SSLV120 - Stretching of a parallelepiped orthotropic under its own weight

Summary:

This test of mechanics of the structures allows the evaluation of displacements and the constraints of a parallelepiped becoming deformed under its own weight. The material is elastic linear orthotropic. Modeling is three-dimensional. The model is similar to test VPCS SSLV07 (but in this case the material is isotropic) and with test SSLV121 (in this case the material is isotropic transverse).

Variations of the results got by Aster range between 0.00 and 0.5% of the analytically calculated reference.
1 Problem of reference

1.1 Geometry

Coordinates of the points (in meters):

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>0.0</td>
<td>0.0</td>
<td>0.5</td>
<td>0.5</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>y</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.5</td>
<td>0.0</td>
</tr>
<tr>
<td>z</td>
<td>3.0</td>
<td>0.0</td>
<td>0.5</td>
<td>3.0</td>
<td>1.5</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Hauteur : L = 3 m  Largeur : a = 1 m  Epaisseur : b = 1 m

1.2 Material properties

Young moduli in the directions $x$, $y$ and $z$:

$$E_L = 5 \times 10^{11} \text{ Pa}, \quad E_T = 5 \times 10^{11} \text{ Pa}, \quad E_N = 2 \times 10^{11} \text{ Pa}.$$

Poisson's ratios in the plans $xy$, $xz$ and $yz$:

$$\nu_{LT} = 0.1, \quad \nu_{LN} = 0.3, \quad \nu_{TN} = 0.1.$$

Moduli of rigidity in the plans $xy$, $xz$ and $yz$:

$$G_{LT} = 7.69231 \times 10^{10} \text{ Pa}, \quad G_{LN} = 7.69231 \times 10^{10} \text{ Pa},$$
$$G_{TN} = 7.69231 \times 10^{10} \text{ Pa}.$$

Density: $\rho = 7800 \text{ kg/m}^3$.

1.3 Boundary conditions and loadings

Not $A : (u=v=w=0, \quad \theta_x = \theta_y = \theta_z = 0)$

Actual weight following the axis $z : \rho g z$

Uniform constraint with traction for the higher face:
2 Reference solution

2.1 Method of calculating used for the reference solution

The reference solution is resulting from that given in card SSLV07/89 of guide VPCS (while considering in more one orthotropic elastic matrix). The analytical expression of the solution is the following one:

Displacements:

\[
\begin{align*}
    u &= -\frac{v_{NL} \rho g x z}{E_N} \\
    v &= -\frac{v_{NT} \rho g y z}{E_N} \\
    w &= \frac{\rho g z^2}{2E_N} + \frac{\rho g}{2E_N} (v_{NL} x^2 + v_{NT} y^2) - \frac{\rho g L^2}{2E_N}
\end{align*}
\]

Constraints:

\[\sigma_{zz} = \rho g z = \sigma_{yy} = \sigma_{xx} = \sigma_{xy} = \sigma_{xz} = \sigma_{yz} = 0\]

2.2 Results of reference

Displacement of the points \(B\), \(C\), \(D\), \(E\) and \(X\).

Constraints \(\sigma_{zz}\) in \(A\) and \(E\)

2.3 Uncertainty on the solution

Exact analytical results.

2.4 Bibliographical references

3 Modeling A

3.1 Characteristics of modeling

3D meshes hexa20

Cutting:

3 elements in height
2 elements in width and thickness

Limiting conditions:
on the axis \( AB \)
in \( A \) and \( D \)

DDL_IMPO: (GROUP_NO: ABsansA \( \text{DX}=0., \text{DY}=0. \) )

(NODE: With \( \text{DX}=0., \text{DY}=0., \text{DZ}=0. \))

(NODE: \( D \) \( \text{DY}=0. \))

Names of the nodes:

\( A = N59 \)
\( B = N53 \)
\( C = N12 \)
\( D = N18 \)
\( E = N56 \)
\( X = N70 \)

3.2 Characteristics of the grid

Many nodes: 111
Many meshes and types: 12 HEXA20

3.3 Values tested

<table>
<thead>
<tr>
<th>Identification</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_B )</td>
<td>0.</td>
</tr>
<tr>
<td>( V_B )</td>
<td>0.</td>
</tr>
<tr>
<td>( W_B )</td>
<td>-1.721655E-6</td>
</tr>
<tr>
<td>( U_C )</td>
<td>0.</td>
</tr>
<tr>
<td>( V_C )</td>
<td>0.</td>
</tr>
<tr>
<td>( W_C )</td>
<td>-1.715916E-6</td>
</tr>
<tr>
<td>( U_D )</td>
<td>-6.88662E-08</td>
</tr>
<tr>
<td>( V_D )</td>
<td>0.</td>
</tr>
</tbody>
</table>
3.4 Remarks

Modeling in HEXA20 is completely acceptable for this coarse grid.
Summary of the results

The results concerning displacements and the constraints are very close to the analytical solution with adopted modeling (< 0.2% for displacements, < 0.5% for the constraints).

The elastic coefficients in the 3 directions of orthotropism were selected so as to obtain the same values of displacements at the points $B$, $C$, $D$ and $E$ that those calculated for an isotropic material (test SSLV007) or isotropic transverse (test SSLV121). Numerically, these values are very close to those of these tests at the points considered (about $10^{-6}$) the difference resulting from the method of construction of the matrices from rigidity in the various cases. At the point $X$, these values differ but correspond well to the reference solution.