SSLL103 - Elastic buckling of an angle

Summary:

A right beam (corner with equal wings) biarticulée is subjected to a normal effort (excentré or not) or to one bending moment.

One seeks the critical loads of elastic buckling.
• linear elastic mechanics,
• buckling of a beam,
• offsetting of the center of torsion,
• interest of the test: calculation of the geometrical matrix of rigidity of the elements POU_D_TG and POU_D_T,
• 2 modelings.

An uncertainty persists on the number of modes of buckling of the reference solution [§5].
### 1 Problem of reference

#### 1.1 Geometry

![Geometry Diagram]

### Caractéristiques de la section

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1856 mm²</td>
</tr>
<tr>
<td>I_y</td>
<td>4167339 mm⁴</td>
</tr>
<tr>
<td>I_z</td>
<td>1045547 mm⁴</td>
</tr>
<tr>
<td>J</td>
<td>39595 mm⁴</td>
</tr>
<tr>
<td>I_yr2</td>
<td>84948392 mm⁴</td>
</tr>
<tr>
<td>y_c</td>
<td>-41.012 mm</td>
</tr>
<tr>
<td>z_c</td>
<td>0</td>
</tr>
</tbody>
</table>

#### a = 120 mm
e = 8 mm
CG = 41.012 mm

#### 1.2 Material properties

Young modulus: \( E = 2.10 \times 10^{-5} \) MPa
Poisson's ratio: \( \nu = 0.3 \)

#### 1.3 Boundary conditions and loadings

- **A1**: \( DX = DY = DZ = DRX = 0 \)
- **A2**: \( DY = DZ = DRX = 0 \)

Loading
- case 1: axial load \( P \) in \( G \)
- case 2: axial load \( P \) in \( C \)
- case 3: axial load \( P \) in \( A \)
- case 4: bending moment \( M \)

#### 1.4 Remarks

For cases 2 and 3, one applies in \( A2 \) an effort in \( G \), then one superimposes in \( A1 \) and \( A2 \) one bending moment (according to \( o_z \) for case 2, following \( o_y \) for case 3) to offset the effort in \( C \) (or in \( A \)).
2 Reference solution

2.1 Method of calculating used for the reference solution

With taking into account of warping, the calculations done by V. Of City De Goyet [bib1] give:

that is to say:

\[
I_y = \int_A z^2 dA \quad I_z = \int_A y^2 dA \quad I_{yz} = \int_A yz \left( y^2 + z^2 \right) dA
\]

\[
P_{cry} = \frac{\pi^2 EI_z}{L^2} \quad P_{crz} = \frac{\pi^2 EI_y}{L^2} \quad P_{crx} = \left( \frac{GJ + \pi^2 EI_{infty}}{L^2} \right) Ar_a
\]

\[
Ar_c = \left( I_y + I_z \right) + y_c^2 + z_c^2 + y_c \left( \frac{I_{yz}}{I_z} - 2y_c \right) + z_c \left( \frac{I_{yz}}{I_z} - 2z_c \right)
\]

\[
Ar_a = \left( I_y + I_z \right) + y_a^2 + z_a^2 + y_a \left( \frac{I_{yz}}{I_z} - 2y_a \right) + z_a \left( \frac{I_{yz}}{I_z} - 2z_a \right)
\]

with:

\[
\begin{align*}
y_{a}, z_{a} & : \text{coordinates of the point of load application} \\
y_{c}, z_{c} & : \text{coordinates of the center of torsion}
\end{align*}
\]

Case 1,2,3:

One obtains 3 critical loads by solving the equation of the 3° degree in \( P \):

\[
Ar_a P_{cry} - P \left| P_{crz} - P \right| P_{crx} - P - P^2 \left| P_{crx} - P \right| (z_c - z_a)^2 - P^2 \left| P_{cry} - P \right| (y_c - y_a)^2 = 0
\]

Case 4:

Critical moment \( M_{cr} \) (around the axis \( y \)) is worth:

\[
M_{cr} = \pm \left( \frac{GJ + \pi^2 EI_{infty}}{L^2} \right)^{1/2} P_{cry}
\]

By neglecting warping: the analytical solution of reference is given in [bib2] [bib3].

2.2 Results of reference

Values of the critical loads corresponding to the first modes of buckling for the various loading cases.

2.3 Uncertainty on the solution

Analytical solution. The values of reference are obtained using NAG (routine COSAGF, \( EPS = 10^{-8} \)).

2.4 Bibliographical references

1. V. OF TOWN OF GOYET "Analyzes static nonlinear by the finite element method of the formed space structures by beams with nonsymmetrical section" - Doctorate University of Liege, MSM, academic year (1988-1989).

2. P. PENSERINI "elastic Instability of the beams with open mean profile: theoretical and digital aspects" Notes EDF/DER/HM77/112.

3. J. CHERRY TREE "Propagation of two cases tests of modeling of the calculation of the beams in elastic buckling in Code_Aster"HM77/184
3 Modeling A

3.1 Characteristics of modeling

8 elements `POU_D_TG`

![Diagram of modeling elements](image)

3.2 Characteristics of the grid

Many nodes: 9
Many meshes and types: 8 `SEG2`

3.3 Sizes tested and results

<table>
<thead>
<tr>
<th>Identification</th>
<th>Reference</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>mode 1</td>
<td>6.92531E+05</td>
<td>3.0E-3</td>
</tr>
<tr>
<td>mode 2</td>
<td>1.50487E+06</td>
<td>1.0E-2</td>
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<tr>
<td>mode 3</td>
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<td>0.04</td>
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<tr>
<td>Case 2</td>
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<td></td>
</tr>
<tr>
<td>mode 1</td>
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<td>3.0E-3</td>
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<tr>
<td>mode 2</td>
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<td>Case 3</td>
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<td>Case 4</td>
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<tr>
<td>mode 1</td>
<td>-7.00631E+07</td>
<td>5.0E-3</td>
</tr>
</tbody>
</table>

3.4 Remarks

The precision is excellent with 8 elements in the length.
4 Modeling B

4.1 Characteristics of modeling
8 elements $POU_D_T$

4.2 Characteristics of the grid
Many nodes: 9
Many meshs and types: 8 $SEG2$

4.3 Sizes tested and results

<table>
<thead>
<tr>
<th>Identification</th>
<th>Reference</th>
<th>Error</th>
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</thead>
<tbody>
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<td>Case 1</td>
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<tr>
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<td>Case 2</td>
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<td>mode 1</td>
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</table>

4.4 Remarks
The precision is rather good with 8 elements in the length. The solution differs a little that obtained with warping (modeling A).
5 Summary of the results

The analytical solution gives us 3 modes of buckling of which the critical loads are roots of an equation of the 3rd degree.

Y-a it of other critical loads inserted between the 3 found values?

Aster find the good critical loads, but in the middle of much of others… for example for case 3, the 3 sought critical loads correspond to NUME_MODE : 1.10 and 19.

This is true for two modelings.