SSLV155 – Crack lens in traction

Summary:

The purpose of this test is to validate the calculation of the stress intensity factors (SIFs) along the bottom of a nonplane crack 3D, within the framework of linear elasticity.

This test formats concerned a cube with a crack of lens, subjected to a hydrostatic tension.

This test contains 3 modelings:

- Modeling a: the crack is with a grid in 2D-axi (FEM);
- Modeling b: the crack is not with a grid, it is represented by level sets (X-FEM) in 2D-axi;
- Modeling C: the crack is not with a grid, it is represented by level sets (X-FEM) in 3D;
- Modeling D: the crack is not with a grid, it is represented by level sets (X-FEM) in 2D-axi and one uses a structured grid;
- Modeling E: the crack is not with a grid, it is represented by level sets (X-FEM) in 2D-axi with quadratic elements;
- Modeling F: the crack is not with a grid, it is represented by level sets (X-FEM) in 2D-axi with quadratic elements and a structured grid is used;

There is no modeling FEM in 3D because the creation of the grid is extremely difficult.

For each modeling, SIFs are evaluated by the orders \texttt{POST\_K1\_K2\_K3} and \texttt{CALC\_G}.

The digital values are compared with the analytical values.
1 Problem of reference

1.1 Geometry

One considers a cube of with dimensions $2L$ and a crack in the shape of lens (Lens shaped ace) of ray $R$ such as $\frac{L}{R} = 5$ and of angle in the center $\alpha = \frac{\pi}{4}$ (see Figure 1).

The equation characteristic of the form of the surface of the crack is:

$$x^2 + y^2 + (z - R)^2 = R^2 \text{ with } 0 \leq z \leq (1 - \cos \alpha) R.$$ 

One poses $a = R \sin \alpha$.

The equation characteristic of the bottom of crack is:

$$x^2 + y^2 = a^2 \text{ with } z = (1 - \cos \alpha) R.$$

![Figure 1: geometry of the fissured cube](image)

1.2 Material properties

The material is elastic isotropic properties:

- $E = 210000 \text{ MPa}$
- $\nu = 0.22$

1.3 Boundary conditions and loadings

The cube is subjected to a hydrostatic tension $\sigma$. 

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2 Reference solution

2.1 Method of calculating used for the reference solution

The reference solution (1) for a crack in the shape of lens of ray $R$ in an infinite medium, subjected to a uniform pressure $\sigma$ rather far away from the crack, shows that the stress intensity factors are constant along the bottom of crack and are worth:

$$K_I = 0.877 \left( \frac{2}{\pi} \right) \sigma \sqrt{\pi a}$$
$$K_{II} = 0.235 \left( \frac{2}{\pi} \right) \sigma \sqrt{\pi a}$$
$$K_{III} = 0$$

with $a = R \sin \alpha$.

2.2 Results of reference

For the loading considered $\sigma = 1 \text{ MPa}$ and following geometrical characteristics:
$L = 10 \text{ m}, R = 2 \text{ m}, a = \sqrt{2}$, one finds:
$$K_I = 1.177 \text{ MPa} \sqrt{m}$$
$$K_{II} = 0.3153 \text{ MPa} \sqrt{m}$$
$$K_{III} = 0$$

2.3 Bibliographical references

3 Modeling a: Modeling FEM 2D-axi

3.1 Characteristics of modeling

In this modeling, the crack is with a grid (case FEM) and the structure is modelled in 2D-axisymmetry.

3.2 Characteristics of the grid

Number of nodes: 5211  
Number of meshes and type: 2550 TRIA6

The length characteristic of an element close to the bottom to crack is of \( H = 0.025 \text{m} \).

The nodes mediums of the edges of the elements touching the bottom of crack are moved with the quarter of these edges (elements of Barsoum).

3.3 Boundary conditions and loadings

- A surface effort of traction is applied to the faces higher, lower and that of right-hand side;
- Displacements according to \( \Omega \alpha \) nodes of the axis of rotation are blocked, as that is advised for axisymmetric modelings;
- Rigid mode of displacement following the axis \( \Omega \gamma \) is blocked via the blocking of a node following this axis.

3.4 Sizes tested and results

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One tests the values of $K_I$ and $K_{II}$ in bottom of crack obtained with the operators \texttt{CALC\_G} and \texttt{POST\_K1\_K2\_K3}. These values are compared with the analytical solution.

Crowns of integration of the field theta for the order \texttt{CALC\_G} are:

$$R_{\text{inf}} = 2h \quad \text{and} \quad R_{\text{sup}} = 5h.$$  

The parameter \texttt{ABS\_CURV\_MAXI} of the operator \texttt{POST\_K1\_K2\_K3} by default is selected.

### 3.4.1 Values resulting from \texttt{CALC\_G}

The values are in $Pa \cdot \sqrt{m}$.

<table>
<thead>
<tr>
<th>Identification</th>
<th>Type of reference</th>
<th>Value of reference</th>
<th>% Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_I$</td>
<td>'ANALYTICAL'</td>
<td>$1.177 \times 10^6$</td>
<td>2%</td>
</tr>
<tr>
<td>$K_{II}$</td>
<td>'ANALYTICAL'</td>
<td>$0.3153 \times 10^6$</td>
<td>2%</td>
</tr>
</tbody>
</table>

### 3.4.2 Values resulting from \texttt{POST\_K1\_K2\_K3}

The values are in $Pa \cdot \sqrt{m}$.

<table>
<thead>
<tr>
<th>Identification</th>
<th>Type of reference</th>
<th>Value of reference</th>
<th>% Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_I$</td>
<td>'ANALYTICAL'</td>
<td>$1.177 \times 10^6$</td>
<td>2%</td>
</tr>
<tr>
<td>$K_{II}$</td>
<td>'ANALYTICAL'</td>
<td>$0.3153 \times 10^6$</td>
<td>12%</td>
</tr>
</tbody>
</table>
4 Modeling b: Modeling X-FEM 2D-axi

4.1 Characteristics of modeling

In this modeling, the crack is not with a grid (case X-FEM) and the structure is modelled in 2D-axisymetry.

The crack is represented by level sets:

\[ \text{lsn} = \sqrt{x^2 + (y - R)^2} - R \]
\[ \text{lst} = \sqrt{x^2 + (y - y_h)^2} - R' \]

with \( y_h = R - \frac{R}{\cos(\alpha)} \) and \( R' = R \tan(\alpha) \)

![Figure 4.1-1: level sets](image)

4.2 Characteristics of the grid

The initial healthy grid is relatively coarse: 252 nodes and 442 TRIA3. The size of the meshes is \( h_0 = 1 \text{ m} \). One uses a procedure of successive refinement to lead to a size targets corresponding to half of the size of the meshes of modeling A, that is to say \( h_c = 0.0125 \text{ m} \). Indeed, modeling A uses quadratic elements, one thus needs finer linear elements to obtain an equivalent precision. For that, one calls Lobster in an iterative way. After refinement, the size of the meshes close to the bottom of crack is worth \( h = 0.0078125 \text{ m} \). One refines all the meshes in a disc of ray \( 5h \) around the bottom of crack.

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Figure 4.2-1: initial healthy grid

Figure 4.2-2: refined healthy grid
4.3 Boundary conditions and loadings

- A surface effort of traction is applied to the faces higher, lower and that of right-hand side;
- Displacements according to $Ox$ nodes of the axis of rotation are blocked, as that is advised for axisymmetric modelings;
- Rigid mode of displacement following the axis $Oy$ is blocked via the blocking of a node following this axis.

4.4 Sizes tested and results

The choice of the digital parameters for the postprocessing of SIFs is identical to that done for modeling A: $R_{inf} = 2h$ and $R_{sup} = 5h$, but $h$ less than half is worth here of $h$ modeling A.

4.4.1 Values resulting from CALC_G

The values are in $P_a \cdot \sqrt{m}$.

<table>
<thead>
<tr>
<th>Identification</th>
<th>Type of reference</th>
<th>Value of reference</th>
<th>% Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_i$</td>
<td>‘ANALYTICAL’</td>
<td>$1.177 \times 10^6$</td>
<td>2,0%</td>
</tr>
<tr>
<td>$K_{II}$</td>
<td>‘ANALYTICAL’</td>
<td>$0.3153 \times 10^6$</td>
<td>7,0%</td>
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4.4.2 Values resulting from \texttt{POST_K1_K2_K3}

The values are in $Pa \cdot \sqrt{m}$.

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<thead>
<tr>
<th>Identification</th>
<th>Type of reference</th>
<th>Value of reference</th>
<th>% Tolerance</th>
</tr>
</thead>
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<tr>
<td>$K_1$</td>
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<td>6.0%</td>
</tr>
<tr>
<td>$K_II$</td>
<td>‘ANALYTICAL’</td>
<td>$0.3153 \times 10^6$</td>
<td>8.0%</td>
</tr>
</tbody>
</table>
5 Modeling C: Modeling X-FEM 3D

5.1 Characteristics of modeling

In this modeling, the crack is not with a grid (case X-FEM) and the structure is modelled in 3D. Only a quarter of the structure is modelled, for reasons of symmetry.

The crack is represented by level sets:
\[
\text{lsn} = \sqrt{x^2 + (y-R)^2 + z^2 - R}
\]
\[
\text{lst} = \sqrt{x^2 + (y-y_h)^2 + z^2 - R'}
\]
with \(y_h = R - \frac{R}{\cos(\alpha)}\) and \(R' = R \tan(\alpha)\)

Note:
In 3D, there exists an arbitrary choice of orientation of the local base in bottom of crack. According to the orientation of this base, the sign of \(K_{II}\) and \(K_{III}\) will be different (because related to the base). But that does not affect the physical result (possible angle of junction of the crack expressed in the total reference mark). Here, to find \(K_{II} > 0\) as in the reference solution, it is necessary to define the level set normal like the opposite one of the formula above. In this modeling, one will retain finally \(\text{lsn} = -\sqrt{x^2 + (y-R)^2 + z^2 + R}\). That remains a convention and does not change the physical result!

In 2D, the sign of the level set normal does not influence the sign of \(K_{II}\) because there is no choice to define the local base.

5.2 Characteristics of the grid

The initial healthy grid is relatively coarse: 2508 nodes and 11945 \text{TETRA4} . The size of the meshes is \(h_0 = 1\ m\) . One uses a procedure of successive refinement to lead to a size targets virtually identical to that of modeling A, that is to say \(h_c = 0,025\ m\) . For that, one calls Lobster in an iterative way. After refinement, the size of the meshes close to the bottom of crack is \(h = 0,015625\ m\) . One refines all the meshes in a disc of ray \(5\ h\) around the bottom of crack.

Many nodes: 18166
Number of meshes and type: 103079 \text{TETRA4}
The length characteristic of an element close to the bottom to crack is of 0.0156 Mr.
Figure 5.2-1: initial grid

Figure 5.2-2: refined grid
5.3 Boundary conditions and loadings

- A surface effort of traction is applied to the faces higher, lower and external;
- The conditions of symmetry on the side faces are applied;
- Rigid mode of displacement following the axis \(O_y\) is blocked via the blocking of a node following this axis.

5.4 Sizes tested and results

The choice of the digital parameters for the postprocessing of SIFs is identical to that done for modeling A: \(R_{\text{inf}} = 2h\) and \(R_{\text{sup}} = 5h\).

5.4.1 Values resulting from \texttt{CALC\_G}

The values are in \(P_a \sqrt{m}\).

<table>
<thead>
<tr>
<th>Identification</th>
<th>Type of reference</th>
<th>Value of reference</th>
<th>Tolerance</th>
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</thead>
<tbody>
<tr>
<td>max ((K_I))</td>
<td>‘ANALYTICAL’</td>
<td>(1.177 \times 10^6)</td>
<td>5%</td>
</tr>
<tr>
<td>min ((K_I))</td>
<td>‘ANALYTICAL’</td>
<td>(1.177 \times 10^6)</td>
<td>2%</td>
</tr>
<tr>
<td>max ((K_{II}))</td>
<td>‘ANALYTICAL’</td>
<td>(0.3153 \times 10^6)</td>
<td>15%</td>
</tr>
<tr>
<td>min ((K_{II}))</td>
<td>‘ANALYTICAL’</td>
<td>(0.3153 \times 10^6)</td>
<td>5%</td>
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</table>
5.4.2 Values resulting from \texttt{POST_K1\_K2\_K3}

The values are in $P_a \sqrt{m}$.

<table>
<thead>
<tr>
<th>Identification</th>
<th>Type of reference</th>
<th>Value of reference</th>
<th>Tolerance</th>
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<tbody>
<tr>
<td>$\max(K_I)$</td>
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<td>$1.177 \times 10^6$</td>
<td>2%</td>
</tr>
<tr>
<td>$\min(K_I)$</td>
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<td>14%</td>
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<tr>
<td>$\min(K_{II})$</td>
<td>'ANALYTICAL'</td>
<td>$0.3153 \times 10^6$</td>
<td>2%</td>
</tr>
</tbody>
</table>

5.4.3 Comments

By refining the grid more, one can decrease the error, but the computing time becomes incompatible with that of a CAS-test.

![Comparison graph](Figure_5.4.3-1.png)

Figure 5.4.3-1: comparison of $K$ enter the various methods
6 Modeling D: Modeling X-FEM 2D-axi with structured grid

6.1 Characteristics of modeling

Here, one takes the same model as that of modeling B, but a structured grid is used.

6.2 Characteristics of the grid

The initial healthy grid is a grid structured with 192 nodes and 324 elements TRIA3. The size of the meshes is \( h_0 = 1 \text{ m} \). One uses a procedure of successive refinements to lead to a size targets corresponding to half of the size of the meshes of modeling A, that is to say \( h_c = 0.0125 \text{ m} \). Indeed, modeling A uses quadratic elements: one thus needs finer linear elements to obtain an equivalent precision. For that, one calls Lobster in an iterative way. After refinement, the size of the meshes close to the bottom of crack is worth \( h = 0.0078125 \text{ m} \). One refines all the meshes in a disc of ray \( 5h \) around the bottom of crack.
Figure 6.2-2: refined healthy grid

Figure 6.2-3: zoom on the refined part
Many nodes: 324  
Number of meshes and type: 587 TRIA3  
The length characteristic of an element close to the bottom to crack is of 0.0078 Mr.

6.3 Boundary conditions and loadings

- A surface effort of traction is applied to the faces higher, lower and that of right-hand side;
- Displacements according to $Ox$ nodes of the axis of rotation are blocked, as that is advised for axisymmetric modelings;
- Rigid mode of displacement following the axis $Oy$ is blocked via the blocking of a node following this axis.

6.4 Sizes tested and results

The choice of the digital parameters for the postprocessing of SIFs is identical to that done for modeling A: $R_{\text{inf}} = 2h$ and $R_{\text{sup}} = 5h$, but $h$ less than half is worth here of $h$ modeling A.

6.4.1 Values resulting from CALC_G

The values are in $P \sqrt{m}$.

<table>
<thead>
<tr>
<th>Identification</th>
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<th>Value of reference</th>
<th>% Tolerance</th>
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<tbody>
<tr>
<td>$K_I$</td>
<td>‘ANALYTICAL’</td>
<td>1.177 $10^6$</td>
<td>2%</td>
</tr>
<tr>
<td>$K_{II}$</td>
<td>‘ANALYTICAL’</td>
<td>0.3153 $10^6$</td>
<td>5%</td>
</tr>
</tbody>
</table>

6.4.2 Values resulting from POST_K1_K2_K3

The values are in $P \sqrt{m}$.

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<tr>
<th>Identification</th>
<th>Type of reference</th>
<th>Value of reference</th>
<th>% Tolerance</th>
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<tbody>
<tr>
<td>$K_I$</td>
<td>‘ANALYTICAL’</td>
<td>1.177 $10^6$</td>
<td>2.0%</td>
</tr>
<tr>
<td>$K_{II}$</td>
<td>‘ANALYTICAL’</td>
<td>0.3153 $10^6$</td>
<td>5.0%</td>
</tr>
</tbody>
</table>
7 Modeling E: Modeling X-FEM 2D-axi with quadratic elements

7.1 Characteristics of modeling

Here, one takes the same model as that of modeling B, but quadratic elements are used.

7.2 Characteristics of the grid

The initial healthy grid is the same grid as that of modeling B with 252 nodes and 442 elements TRIA3. Here, we use CREA_MAILLAGE to create a quadratic grid starting from the linear grid. This new grid has 942 nodes and 442 elements TRIA6. The size of the meshes is $h_0 = 1 \text{ m}$. One uses a procedure of successive refinements to lead to a size targets corresponding to half of the size of the meshes of modeling A, that is to say $h_c = 0,0125 \text{ m}$. For that, one calls Lobster in an iterative way. After refinement, the size of the meshes close to the bottom of crack is worth $h = 0,0078125 \text{ m}$. One refines all the meshes in a disc of ray $5h$ around the bottom of crack.

Figure 7.2-1: initial healthy grid

Figure 7.2-2: refined healthy grid
Many nodes: 1517
Number of meshes and type: 728 TRIA6
The length characteristic of an element close to the bottom to crack is of 0.0078 Mr.

7.3 Boundary conditions and loadings

- A surface effort of traction is applied to the faces higher, lower and that of right-hand side;
- Displacements according to $Ox$ nodes of the axis of rotation are blocked, as that is advised for axisymmetric modelings;
- Rigid mode of displacement following the axis $Oy$ is blocked via the blocking of a node following this axis.

7.4 Sizes tested and results

The choice of the digital parameters for the postprocessing of SIFs is identical to that done for modeling A: $R_{\text{int}} = \frac{1}{6}h$ and $R_{\text{sup}} = \frac{6}{6}h$, but $h$ less than half is worth here of $h$ modeling A.

7.4.1 Values resulting from CALC_G

The values are in $Pa \sqrt{m}$.

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<thead>
<tr>
<th>Identification</th>
<th>Type of reference</th>
<th>Value of reference</th>
<th>% Tolerance</th>
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<tbody>
<tr>
<td>$K_I$</td>
<td>‘ANALYTICAL’</td>
<td>$1.177 \times 10^6$</td>
<td>2.0%</td>
</tr>
<tr>
<td>$K_{II}$</td>
<td>‘ANALYTICAL’</td>
<td>$0.3153 \times 10^6$</td>
<td>5.0%</td>
</tr>
</tbody>
</table>
7.4.2 Values resulting from POST_K1_K2_K3

The values are in \( P_a \sqrt{m} \).

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<th>Identification</th>
<th>Type of reference</th>
<th>Value of reference</th>
<th>% Tolerance</th>
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<tbody>
<tr>
<td>( K_I )</td>
<td>'ANALYTICAL'</td>
<td>1.177 ( \times 10^6 )</td>
<td>2.0%</td>
</tr>
<tr>
<td>( K_{II} )</td>
<td>'ANALYTICAL'</td>
<td>0.3153 ( \times 10^6 )</td>
<td>5.0%</td>
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</table>

It is noticed that the values of \( K_{II} \) by POST_K1_K2_K3 results much further away from the solution give than the approach by CALC_G.
8 Modeling F: Modeling X-FEM 2D-axi with quadratic elements and structured grid

8.1 Characteristics of modeling

Here, one takes the same model as that of modeling E, but a structured grid is used.

8.2 Characteristics of the grid

The initial healthy grid is a grid structured with 192 nodes and 324 elements TRIA3. Here, we use CREA_MAILLAGE to create a quadratic grid starting from the linear grid. This new grid has 1232 nodes and 587 elements TRIA6. The size of the meshes is \( h_0 = 1 \) m. One uses a procedure of successive refinements to lead to a size targets corresponding to half of the size of the meshes of modeling A, that is to say \( h_c = 0.0125 \) m. For that, one calls Lobster in an iterative way. After refinement, the size of the meshes close to the bottom of crack is worth \( h = 0.0078125 \) m. One refines all the meshes in a disc of ray 5 \( h \) around the bottom of crack.
Figure 8.2-2: refined healthy grid

Figure 8.2-3: zoom on the refined part
Many nodes: 1232
Number of meshes and type: 587 TRIA6
The length characteristic of an element close to the bottom to crack is of 0.0078 Mr.

8.3 Boundary conditions and loadings

- A surface effort of traction is applied to the faces higher, lower and that of right-hand side;
- Displacements according to Ox nodes of the axis of rotation are blocked, as that is advised for axisymmetric modelings;
- Rigid mode of displacement following the axis Oy is blocked via the blocking of a node following this axis.

8.4 Sizes tested and results

The choice of the digital parameters for the postprocessing of SIFs is identical to that done for modeling A: \( R_{\text{inf}} = 1h \) and \( R_{\text{sup}} = 6h \), but \( h \) less than half is worth here of \( h \) modeling A.

8.4.1 Values resulting from CALC_G

The values are in \( Pa \cdot \sqrt{m} \).

<table>
<thead>
<tr>
<th>Identification</th>
<th>Type of reference</th>
<th>Value of reference</th>
<th>% Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_I )</td>
<td>‘ANALYTICAL’</td>
<td>1.177 ( 10^6 )</td>
<td>0.00152%</td>
</tr>
<tr>
<td>( K_{II} )</td>
<td>‘ANALYTICAL’</td>
<td>0.3153 ( 10^6 )</td>
<td>2.6%</td>
</tr>
</tbody>
</table>

8.4.2 Values resulting from POST_K1_K2_K3

The values are in \( Pa \cdot \sqrt{m} \).

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<thead>
<tr>
<th>Identification</th>
<th>Type of reference</th>
<th>Value of reference</th>
<th>% Tolerance</th>
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</thead>
<tbody>
<tr>
<td>( K_I )</td>
<td>‘ANALYTICAL’</td>
<td>1.177 ( 10^6 )</td>
<td>1.0%</td>
</tr>
<tr>
<td>( K_{II} )</td>
<td>‘ANALYTICAL’</td>
<td>0.3153 ( 10^6 )</td>
<td>19.0%</td>
</tr>
</tbody>
</table>

It is noticed that the values of \( K_{II} \) by POST_K1_K2_K3 results much further away from the solution give than the approach by CALC_G.
9 Summary of the results

This CAS-test validates the calculation of the stress intensity factors of a nonplane crack in 2D and 3D.

Results of $K_{II}$ by POST_K1_K2_K3 are not good for modeling E and F and differ from 10 and 15% of the value desired for modeling S With, C and D.