SSLS143 - Pin addition to cantilever with offset heart

Summary:

The objective of this test is to validate the calculation of the options EPSI_ELGA and DEGE_ELNO for the multifibre beams of Euler-Bernoulli POU_D_EM, including if the reference axis is not confused with the locus of elastic centres.

The case test also validates the calculation of the elastic matrix (RIGI_MECA) when the axis of reference is not confused with the locus of elastic centres (offsetting).
1 Problem of reference

1.1 Geometry

A beam comforts length \( L = 1 \text{ m} \) (see Figure 1).

![Figure 1: Beam comforts](image)

1.2 Properties of material

The cross-section is rectangular \( b \times h = 0.4 \times 1 \text{ m}^2 \), and homogeneous with a material of modulus Young \( E = 3 \times 10^{10} \text{ Pa} \).

1.3 Boundary conditions and loadings

The beam is embedded at an end and is charged by a force \( F = 10^6 \text{ N} \) at its other end (see Figure 1)
2 Reference solution

2.1 Analytical expressions

On the basis of the embedded end, the expression of the bending moment is: \( M(x) = F(x - L) \)

The arrow at the end charged with the beam is \( f = \frac{FL^3}{EI G_0} \) where \( I G_0 \) is the quadratic moment calculated with the barycentre of the section \( G_0 : I G_0 = \int_S (y - y_{G_0})^2 dS \).

Curve in a point located at a distance \( x \) embedding is \( \chi_s(x) = -\frac{M(x)}{EI G_0} \). Because of the offsetting of the reference axis, the lengthening of the beam (on the level of this axis) is worth:\n\[ e_s(x) = -\frac{A G}{S} \chi_s(x) \] where \( S \) is the surface of the section and \( A G \) static moment of the section compared to an axis passing by \( G : A G = \int_S z dS \).

Deformation of a point of coordinates \((x, y, z)\) is: \( \epsilon = e_s(x) - \chi_s(x) z \), and the constraint at the same point is worth: \( \sigma = E \epsilon \).

2.2 Calculation of the characteristics of the cross-section

In order to eliminate uncertainty from the approximate digital calculation of the characteristics geometrical of cross-section (low number of fibres), the values used in the reference solution are calculated as in digital calculation:

\[ S = \sum_{fibres} S_i \quad A_G = \sum_{fibres} z_i S_i \quad I_G = \sum_{fibres} z_i^2 S_i \quad I_{G_0} = \sum_{fibres} (z_i - z_{G_0})^2 S_i \]

where \( z_i \) is the ordinate of the center of fibre \( i \) and \( S_i \) the surface of this fibre.

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3 Modeling A

3.1 Characteristics of modeling

A modeling is used $POU\_D\_EM$.

the beam is modelled with 1 finite element and the section is discretized with 8 fibres. The reference axis east chooses voluntarily different from the barycentre, excentré of $h/2$ to the bottom, with the keyword $COOR\_AXE\_POUTRE$ of $DEFI\_GEOM\_FIBRE$.

The section “is cut out” in 8 fibres (Figure 2). The coordinates of the centers of fibres and their surfaces are given in table 1.

![Figure 2: Section division in 8 fibres](image)

<table>
<thead>
<tr>
<th>Fibres</th>
<th>$y_i$</th>
<th>$z_i$</th>
<th>$S_i$</th>
<th>Fibres</th>
<th>$y_i$</th>
<th>$z_i$</th>
<th>$S_i$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.875</td>
<td>0.05</td>
<td>5</td>
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<td>0.875</td>
<td>0.05</td>
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<tr>
<td>2</td>
<td>0.1</td>
<td>0.625</td>
<td>0.05</td>
<td>6</td>
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<td>0.625</td>
<td>0.05</td>
</tr>
<tr>
<td>3</td>
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<td>0.375</td>
<td>0.05</td>
<td>7</td>
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<td>0.375</td>
<td>0.05</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>0.125</td>
<td>0.05</td>
<td>8</td>
<td>-0.1</td>
<td>0.125</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 1: Characteristics of fibres

3.2 Characteristics of the grid

The grid contains 1 elements of the type $SEG2$.

3.3 Sizes tested and results

The digital calculation of the sizes of the cross-section gives:

$S=0.4 \, m^2 \quad A_G=0.2 \, m^3 \quad I_G=0.13125 \, m^4$ and $I_G^0=0.03125 \, m^4$

The arrow at the end charged with the beam is: $f=0.00035555 \, m$

On the level of embedding $x=0$ one has the following values:

$M=-10^{-6} \, Nm \quad \chi_z=0.00106666 \, m^{-1} \quad \epsilon_z=-0.00053333 \, m^{-1}$

On the level of the first point of Gauss $x=(1-\frac{1}{\sqrt{3}})/2=0.21132486540518503 \, m$ one has the following values:

$M=-788675.13 \, Nm \quad \chi_z=0.00084125 \, m^{-1} \quad \epsilon_z=-0.0004206 \, m^{-1}$

for the fibre $n^1$ ($z=0.875 \, m$): $\epsilon=0.00031547 \, Pa$ and $\sigma=9464101.6 \, Pa$ and for the fibre $n^4$ with $z=0.125 \, m$: $\epsilon=-0.00031547 \, Pa$ and $\sigma=-9464101.6 \, Pa$
Arrow at the end of the beam (**DEPL**):

<table>
<thead>
<tr>
<th>Not</th>
<th>Component</th>
<th>Value of reference</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUPPORT</td>
<td>D2</td>
<td>−3.55555555555555E-4</td>
<td>1.E-6</td>
</tr>
</tbody>
</table>

Deformations generalized with embedding (**DEGE_ELNO**):

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Node</th>
<th>Component</th>
<th>Value of reference</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
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<td>1.E-6</td>
</tr>
<tr>
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<td>N1</td>
<td>EPXX</td>
<td>−5.3333333333333333E-3</td>
<td>1.E-6</td>
</tr>
</tbody>
</table>

Strains and stresses in fibres (**EPSI_ELGA** and **SIEF_ELGA**):

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Not</th>
<th>Under-point</th>
<th>Component</th>
<th>Value of reference</th>
<th>Tolerance</th>
</tr>
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<tr>
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<td>4</td>
<td>SIXX</td>
<td>−9.46410161513778E6</td>
<td>1.E-6</td>
</tr>
</tbody>
</table>
4 Summary of the results

Subject coarsely using the approximate characteristics of the cross-section with a grid for the calculation of the analytical values of reference, the arrow, the constraints and the deformations numerically calculated are identical to these values of reference.