SSLL118 – Fixed beam subjected to the displacements defined in a local reference mark

Summary:

This case test makes it possible to validate the option `DDL_POUTRE` order `AFFE_CHAR_MECA`, which makes it possible to impose the displacements defined in a local reference mark related to the beam.

Conditions on the degrees of freedom \((dx, dy, dz, drx, dry, drz)\) expressed in the local reference mark are translated into conditions on the degrees of freedom \((DX, DY, DZ, DRX, DRY, DRZ)\) expressed in the total reference mark.

This case test in particular makes it possible to validate the fact, that to impose a null displacement in a local reference mark is correctly translated in the total reference mark.
1 Problems of reference

1.1 Geometry

There are 4 beams length $L$ modelled with POU_D_E.

Pout01, Pout02: they are along the total axis $X$.

Pout03, Pout04: their axis is in the direction $(1.0,1.0,0.0)$.

1.2 Properties of material

Characteristics of material

$E = 2.0E11 Pa$

$\nu = 0.30$

1.3 Geometrical characteristics

All the beams have the same rectangular section:

$HY = 0.2$

$HZ = 0.1$

The length of the beams is of 2m.

1.4 Boundary conditions and loadings

All the beams are embedded in the beginning: nodes $N1, N2, N3, N4$.

A displacement is imposed at the other end: nodes $NA, NB, NC, ND$.

An effort can be imposed on the node medium: nodes $IA, IB, IC, ID$. 
2 Reference solution

2.1 Method of calculating

The reference solution uses the results below:

\[ \delta c \quad \text{and} \quad \delta b \quad \text{represent displacement at the points} \quad C \quad \text{and} \quad B. \]

\[ EI. \delta c = \frac{F_c . L}{3} + \frac{5 . F_b . L^3}{48} \quad [1] \]

\[ EI. \delta b = \frac{5 . F_c . L^3}{47} + \frac{F_b . L^3}{24} \quad [2] \]

Reaction of support \( F_a = -F_c - F_b \quad [3] \)

The boundary conditions which is used during the validation are: a displacement in \( C : \delta c \) and a possible effort in \( B : F_b \). The resolution of the equations \([1],[2],[3]\) give the following expressions:

\[ \delta b = \frac{7 . F_b . L^3}{768 . EI} + \frac{5 . \delta c}{16} \]

\[ F_c = \frac{3 . \delta c . EI}{L^3} - \frac{5 . F_b}{16} \]

\[ F_a = -\frac{3 . \delta c . EI}{L^3} - \frac{11 . F_b}{16} \]

2.2 Sizes and results of reference

The calculations carried out on the 4 beams differ is by boundary conditions imposed on their nodes \( B \) and \( C \), that is to say by a definition different from the local reference mark. The boundary conditions imposed in displacement or effort make it possible to determine the theoretical solutions of the various cases considered.

Characteristics of the beam:

\[ 3 . EI_y = 10E+07 \text{ N.m}^2 \quad 3 . EI_z = 4.0E+07 \text{ N.m}^2 \]

Name of the beam: \( Pout01 \)

Data:

\[
\begin{align*}
\text{VECT}_y & : (0.0, 1.0, 0.0) \\
\delta y \text{ in } C \text{ (in the local reference mark)} & : 2.0E-03 \text{ m} \\
\delta z \text{ in } C \text{ (in the local reference mark)} & : 1.0E-03 \text{ m}
\end{align*}
\]

Theoretical results:

\[ FY \text{ in } A \text{ (in the total reference mark)} : \frac{-3 . \delta y . EI_z}{L^3} \]
Name of the beam: *Pout02*

Data:

\[
\text{VECT}_Y : (-0.0, 0.0, 1.0) \\
\delta y \text{ in } C : 2.0\times10^{-3} \text{ m} \\
\delta z \text{ in } C : 1.0\times10^{-3} \text{ m}
\]

Theoretical results:

\[
FY \text{ in } A : \frac{3.\delta z. EI_z}{L^3} \\
FZ \text{ in } A : \frac{-3.\delta z. EI_y}{L^3}
\]

Name of the beam: *Pout03*

Data:

\[
\text{VECT}_Y : (-1.0, 1.0, 0.0) \\
\delta y \text{ in } C : 2.0\times10^{-3} \text{ m} \\
\delta z \text{ in } C : 1.0\times10^{-3} \text{ m}
\]

Theoretical results:

\[
FX \text{ in } A : -\frac{3\sqrt{2}.\delta y. EI_z}{2L^3} \\
FY \text{ in } A : \frac{3\sqrt{2}.\delta y. EI_z}{2L^3} \\
FZ \text{ in } A : \frac{-3.\delta z. EI_y}{L^3}
\]

Name of the beam: *Pout04*

Data:

\[
\text{VECT}_Y : (-1.0, 1.0, 0.0) \\
\delta y \text{ in } C : 2.0\times10^{-3} \text{ m} \\
\delta z \text{ in } C : 0 \\
FZ \text{ in } B : 1000.0
\]

Theoretical results:

\[
FX \text{ in } A : -\frac{3\sqrt{2}.\delta y. EI_z}{2L^3} \\
FY \text{ in } A : -\frac{3\sqrt{2}.\delta y. EI_z}{2L^3} \\
FZ \text{ in } A : \frac{-11.Fb}{16} \\
DZ \text{ in } B : \frac{7.Fb.L^3}{768EI}
\]
3 Modeling A

3.1 Characteristics of the grid and modeling

Many nodes : 12
Number of SEG2 : 8
Number of group of meshes : 4

The beams are modelled with POU_D_E.

3.2 Sizes tested and results

The sizes tested are the reactions to embedding and the displacement of the node $ID$.

<table>
<thead>
<tr>
<th>Beam</th>
<th>Type of field</th>
<th>Component</th>
<th>Node</th>
<th>Values of Reference</th>
<th>Error Relative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pout01</td>
<td>FORC_NODA</td>
<td>DY</td>
<td>N1</td>
<td>-1000.0</td>
<td>0.0%</td>
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<tr>
<td></td>
<td></td>
<td>DZ</td>
<td>N1</td>
<td>-1250.0</td>
<td>0.0%</td>
</tr>
<tr>
<td>Pout02</td>
<td>FORC_NODA</td>
<td>DY</td>
<td>N2</td>
<td>5000.0</td>
<td>0.0%</td>
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<tr>
<td></td>
<td></td>
<td>DZ</td>
<td>N2</td>
<td>-2500.0</td>
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<tr>
<td>Pout03</td>
<td>FORC_NODA</td>
<td>DX</td>
<td>N3</td>
<td>7071.0678</td>
<td>1.34E-05%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DZ</td>
<td>N3</td>
<td>-7071.0678</td>
<td>1.34E-05%</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>Pout04</td>
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<td>DX</td>
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<td>1.34E-05%</td>
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<tr>
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<td>ID</td>
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4 Modeling B

4.1 Characteristics of the grid and modeling

Many nodes : 16
Number of SEG2 : 12
Number of group of meshes : 6

The beams are modelled with POU_D_E.

4.2 Sizes tested and results

The sizes tested are the reactions to embedding and the displacement of the node ID.

<table>
<thead>
<tr>
<th>Beam</th>
<th>Type of field</th>
<th>Component</th>
<th>Node</th>
<th>Type of Reference</th>
<th>Values of Reference</th>
<th>Tolerance</th>
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</thead>
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<tr>
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<td></td>
<td>DZ</td>
<td>N1</td>
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<td>-1250.0</td>
<td>0.1%</td>
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<tr>
<td>Pout02</td>
<td>FORC_NODA</td>
<td>DY</td>
<td>N2</td>
<td>'ANALYTICAL'</td>
<td>5000.0</td>
<td>0.1%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DZ</td>
<td>N2</td>
<td>'ANALYTICAL'</td>
<td>-2500.0</td>
<td>0.1%</td>
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<tr>
<td>Pout03</td>
<td>FORC_NODA</td>
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<td>N3</td>
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<td>'ANALYTICAL'</td>
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</tr>
<tr>
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<td></td>
<td>DZ</td>
<td>N3</td>
<td>'ANALYTICAL'</td>
<td>1250.0</td>
<td>0.1%</td>
</tr>
<tr>
<td>Pout04</td>
<td>FORC_NODA</td>
<td>DX</td>
<td>N4</td>
<td>'ANALYTICAL'</td>
<td>7071.0678</td>
<td>0.1%</td>
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<tr>
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<td>N4</td>
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<td>0.1%</td>
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<td>0.1%</td>
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<td>ID</td>
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<td>0.1%</td>
</tr>
</tbody>
</table>

4.3 Remarks

This modeling makes it possible to validate DDL_POUTRE order AFFE_CHAR_MECA with the keyword MAILLE/GROUP_MA, beams Pout01, Pout02, Pout03, Pout04 were lengthened of 1m.

The place of application of boundary conditions and of the loadings did not change.
5 Summary of the results

This case test makes it possible to validate the use of DDL_POUTRE order AFFE_CHAR_MECA. One validates in particular the fact that to impose a null displacement in a local reference mark is correctly translated in the total reference mark.