left( \frac{2\pi \omega_0}{a} L \right) \right]$$
2.1.2 Specific loadings

A beam comforts length \( L \) working only in traction compression (or torsion) is subjected to a sinusoidal force in time, (or a moment) applied at its loose lead.

2.1.2.1 Traction

\[
\rho S \frac{\partial^2 u}{\partial t^2} - ES \frac{\partial^2 u}{\partial x^2} = 0
\]

\[u(0) = 0, \quad \frac{\partial u}{\partial x}(L) = \frac{1}{ES} f(t).\]

The technique of resolution is equivalent to that of the paragraph [§ 2.1.1.1].

For \( f(t) = F \cos(2\pi \omega_0 t) \), we have:

\[
u(x, t) = \frac{a F}{ES2\pi \omega_0} \sin\left[\frac{2\pi \omega_0}{a} x\right] \cos(2\pi \omega_0 t)
\]

\[
\text{avec } a^2 = \frac{E}{\rho}
\]

and \( N(x, t) = F \frac{\cos\left[\frac{2\pi \omega_0}{a} x\right]}{\cos\left[\frac{2\pi \omega_0}{a} L\right]} \cos(2\pi \omega_0 t)\)

2.1.2.2 Torsion

\[
G I_\rho \frac{\partial^2 \theta}{\partial x^2} - I_{\theta_x} \frac{\partial^2 \theta_x}{\partial t^2} = f(t)
\]

\[u(0) = 0, \quad u(L) = 0\]

\[G = \frac{E}{2(1+v)}\]

\[I_\rho = \frac{\pi 0.014^4}{2} m^4, \quad \theta_x(x, t) = \frac{b F}{G I_\rho} \sin\left[\frac{2\pi \omega_0}{b} x\right] \cos(2\pi \omega_0 t)\]

\[
M_T(x, t) = F \frac{\cos\left[\frac{2\pi \omega_0}{b} x\right]}{\cos\left[\frac{2\pi \omega_0}{b} L\right]} \cos(2\pi \omega_0 t)
\]

\[
\text{avec } b = \frac{G}{\rho}
\]
2.2 **Results of reference**

Interior efforts ($N$ and $MT$)

2.3 **Uncertainty on the solution**

Analytical solution.

2.4 **Bibliographical references**

1) Report n° 2314/A of the Institute Aerotechnics “Proposal and realization for new cases tests missing with the validation of the beams ASTER”
3 Modeling A

3.1 Characteristics of modeling

The model is composed of 2 elements right beam of Timoshenko.

3.2 Characteristics of the grid

2 elements $POU_D_T$

3.3 Sizes tested and results

3.3.1 Distributed load in traction

<table>
<thead>
<tr>
<th>Analytical results</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal effort for $x=0$, $t=1/3,s$</td>
<td>472.478 NR</td>
</tr>
<tr>
<td>$t=2/3,s$</td>
<td>392.944 NR</td>
</tr>
</tbody>
</table>

| Normal effort for $x=L/2$, $t=1/3\,s$ | 0 NR | 1E-6 NR (*) |
| $t=2/3\,s$ | 0 NR | 1E-6 NR (*) |

(*) absolute Deviation

3.3.2 Concentrated loading

3.3.2.1 Loading in traction

Normal effort for $x=0$

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<td>$t=1/3,s$</td>
<td>944.957 NR</td>
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<tr>
<td>$t=2/3,s$</td>
<td>785.887 NR</td>
</tr>
</tbody>
</table>

3.3.2.2 Loading in torsion

Torque for $x=0$

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<td>$t=1/3,s$</td>
<td>944.957 N.m</td>
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<td>785.887 N.m</td>
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4 Modeling B

It is noted that the beam is very stiff on its modes of traction and compression:

\[ f_{\text{traction/compression}} = \frac{1}{2\pi} \sqrt{\left( \frac{a}{\rho c} \right)} = \frac{1}{2\pi} \sqrt{\frac{E}{\rho c}} = 806 \text{ Hz} \]

In comparison, the frequency of the efforts of traction and compression with \( \frac{1}{2\pi} \text{Hz} \) can be regarded as quasi-static. It is what is made in modeling b: one compares the results of a calculation with the linear operator of statics of Code_Aster with the results of modeling A on the case of the force distributed.

Modeling finite elements being identical to that of modeling A, the results expected are the same ones:

Distributed load in traction

<table>
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<td>Normal effort for ( x = 0 ) ( t = 1/3 \text{ s} )</td>
<td>472.478 NR</td>
<td>1E-3 %</td>
</tr>
<tr>
<td></td>
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<td>1E-3 %</td>
</tr>
<tr>
<td>Normal effort for ( x = L/2 ) ( t = 1/3 \text{ s} )</td>
<td>0 NR</td>
<td>1E-6 NR (*)</td>
</tr>
<tr>
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<td>0 NR</td>
<td>1E-6 NR (*)</td>
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(*) absolute Deviation

5 Summary of the results

This test makes it possible to check that the internal efforts of the elements of beam in dynamics are correct. The results show very a good agreement with the analytical solution, for a grid only made up of two elements \textsc{POU_D_T}.

It is also shown that for a very stiff system compared to the frequency of the request, a quasi-static calculation gives as good performances as dynamic calculation.