SDLS503 - Vibrations of inflection of a beam sandwich

Summary:

This test represents a calculation in modal analysis of a sandwich beam simply supported. This test makes it possible to validate:

- modeling finite elements DKT with meshes QUAD4 and TRIA3,
- modeling finite elements DST with meshes QUAD4 and TRIA3,
- the taking into account of rigidity in transverse shearing,
- the taking into composite material account.

The frequencies and the modes obtained are compared with an analytical reference solution. It should be noted that modeling DKT, whose formulation does not take into account transverse shearing, is not adapted to model this CAS-test. Thus, the errors are very important for modelings A and B. The results got with DST are satisfactory.
1 Problem of reference

1.1 Geometry

![Diagram of the beam with dimensions and labels]

- $L = 1.0 \text{ m}$
- $d = 0.1 \text{ m}$
- $h_1 = 0.1 \text{ m}$
- $h_2 = 0.05 \text{ m}$

1.2 Properties of material

- **Coatings:**
  - $E_z = 4.10^{10} \text{ Pa}$
  - $G_{xz} = 4.10^9 \text{ Pa}$
  - $\nu_{xz} = 0.3$
  - $\rho_1 = 2000 \text{ kg/m}^3$

- **Heart:**
  - $E_z = 4.10^7 \text{ Pa}$
  - $G_{xz} = 1.5.10^7 \text{ Pa}$
  - $\nu_{xz} = 0.3$
  - $\rho_2 = 50 \text{ kg/m}^3$

Coefficient of shearing $K = 1/K = 110.8$

The Poisson's ratios are identical: $\nu_{xz} = \nu_{xy} = \nu_{yz}$

1.3 Boundary conditions and loadings

The beam rests simply on with dimensions ones $AB$ and $CD$.

1.4 Initial conditions

Without object
2 Reference solution

2.1 Method of calculating used for the reference solution

Calculation is carried out starting from the relations of dynamic balance and behavior [bib2] pointed out Ci - afterwards:

\[
\frac{\partial M_x}{\partial X} + T_y = \langle \rho I \rangle \frac{\partial^2 \theta}{\partial t^2} \quad \frac{\partial T_y}{\partial X} = \langle \rho S \rangle \frac{\partial^2 v}{\partial t^2}
\]

\[
M_z = \langle EI \rangle \frac{\partial^2 z}{\partial X^2} \quad T_y = K \langle GS \rangle \frac{\partial v}{\partial X} - \frac{\partial T_y}{\partial X} \theta_z
\]

These relations make it possible to write the equation of the movement of dynamic inflection transverse \( \nu(x,t) \). One obtains the equation at the Eigen frequencies after having associated the boundary conditions.

The equation at the Eigen frequencies is written:

\[
\sin(X_2) = 0 \quad \text{with} \quad X_2 = \frac{\omega^2 (1+a)^2}{2} + \sqrt{\omega^2 \left( \frac{1-a^2}{2} \right)^2 + \frac{1}{r^2}}}^{1/2}
\]

and

\[
\omega^2 = \frac{\langle \rho I \rangle \omega^2 l^2}{\langle EI \rangle} ; \quad r^2 = \frac{\langle \rho I \rangle}{\langle \rho S \rangle} ; \quad a = \frac{\langle \rho S \rangle \langle EI \rangle}{K \langle \rho I \rangle \langle GS \rangle}
\]

The solutions of the equation at the Eigen frequencies are written then:

\[
X_2 = n \pi \quad (n=1,2,3,...)
\]

2.2 Results of reference

the first 5 frequencies and clean modes of inflection associated.

- Frequency mode 1: 64.476 Hz
- Frequency mode 2: 131.918 Hz
- Frequency mode 3: 198.734 Hz
- Frequency mode 4: 265.383 Hz
- Frequency mode 5: 331.963 Hz

2.3 Uncertainties on the solution

The reference solution is calculated within the framework of the assumptions of the theory of the beams [bib2]: \( \sigma_y = \sigma_z = 0 \).

2.4 Bibliographical references


3 Modeling A

3.1 Characteristics of modeling

Modelisation DKT (TRIA3)

- Plaque située dans le plan x = 0.33
- Conditions aux limites : Côtes AB et CD : u=0

3.2 Characteristics of the grid

Many nodes: 22
Number of meshes and type: 20 TRIA3

3.3 Sizes tested and results

<table>
<thead>
<tr>
<th>Identification</th>
<th>Reference</th>
<th>Aster</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency mode 1</td>
<td>64,476</td>
<td>277,449</td>
<td>330.</td>
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<tr>
<td>Frequency mode 2</td>
<td>131,918</td>
<td>1105.83</td>
<td>738.</td>
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<td>Frequency mode 3</td>
<td>198,734</td>
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<tr>
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<td>265,383</td>
<td>4363.97</td>
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<tr>
<td>Frequency mode 5</td>
<td>331,963</td>
<td>6753.904</td>
<td>1.93E3</td>
</tr>
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</table>

3.4 Remarks

In the table of results, we deferred the frequencies whose modes are identical to the modes of reference.

- the effects of transverse shearing are neglected in modeling DKT,
- the Aster results are much higher than the results of reference,
- appearance of a mode of membrane enters modes 2 and 3 and between modes 5 and 6 of reference.

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4 Modeling B

4.1 Characteristics of modeling

![Modélisation DKT (QUAD4)](image)

- Plaque située dans le plan x= 0.33
- Conditions aux limites : Cotés AB et CD : u=0

4.2 Characteristics of the grid

Many nodes: 22
Number of meshes and type: 10 QUAD4

4.3 Sizes tested and results

<table>
<thead>
<tr>
<th>Identification</th>
<th>Reference</th>
<th>Aster</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency mode 1</td>
<td>64,476</td>
<td>277,788</td>
<td>331</td>
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<td>Frequency mode 2</td>
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<td>1111.225</td>
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<td>Frequency mode 3</td>
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<td>1.52E3</td>
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<td>Frequency mode 5</td>
<td>331,963</td>
<td>6960.324</td>
<td>2.00E3</td>
</tr>
</tbody>
</table>

4.4 Remarks

In the table of results, we deferred the frequencies whose modes are identical to the modes of reference.

- the effects of transverse shearing are neglected in modeling DKT,
- the Aster results are much higher than the results of reference,
- appearance of a mode of membrane enters modes 2 and 3 and between modes 5 and 6 of reference.
5 Modeling C

5.1 Characteristics of modeling

![Modélisation DST (TRIA3)](image)

- Plaque située dans le plan x = 0.33
- Conditions aux limites : Cotés AB et CD : u = 0

5.2 Characteristics of the grid

Many nodes: 22
Number of meshes and type: 20 TRIA3

5.3 Sizes tested and results

<table>
<thead>
<tr>
<th>Identification</th>
<th>Reference</th>
<th>Aster</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency mode 1</td>
<td>64,476</td>
<td>64,573</td>
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<td>Frequency mode 2</td>
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<td>Frequency mode 3</td>
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<td>Frequency mode 5</td>
<td>331,963</td>
<td>365,919</td>
<td>10,229</td>
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6 Modeling D

6.1 Characteristics of modeling

![Modélisation DST (QUAD4)](image)

- Plaque située dans le plan x= 0.33
- Conditions aux limites : Cotés AB et CD : u=0

6.2 Characteristics of the grid

Many nodes: 22
Number of meshes and type: 10 QUAD4

6.3 Sizes tested and results

<table>
<thead>
<tr>
<th>Identification</th>
<th>Reference</th>
<th>Aster</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency mode 1</td>
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<td>0.184</td>
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<td>Frequency mode 3</td>
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<td>Frequency mode 5</td>
<td>331,963</td>
<td>320,409</td>
<td>-3.480</td>
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</table>
Summary of the results

Modeling DKT is not adapted to model this CAS-test, the errors are very important. The formulation DKT does not take into account transverse shearing contrary to modeling DST. For this kind of example, where the structure consists of a composite and relatively thick material (\( h/L = 0.1 \)), it is preferable to use modeling DST.

Results got with DST are:

- satisfactory for the first 3 frequencies with the mesh TRIA3 and for the first 5 frequencies for the mesh QUAD4 with a better precision for the mesh QUAD4,
- the error of 10% for the 4\textsuperscript{ième} and 5\textsuperscript{ième} frequency with the mesh TRIA3 is significant. A finer grid should make it possible to improve the results by having a better representation of the last modes.