SDND100 - To release of a rubbing shoe with friction of the Coulomb type

Summary

One considers the one-way system with a degree of freedom made up of a mass in contact rubbing of Coulomb type on a rigid level, and of a spring attaching it to a fixed point. The mass is released in an initial position except balance. It oscillates until the complete stop at the end of a finished time.
The first two modelings correspond to the transitory answer by modal recombination of the rubbing shoe, the third corresponds to its direct transitory answer. Three calculations are compared with the analytical solution.

1 Problem of reference

1.1 Geometry

Direction of displacement: $\theta=45°$ in the plan XY

1.2 Material properties

Stiffness of the spring:	$k=10\,000\,N/m$
Specific mass:	$m=1\,kg$
Gravity:	$g=10\,m/s^2$
Coefficient of Coulomb:	$\mu=0,1$

1.3 Boundary conditions and loadings

The system rests on the plan $Z=0$ on which it can slip with a coefficient of friction of Coulomb of $\mu=0,1$.

1.4 Initial conditions

Initial displacement of the mass: $r_0=0,85\,mm$ according to the direction θ .

Worthless initial speed.

2 Reference solution

2.1 Method of calculating used for the reference solution

For a system without damping, the differential equation to solve is written:

$$\begin{cases} m\ddot{r}+k\,r=\mu\,|F_n| \\ r(t=0)=r_0\geq 0 \\ \dot{r}(t=0)=0 \end{cases} \qquad \text{with } F_n=-mg\,sign(\dot{r})$$

It is shown [bib1] that the solution of the differential equation is written:

$$r(t)=\frac{\mu|F_n|}{k}+\left(r_0-\frac{\mu|F_n|}{k}\right)\cos\omega_0 t$$

The amplitude of the extrema, which all come them $t_{n+1}=\frac{n\pi}{\omega_0}$, obeys the law of following recurrence:

$$r(t_{n+1})=(-1)^n\left[r_0-\frac{\mu|F_n|}{k}\right]\cos\omega_0 t$$

with $n=1,2,\ldots,N$ such as $\left|\frac{r(t_{n+1})}{r_0}\right|<\frac{\mu|F_n|}{k\,r_0}$

The movement stops when $\left|\frac{r(t_{n+1})}{r_0}\right|<\frac{\mu|F_n|}{k\,r_0}$ with the position $r(t_{n+1})$.

2.2 Results of reference

Values of displacements in the direction θ for the moments of change of sign speed ($r(t_1), r(t_2), \ldots, r(t_5)$ benches above).

2.3 Uncertainty on the solution

Analytical solution.

2.4 Bibliographical references

F. AXISA - Methods of analysis in nonlinear dynamics of the structures: non-linearities of contact - Course IPSI from May 28th to May 30th, 1991

3 Modeling A

3.1 Characteristics of modeling

An element of the type DIS_T on a mesh POI1 is used to model the system.
Conditions of relations between degrees of freedom are employed to force the movement to be one-way in the direction θ :

```
LIAISON_DDL = _F (NODE = ('NO1', 'NO1'),
                  DDL = ('DX' 'DY'),
                  COEF_MULT = (0,707, -0,707),
                  COEF_IMPO = 0.)
```

An obstacle of the type PLAN_Z (two parallel plans separated by a game) is used to simulate the slip surface. One chooses to take for generator of this plan the axis Oy, that is to say NORM_OBST = (0. , 1. , 0.). The origin of the obstacle is ORIG_OBST = (0. , 0. , 1.). It remains to define its game which gives the half - spacing between the plans.

So that there exists a force of reaction of the plan on the system it is necessary that this last is slightly inserted in the obstacle plan of a distance δn such as: $F_n = K_n \cdot \delta n$
Like $F_n = mg$, one has then $\delta n = mg / K_n$.
One considered a normal stiffness of shock of $20\, N/m$ (fictitious stiffness which has direction only to generate a force of reaction of the plan on the system), one thus has $\delta n = 0{,}5$. The obstacle PLAN_Z having for origin $Z=1$ and the solid being in $Z=0$; a game of $0{,}5\, m$ will create a depression $\delta n = 0{,}5\, m$ from where GAME: 0.5

Tangential stiffness of shock: $K_T = 400\,000\, N/m$: it is large in front of the stiffness of the oscillator so that the phase of stop is modelled correctly.

Pas de time used for temporal integration: 5.10^{-4s} .

3.2 Characteristics of the grid

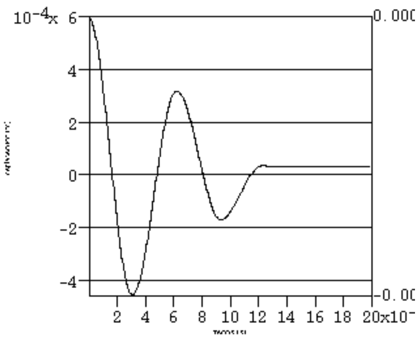
Many nodes: 1
Many meshs and types: 1 POI1

3.3 Sizes tested and results

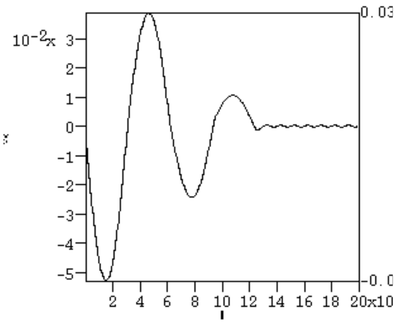
Values of displacements (in meters) in the direction θ for the moments of change of sign speed over the period of time $(0 ; 0.3 s)$.

Identification	moment (S)	Reference
$DY = r2\ cos45$	$\pi \times 10^{-2}$	– 4.596E-4
$DY = r3\ cos45$	$2\pi \times 10^{-2}$	3.182E-4
$DY = r4\ cos45$	$3\pi \times 10^{-2}$	– 1.768E-4
$DY = r5\ cos45$	$4\pi \times 10^{-2}$	3.536E-5

One presents Ci below the evolution of displacement and speed to the point $NO1$



Displacement of the point $NO1$



Speed of the point $NO1$

4 Modeling B

4.1 Characteristics of modeling

In modeling B, one regards the shoe and the plan as two mobile structures. Each structure is then modelled by a node and an element of the type `POI1`. The node $NO2$ is supposed to be blocked, it materializes the plan of friction. One imposes conditions of relations between degrees of freedom on the node $NO1$ (which models the shoe) so that the movement is one-way in the direction θ.

An obstacle of the type `BI_PLAN_Z` (two mobile parallel plans separated by a game) is used to simulate the slip surface. One chooses to take for generator of this plan axis OY, that is to say `NORM_OBST = (0. , 1. , 0.)`. By default, the origin of the obstacle is located at semi distance from the nodes $NO1$ and $NO2$. It remains to define the parameters `DIST_1` and `DIST_2` who represent the thickness of matter around the nodes of shock.

So that there exists a force of reaction of the plan on the system it is necessary that this last is slightly inserted in the obstacle plan of a distance δn such as: $F_n = K_n \cdot \delta n$.
Like $F_n = mg$, one has then $\delta n = mg / K_n$.
One considered a normal stiffness of shock of $20\,N/m$ (fictitious stiffness which has direction only to generate a force of reaction of the plan on the system), one thus has $\delta n = 0{,}5\,m$. Knowing that two nodes $NO1$ and $NO2$ are geometrically confused, one chooses for example `DIST_1` = `DIST_2` = $\delta n/2$.

Tangential stiffness of shock: $K_T = 400\,000\,N/m$: it is large in front of the stiffness of the oscillator so that the phase of stop is modelled correctly.

Pas de time used for temporal integration: $5.10^{-4}\,s$.

4.2 Characteristics of the grid

Many nodes: 2
Many meshs and types: 2 `POI1`

4.3 Sizes tested and results

Values of displacements (in meters) in the direction of the oscillator for the moments of change of sign speed over the period of time $(0\,;\,0.3\,s)$.

Identification	moment (S)	Reference
$DY = r2\cos 45$	$\pi \times 10^{-2}$	– 4.596E-4
$DY = r3\cos 45$	$2\pi \times 10^{-2}$	3.182E-4
$DY = r4\cos 45$	$3\pi \times 10^{-2}$	– 1.768E-4
$DY = r5\cos 45$	$4\pi \times 10^{-2}$	3.536E-5

5 Modeling C

5.1 Characteristics of modeling

This modeling corresponds to the direct transitory answer of the rubbing shoe.

The normal direction of contact is the local axis X who corresponds in the case test to the total axis Z. The slip surface is the local plan (Y, Z) that is to say the plan (X, Y) in the total reference mark. One thus directs the element of shock to a node, with the keyword ORIENTATION of the operator AFFE_CARA_ELEM in the following way:

```
ORIENTATION= _F (MAILLE=' EL1',  CARA: 'VECT_X_Y',
                 VALE = (0. , 0. , -1. , 0. , 1. , 0. ))
```

To be able to obtain a force of reaction of the plan on the system it is necessary that this last is slightly inserted in the obstacle plan of a distance δn such as: $F_n = K_n \cdot \delta n$.
The reaction balances the weight of the shoe, one thus has: $F_n = mg$ i.e. $\delta n = mg / K_n$.
One considered a normal stiffness of shock of $20\,N/m$ (fictitious stiffness which has direction only to generate a force of reaction of the plan on the system), one thus has $\delta n = 0{,}5$ from where DIST_1 = 0.5.

The tangential stiffness of shock considered is $K_T = 400\,000\,N/m$, the coefficient of Coulomb is worth 0.1.

The law of behavior of shock is thus in the following way defined in DEFI_MATERIAU :

```
DIS_CONTACT =_F (RIGI_NOR= 20. ,
                   DIST_1 = 0.5,
                   RIGI_TAN = 400000. ,
                   COULOMB = 0.1)
```

One uses a step of time of $5.10^{-4}s$ for temporal integration.

5.2 Characteristics of the grid

Many nodes: 1
Many meshs and types: 1 POI1

5.3 Sizes tested and results

Values of displacements in the direction oscillator for the approximate moments of change of sign speed over the period of time $(0; 0.2s)$.

Identification	moments (S)	Reference
$DY = r2\,cos45$	$\pi \times 10^{-2}$	– 4,585E-04
$DY = r3\,cos45$	$2\pi \times 10^{-2}$	3,173E-04
$DY = r4\,cos45$	$3\pi \times 10^{-2}$	– 1,754E-04
$DY = r5\,cos45$	$4\pi \times 10^{-2}$	3,550E-05

6 Modeling D

6.1 Characteristics of modeling

This modeling corresponds to modeling B, in which the option of one-way friction is activated. One regard the shoe and the plan as two mobile structures. Each structure is then modelled by a node and an element of the type POI1. The node $NO2$ is supposed to be blocked, it materializes the plan of friction. One imposes conditions of relations between degrees of freedom on the node $NO1$ (which models the shoe) so that the movement is one-way in the direction θ .

An obstacle of the type BI_PLAN_Z (two mobile parallel plans separated by a game) is used to simulate the slip surface. One chooses to take for generator of this plan axis OY, that is to say NORM_OBST = (0. , 1. , 0.) . By default, the origin of the obstacle is located at semi distance from the nodes $NO1$ and $NO2$. It remains to define the parameters DIST_1 and DIST_2 who represent the thickness of matter around the nodes of shock.

One-way friction implies that the coefficient of friction is not isotropic any more in the plan defined previously. It is worth 0 along the axis indicated in NORM_OBST and μ in the perpendicular direction. For an angle θ of $\pi/2$, this corresponds to divide the coefficient of friction by $\sqrt{(2)}$, compared to modeling B.

So that there exists a force of reaction of the plan on the system it is necessary that this last is slightly inserted in the obstacle plan of a distance δn such as: $F_n=K_n\cdot\delta n$ ·
Like $F_n=mg$, one has then $\delta n=mg/K_n$.
One considered a normal stiffness of shock of $20\,N/m$ (fictitious stiffness which has direction only to generate a force of reaction of the plan on the system), one thus has $\delta n=0{,}5\,m$. Knowing that two nodes $NO1$ and $NO2$ are geometrically confused, one chooses for example DIST_1 = DIST_2 = $\delta n/2$.

Tangential stiffness of shock: $K_T=400\,000\,N/m$: it is large in front of the stiffness of the oscillator so that the phase of stop is modelled correctly.

Pas de time used for temporal integration: $5.10^{-4}s$.

6.2 Characteristics of the grid

Many nodes: 2
Many meshs and types: 2 POI1

6.3 Sizes tested and results

Values of displacements in the direction oscillator for the approximate moments of change of sign speed over the period of time $(0;0.2s)$.

Identification	moments (S)	Reference
$DY=r2\,cos45$	$\pi\times10^{-2}$	– 5,010E-04
$DY=r3\,cos45$	$2\pi\times10^{-2}$	4.010E-04
$DY=r4\,cos45$	$3\pi\times10^{-2}$	–3.010E-04
$DY=r5\,cos45$	$4\pi\times10^{-2}$	2.010E-04

7 Summary of the results

The analytical solution of the problem with friction is reproduced with a very good precision ($<0.5\%$). That asks for nevertheless the use of a parameter of tangent stiffness rather high compared to the rigidity of the system as well as a step of relatively reduced time of integration.
On this example, direct nonlinear calculation is much more expensive in computing times than that on modal basis.