SSLS133 - Flexbeam with variable thickness

Summary:

This test represents a quasi-static calculation of a flexbeam with variable thickness. It is embedded at an end, and is subjected to a vertical force at the other end. This test makes it possible to test the elements of voluminal hull SHB8 and SHB20 to manage the variations thicknesses. Four modelings are tested:

Finite elements SHB8 for a linear variation thickness of the plate (modeling A).
Finite elements SHB20 for a linear variation thickness of the plate (modeling B).
Finite elements SHB8 for a quadratic variation thickness of the plate (modeling C).
Finite elements SHB20 for a quadratic variation thickness of the plate (modeling D).

Displacements obtained are compared with the elastic analytical solution of a beam in inflection. This test makes it possible to show the capacities and the limits of the elements SHB8 and SHB20 to manage the variations thicknesses.
1 Problem of reference

1.1 Geometry

1.1.1 Plate with thickness varying linearly

Length $L = 100\, m$, width $l = 100\, m$.

The thickness $h$ vary linearly:

$$h(x) = ax + b$$

We pose $h(x=0) = h_0 = 10\, m$ and $h(x=L) = h_2 = 5\, m$ what gives us:

$$a = (h_2 - h_0)/L$$ and $b = h_0$.

1.1.2 Plate with thickness varying quadratically

The thickness $h$ vary in a quadratic way:

$$h(x) = ax^2 + bx + c$$

We pose $h(x=0) = h_0 = 10\, m$, $h(x=L) = h_2 = 5\, m$ and $h(x=L/2) = h_{12} = 6.25\, m$ what gives us:

$$a = (2(h_0 + h_2) - 4h_{12})/L^2, b = (4h_{12} - h_2 - 3h_0)/L$$ and $c = h_0$.

1.2 Material properties

Young modulus: $E = 2.10^{11}\, Pa$

Poisson's ratio: $\nu = 0.0$
1.3 Boundary conditions and loadings

Boundary conditions:
Embedded on the side \( OC \) : \( u = v = w = 0 \), \( \theta_x = \theta_y = \theta_z = 0 \)

Loading:
At the end \( AB \), one load uniformly distributed of resultant:
Force parallel with the axis \( Z \); \( F_z = 1 \, N \)
2 Reference solution

2.1 Method of calculating used for the reference solution

The results of reference are got by the theory of the elastic beams.

In the case of a linear variation thickness, vertical displacement at the end $AB$ is given by [1]:

$$w(x) = -\frac{FL^2}{2EI_y c^3} \left[ \frac{2LCx + c^2 x^2 - c^3 x^2 + 2L|L + cx| \ln \left( \frac{L}{L + cx} \right)}{|L + cx|} \right]$$

With

$$c = \left( \frac{I_{y_2}}{I_{y_1}} \right)^{\frac{1}{3}} - 1 \quad \text{and} \quad I_{y_1} = \frac{bh_1^3}{12}$$

In the case of a quadratic variation thickness, it is possible to find a formula exact of displacement. However its general expression is sufficiently complex not to be able to be written here. We formulated the approximate function of vertical displacement according to $x$ of our case:

$$w(x) = 3.10^{-8} \frac{2x - 200}{x^2 - 200x + 20000} + 6.10^{-10} \arctan(0.01x - 1) - 3.10^{-12} x + 7.71238 \cdot 10^{-10} \text{ m}$$

2.2 Sizes and results of reference

Displacement of the points $A$ and $B$ according to $Z$.

2.3 Bibliographical references

[1] [V3.01.400] SSLL400 – non-prismatic Beam, subjected to efforts specific or distributed.
3 Modeling A

3.1 Characteristics of modeling

Element SHB8 and thickness varying linearly

Cutting: a regular grid is considered in this modeling.

Regular grid:
100 meshes SHB8: 10 according to the width, 10 according to the length, 1 according to the thickness

Boundary conditions:
All nodes inside the side $P_1P_2P_6P_5$: following blocked displacement $X$
All nodes on the edge $P_1P_5$: following blocked displacement $Y$
All nodes on the edge $P_2P_1$: following blocked displacement $Z$

Loading:
Force linearly distributed on the edge $P_8P_7$: $F = 1$

Name of the nodes:
Not $P_1$ N022 Not $P_5$ N020
Not $P_2$ N002 Not $P_6$ N001
Not $P_3$ N102 Not $P_7$ N100
Not $P_4$ N172 Not $P_8$ N171

3.2 Characteristics of the grid

Many nodes: 242
Many meshes and types: 100 SHB8

3.3 Sizes tested and results

Regular grid:
One also tests analytical fields of equivalent constraints of Von Mises.

4 Modeling B

4.1 Characteristics of modeling

Element SHB20 and thickness varying linearly

Cutting: a regular grid is considered in this modeling.

Regular grid:

100 meshes SHB20: 10 according to the width, 10 according to the length, 1 according to the thickness

Boundary conditions:

All nodes inside the side \( P_1 P_2 P_3 P_5 \) : following blocked displacement \( X \)

All nodes on the edge \( P_1 P_5 \) : following blocked displacement \( Y \)

All nodes on the edge \( P_2 P_1 \) : following blocked displacement \( Z \)

Loading:

Force linearly distributed on the edge \( P_8 P_7 \) : \( F = 1 \)

Name of the nodes:
4.2 Characteristics of the grid

Many nodes: 803
Many meshes and types: 100 \( \text{SHB20} \)

4.3 Sizes tested and results

Regular grid:

<table>
<thead>
<tr>
<th>Not</th>
<th>Size in unit</th>
<th>Reference</th>
<th>Aster</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_7 )</td>
<td>displacement ( W (m) )</td>
<td>( 3.2710 \times 10^{-10} )</td>
<td>( 3.2866 \times 10^{-10} )</td>
<td>+0.476</td>
</tr>
<tr>
<td>( P_8 )</td>
<td>displacement ( W (m) )</td>
<td>( 3.2710 \times 10^{-10} )</td>
<td>( 3.2866 \times 10^{-10} )</td>
<td>+0.476</td>
</tr>
</tbody>
</table>

5 Modeling C

5.1 Characteristics of modeling

Element \( \text{SHB8} \) and thickness varying quadratically

![Grid of modeling C](image)

Figure 5.1-1: Grid of modeling C

The characteristics are the same ones as for modeling \( A \)

Name of the nodes:

- Not \( P_1 \): N005, Not \( P_5 \): N003
- Not \( P_2 \): N006, Not \( P_6 \): N004
- Not \( P_3 \): N008, Not \( P_7 \): N002
- Not \( P_4 \): N007, Not \( P_8 \): N001

5.2 Characteristics of the grid

The grid is the same one as modeling \( A \) except for the thickness which varies here in a quadratic way.
5.3 Sizes tested and results

Regular grid:

<table>
<thead>
<tr>
<th>Not</th>
<th>Size in unit</th>
<th>Reference</th>
<th>Aster</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_r$</td>
<td>displacement $W$ ($m$)</td>
<td>$4.7124 \times 10^{-10}$</td>
<td>$5.1212 \times 10^{-10}$</td>
<td>9%</td>
</tr>
<tr>
<td>$P_h$</td>
<td>displacement $W$ ($m$)</td>
<td>$4.7124 \times 10^{-10}$</td>
<td>$5.1212 \times 10^{-10}$</td>
<td>9%</td>
</tr>
</tbody>
</table>
6 Modeling D

6.1 Characteristics of modeling

Element SHB20 and thickness varying quadratically

Figure 6.1-1: Grid of modeling D

The characteristics are the same ones as for modeling B

Name of the nodes:

Not P₁ N005 Not P₅ N003
Not P₂ N006 Not P₆ N004
Not P₃ N008 Not P₇ N002
Not P₄ N007 Not P₈ N001

6.2 Characteristics of the grid

The grid is the same one as modeling B except for the thickness which varies here in a quadratic way.

6.3 Sizes tested and results

Regular grid:

<table>
<thead>
<tr>
<th>Not</th>
<th>Size in unit</th>
<th>Reference</th>
<th>Aster</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₇</td>
<td>displacement W ( m )</td>
<td>4.7124 (10^{-10})</td>
<td>4.6754 (10^{-10})</td>
<td>-0.784</td>
</tr>
<tr>
<td>P₈</td>
<td>displacement W ( m )</td>
<td>4.7124 (10^{-10})</td>
<td>4.6754 (10^{-10})</td>
<td>-0.784</td>
</tr>
</tbody>
</table>
7 Summary of the results

In the case of a variation linear thickness of the plate, good solutions are obtained using some finite element method (SHB8 or SHB20).

When the geometrical variation is of a quadratic nature, elements SHB20 provide more precise results (error < 1\% ) than elements SHB8 (error < 9\% ).