SDLS115 – Comparison with the analytical solution of a plate in traction

Summary:

This test validates the basic operation of modeling DKT for a transitory calculation with a diagram clarifies digital integration by the operator DYNA_NON LINE. The plate is subjected to the boundary conditions corresponding to a simple traction, making it possible to find the analytically calculated answer.
1 Problem of reference

1.1 Geometry

![Square plate diagram]

Square plate:
- Length: \( l = 1.0 \) m
- Thickness: \( e = 0.1 \) m

1.2 Properties of material

- Young modulus, \( E = 4.388 \times 10^{10} N/m^2 \)
- Poisson's ratio, \( \nu = 0.0 \)
- Density, \( \rho = 2500.0 \) kg/m\(^3\)

1.3 Boundary conditions and loadings

**Boundary conditions:**

<table>
<thead>
<tr>
<th>Localisation</th>
<th>Blocked components</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>DX, DY, DZ, DRX, DRY MARTINI, DRZ</td>
</tr>
<tr>
<td>With C</td>
<td>DX, DZ, DRX, DRY MARTINI, DRZ</td>
</tr>
<tr>
<td>C</td>
<td>DZ, DRX, DRY MARTINI, DRZ</td>
</tr>
</tbody>
</table>

**Loadings:**

One applies the linear force to with dimensions B in the direction \( x \), which depends on time like,

\[
F(t) = Q_0E K e \cos(Kl) \sin(\omega t),
\]

where the following parameters are used:

- \( Q_0 \) - amplitude of the loading
- \( E \) - Young modulus defined above (in \( N/m^2 \))
• $e$ – the thickness defined above (in \( m \))
• $l$ – the dimension of the plate defined above (in \( m \))
• $K \left( \frac{\pi}{8l} \right)$ the number of wave of the analytical solution (in \( m^{-1} \))
• $\omega$ – frequency (time \( 2\pi \)), related to the number of wave $K$, $K = \omega / c$, \( c \) being the celerity of the waves in the structure, \( c = \sqrt{\frac{E}{\rho}} \)

The introduced parameter setting makes it possible to apply the loading right to obtain the analytical solution, simply given by the parameters $Q_0$ and $K$, and then by other parameters of dimensions and properties structural material.

1.4 Initial conditions

At the beginning displacements are worth zero everywhere and speeds obey spatial following,

\[ v_0(x, y) = \omega Q_0 \sin(Kx) \]
2 Reference solution

2.1 Method of calculating

One deals here with a problem of structure (quasi)-unidimensional subjected to a force of edge, 
\( F(t) \), where the analytical solution can be written like,

\[ u(x,t) = Q_0 \cos(Kx)\sin(\omega t) \]

In order to obtain this solution one must apply the force and the initial conditions specified above. The parameters are also commented on there.

2.2 Sizes and results of reference

It is displacement \( x \) with the node \( A_2 \) and at the moment, \( t_{\text{max}} = 0.0012 \text{s} \), which must be equal to

\[ u(x_{A_2},t) = Q_0 \cos(Kx_{A_2})\sin(\omega t_{\text{max}}) \]

the value being calculated in the data file starting from the selected values of the parameters.

2.3 Uncertainties on the solution

Exact solution.

2.4 Bibliographical references

S. Timoshenko, *Theory of the vibrations*, 1939
3 Modeling A

3.1 Characteristics of modeling

Modeling: DKT

3.2 Characteristics of the grid

Nodes: 16
Meshes: 9 QUAD4

3.3 Sizes tested and results

Three calculations are carried out with the operator DYNA_NON_LINE, the first with an explicit diagram, the second with an implicit scheme (HHT) slightly deadened and the third with an implicit scheme (HHT) strongly deadened.

All the values tested are taken at the moment \( t_{\text{max}} = 0.0012 \text{s} \).

Calculation 1: explicit

<table>
<thead>
<tr>
<th>Identification</th>
<th>Reference</th>
<th>Type of reference</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement ( D_X ) in ( A_2 )</td>
<td>3.51957D-05</td>
<td>‘ANALYTICAL’</td>
<td>0.5 %</td>
</tr>
<tr>
<td>DEGE_ELNO, (M1 Mesh), node ( A_2 ), Comp. EXX</td>
<td>3.33675621777E-05</td>
<td>‘ANALYTICAL’</td>
<td>5 %</td>
</tr>
<tr>
<td>EFGE_ELNO, (M1 Mesh), node ( A_2 ), Comp. NXX</td>
<td>1.4641686283587E5</td>
<td>‘ANALYTICAL’</td>
<td>5 %</td>
</tr>
<tr>
<td>ENEL_ELNO, (M1 Mesh), node ( A_2 ), Comp. TOTAL</td>
<td>2.4427868872105</td>
<td>‘ANALYTICAL’</td>
<td>10 %</td>
</tr>
<tr>
<td>ENEL_ELNO, (M1 Mesh), node ( A_2 ), Comp. MEMBRANE</td>
<td>2.4427868872105</td>
<td>‘ANALYTICAL’</td>
<td>10 %</td>
</tr>
<tr>
<td>ENEL_ELGA, (M1 Mesh), point 2, comp. TOTAL</td>
<td>2.4427868872105</td>
<td>‘ANALYTICAL’</td>
<td>10 %</td>
</tr>
<tr>
<td>ENEL_ELGA, (M1 Mesh), point 2, comp. INFLECTION</td>
<td>0.</td>
<td>‘ANALYTICAL’</td>
<td>1E-13 (Absolute)</td>
</tr>
</tbody>
</table>

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Calculation 2: implicit slightly deadened

<table>
<thead>
<tr>
<th>Identification</th>
<th>Reference</th>
<th>Type of reference</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement DX in A2</td>
<td>3.51957D-05</td>
<td>‘ANALYTICAL’</td>
<td>0.05 %</td>
</tr>
</tbody>
</table>

Calculation 3: implicit strongly deadened

<table>
<thead>
<tr>
<th>Identification</th>
<th>Reference</th>
<th>Type of reference</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement DX in A2</td>
<td>-</td>
<td>‘NON_REGRESSION’</td>
<td>-</td>
</tr>
</tbody>
</table>
4 Modeling B

4.1 Characteristics of modeling

Modeling: BAR

In this modeling, the plate square is modelled by a rectangular bar of section of one meter broad and a thickness of 0.1m.

4.2 Boundary conditions and loadings

Boundary conditions:

<table>
<thead>
<tr>
<th>Localisation</th>
<th>Blocked components</th>
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</thead>
<tbody>
<tr>
<td>A1</td>
<td>DX, DY, DZ</td>
</tr>
<tr>
<td>C</td>
<td>DX, DZ</td>
</tr>
</tbody>
</table>

Loadings:

The force is affected in A2, it is a nodal force which must be multiplied by the width of the plate to be equivalent to modeling A.

4.3 Characteristics of the grid

Nodes: 4
Meshs: 3 SEG2

4.4 Sizes tested and results

Three calculations are carried out, the first with DYNA_NON LINE in explicit diagram and MASS_DIAG=' NON' (this calculation is cut into two in order to test the good transmission of the table of observation), the second with DYNA_NON LINE in explicit diagram and MASS_DIAG=' OUI' and the third with DYNA_VIBRA without matrix of diagonal mass.

The first calculation:

<table>
<thead>
<tr>
<th>Identification</th>
<th>Reference</th>
<th>Type of reference</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>3.51957D-05</td>
<td>'ANALYTICAL'</td>
<td>0.05 %</td>
</tr>
<tr>
<td>Displacement</td>
<td>0.</td>
<td>'ANALYTICAL'</td>
<td>0.1 %</td>
</tr>
</tbody>
</table>

Warning: The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Displacement $DX$ in $A1$, $\text{INST} = 9\text{th-4}$ (via table of observation)

0. 'ANALYTICAL' 0.1 %

The second calculation:

<table>
<thead>
<tr>
<th>Identification</th>
<th>Reference</th>
<th>Type of reference</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement $DX$ in $A2$, $\text{INST} = 1,2\text{E-3}$</td>
<td>3.51957D-05</td>
<td>'ANALYTICAL'</td>
<td>0.5 %</td>
</tr>
</tbody>
</table>

The third calculation:

<table>
<thead>
<tr>
<th>Identification</th>
<th>Reference</th>
<th>Type of reference</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECIN_ELEM, Mesh M21, Comp. TOTAL</td>
<td>9.10387D-02</td>
<td>'ANALYTICAL'</td>
<td>0.5 %</td>
</tr>
</tbody>
</table>
5 Summary of the results

The purpose of this test is principal to check if the combination of modeling `DKT` and of the operator `DYNA_NON_LINE` function correctly. The difference between the solution Aster and that of the reference is very weak.