SSLP106 – Rectangular solid mass in pure inflection (test of elements QUAD4 under integrated)

Summary:
One tests the finite elements under integrated into a point of Gauss stabilized by the method assumed strain on a calculation of pure inflection in plane deformation.

Warning: The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
Copyright 2020 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)
1 Problem of reference

1.1 Geometry

The geometry is a square on side $L = 100\,\text{mm}$.

![Geometry diagram]

1.2 Properties of material

The material is elastic incompressible and has as properties:

\[
E = 100\,\text{MPa} \\
\nu = 0.4999
\]

1.3 Boundary conditions and loadings

Taking into account the antisymmetric nature of the problem, one models only half of the solid mass with the boundary conditions following:

On $OE$:

\[
DX(OE) = 0
\]

On $OD$:

\[
DX(O) = DY(O) = 0 \\
DX(D) = 0
\]

\[
fsx = \frac{8y}{L} \cdot \sigma_d
\]

\[
fsy = -\left(1 - \frac{4y^2}{L^2}\right) \cdot \sigma_d
\]

On $BC$:

\[
fsy = +\left(1 - \frac{4y^2}{L^2}\right) \cdot \sigma_d
\]

With $\sigma_d$ a given constraint, which one will take equal to 1 in the test. $\sigma_d = 1\,\text{MPa}$
2 Reference solution

2.1 Method of calculating

The reference solution comes from an analytical solution from [Bib1]:

\[ u_x(x, y) = \frac{4(1-\nu^2)}{EL^2} \left[ y \left( x^2 - 2Lx \right) + \frac{2+\nu(1-\nu)}{3} \cdot y \cdot \left( \frac{L^2}{4} - y^2 \right) \right] \cdot \sigma_d \]  
\[ (1) \]

And following \( y \):

\[ u_y(x, y) = \frac{4(1-\nu^2)}{EL^2} \left[ Lx^2 - \frac{x^3}{3} - \nu \cdot (1-\nu) \cdot y^2 \cdot (x-L) + \frac{4+5\nu(1-\nu)}{12} \cdot xL^2 \right] \cdot \sigma_d \]  
\[ (2) \]

2.2 Sizes and results of reference

While applying 1, one finds displacement following \( x \) at the point \( C \):

\[ u_x(L, L/2) = -1.5 \text{ mm} \]

And while applying 2, one finds displacement following \( y \) at the point \( C \):

\[ u_y(L, L/2) = 4.25 \text{ mm} \]

2.3 Bibliographical references

3 Modeling A

3.1 Characteristics of modeling

Modeling in plane deformations on the grid 2D according to:

<table>
<thead>
<tr>
<th>Value tested</th>
<th>Reference</th>
<th>Type</th>
<th>Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement DX in C</td>
<td>-1.25</td>
<td>Analytical</td>
<td>1.7%</td>
</tr>
<tr>
<td>Displacement DY in C</td>
<td>4.25</td>
<td>Analytical</td>
<td>0.3%</td>
</tr>
</tbody>
</table>

3.2 Characteristics of the grid

Many nodes: 25
Numbers and types of meshes: 16 SEG2, 16 QUAD4 with the elements D_PLAN_SI.

3.3 Sizes tested and results

3.4 Remarks

The request is said to dominate inflection. Through this calculation, one shows the difficulty for QUAD4 even under-integrated to represent the modes of deformation in inflection in plane deformation and for a coefficient $\nu$ near 0.5. This results in an excessive rigidity of the element due under the terms of shearing of the operator discretized gradient: it is about a digital blocking.
Titre : SSLP106 - Massif rectangulaire en flexion pure (te[...]
Responsable : FAYOLLE Sébastien

Date : 26/02/2014
Clé : V3.02.106
Révision : 03aa9a0fcbb2
4 Modeling B

4.1 Characteristics of modeling

One takes again the preceding grid which one passes in quadratic elements with an aim of using modeling D_PLAN_INCO_UPG (elements adapted to the incompressible problems).

4.2 Characteristics of the grid

Many nodes: 65
Numbers and types of meshes: 16 SEG3, 16 QUAD8 with the elements D_PLAN_INCO_UPG.

4.3 Sizes tested and results

<table>
<thead>
<tr>
<th>Value tested</th>
<th>Reference</th>
<th>Type</th>
<th>Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement $DX$ in $C$</td>
<td>-1.25</td>
<td>Analytical</td>
<td>&lt;0.01%</td>
</tr>
<tr>
<td>Displacement $DY$ in $C$</td>
<td>4.25</td>
<td>Analytical</td>
<td>&lt;0.01%</td>
</tr>
</tbody>
</table>

4.4 Remarks

These elements adapted to the incompressible problems give a result identical to the analytical solution.
5 Summary of the results

The poor quality of the result of under-integrated elements QUAD4 is explained by the digital phenomenon of blocking which makes the element very rigid. Moreover, its convergence towards the analytical solution is very slow. This phenomenon appears of course also for the completely integrated element. Calculation using incompressible quadratic elements makes it possible to obtain an exact solution.