SDND106 – Shoe rubbing with coefficients of static and dynamic friction

Summary:

The purpose of this CAS-test is to validate the functionality of nonlinearity of friction by penalization, with the use of two coefficients of friction, a statics (for the phases of adherence) and a dynamics (for the phases of slip).

The studied system contains two degrees of freedom. Its evolution contains a transition adherence – slip and a transition slip – adherence.
The reference solution is analytical.

1 Problem of reference

1.1 Geometry

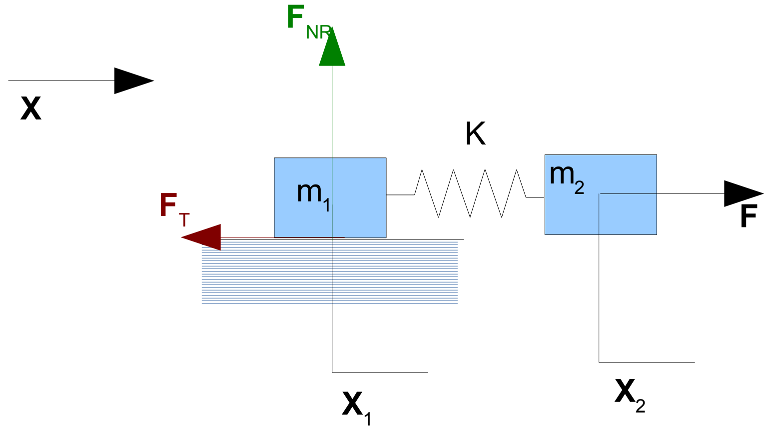

Figure 1.1-a : Diagram of the CAS-test.

1.2 Properties of material

The two masses are identical and are worth $m_1=m_2=m=5\,kg$. The stiffness placed between these masses is of $k=10^4\,N/m$.

1.3 Boundary conditions and loadings

Masses m_1 and m_2 move according to $\vec{x}$ only.
The module of the normal force F_N is constant equal to $F_N=10^4\,N$.
Mass m_1 of a condition of contact – friction is affected. Coefficients of friction 5 are:

- in the static case: $\mu_S=0{,}3$,
- in the dynamic case: $\mu_D=0{,}2$.

In this modeling, the system passes by three phases:

- A phase of adherence, during which the force exerted on the mass m_2 is constant: $F=3\cdot 10^3\,N$.
- A phase of slip, beginner with t_1 , when the tangential force cannot compensate for the force of traction exerted any more by the spring on m_1 . At the end of an arbitrary time ($t_2=0{,}2\,s$), the imposed force is put at 0 and the kinetics of the system tends then towards 0.
- One second phase of adherence, beginner with t_3 , when the speed of the mass m_1 cancel yourself.

1.4 Initial conditions

At the moment $t=0$, the two masses are at rest (displacement and a speed worthless).

2 Reference solution

2.1 Method of calculating

The pulsation of the system is noted ω_0 and is such as $\omega_0^2=\frac{k}{m}$.

Phase of adherence
At the moment $t=0$, mass m_1 check

$$x_1(t)=0$$
$$\dot{x}_1(t)=0$$

and masses it m_2 check

$$x_2(t)=0$$
$$\dot{x}_2(t)=0\,.$$

The basic principle of dynamics applied to the mass m_2 allows to write the following equation

$$\ddot{x}_2+\omega_0^2 x_2=\frac{F}{m}\,,$$

and on the mass m_1 the following equation

$$F_T=-k\,x_2$$

The general solution of x_2 is form:

$$x_2=\tilde{A}\cos(\omega_0 t)+\tilde{B}\sin(\omega_0 t)+\frac{F}{(m\omega_0^2)}$$

where $\tilde{A}$ and $\tilde{B}$ are constants.
By taking account of the initial conditions, the displacement of the mass m_2 is thus written

$$x_2=\frac{F}{k}[1-\cos(\omega_0 t)]\,.$$

This expression is valid until

$$\|F_T\|=\mu F_N$$

In other words, until the moment t_1 checking the following expression

$$F[1-\cos(\omega_0 t_1)]=\mu_S F_N$$

The phase of slip thus starts with t_1 defined by

$$t_1=\frac{1}{\omega_0}\arccos\left(1-\mu_S\frac{F_N}{F}\right)$$

Phase of slip
At the moment $t=t_1$, mass m_1 check

$$x_1(t_1)=0$$
$$\dot{x}_1(t_1)=0$$

The basic principle of dynamics applied to the mass m_1 allows to write the equation

$$\ddot{x}_1+\omega_0^2 x_1=\frac{-\mu_D F_N}{m}+\omega_0^2 x_2\,,$$

and on the mass m_2 the equation

$$\ddot{x}_2+\omega_0^2 x_2=\frac{F}{m}+\omega_0^2 x_1 .$$

By making the change of variables

$$\begin{cases} X=x_1+x_2 \\ Y=x_1-x_2 \\ \Omega_0=\sqrt{2}\,\omega_0 \end{cases}$$

the preceding system is written

$$\ddot{X}=\frac{F-\mu_D F_N}{m}$$

$$\ddot{Y}+\Omega_0^2 Y=-\frac{F+\mu_D F_N}{m}$$

For $t\geq t_1$, the general solution of X is form

$$X=\frac{(F-\mu_D F_N)}{2\mathrm{m}}(t-t_1)^2+\tilde{C}(t-t_1)+\tilde{D} .$$

and that of Y is form

$$Y=\tilde{E}\cos\left(\Omega_0(t-t_1)\right)+\tilde{F}\sin\left(\Omega_0(t-t_1)\right)-\frac{F+\mu_D F_N}{2\mathrm{k}} .$$

By taking account of the initial conditions, expressions of the constants $\tilde{C}$, $\tilde{D}$, $\tilde{E}$ and $\tilde{F}$ are worth:

$$\tilde{C}=\dot{x}_2(t_1)$$

$$\tilde{D}=x_2(t_1)$$

$$\tilde{E}=\frac{F+\mu_D F_N}{2\mathrm{k}}-x_2(t_1)$$

$$\tilde{F}=-\frac{\dot{x}_2(t_1)}{\Omega_0}$$

Displacements x_1 and x_2 are deduced from the preceding expressions.

Phase of slip with a loading $F=0$ from t_2 (arbitrary but higher than t_1) until the moment t_3 (moment of return to the phase adherence)

The basic principle of dynamics presented previously is always valid. For the mass m_1, the checked equation is the same one:

$$\ddot{x}_1+\omega_0^2 x_1=\frac{-(\mu_D F_N)}{m}+\omega_0^2 x_2 ,$$

and for the mass m_2

$$\ddot{x}_2+\omega_0^2 x_2=\omega_0^2 x_1$$

With the same change of variable as previously, the system is written

$$\ddot{X}=-\frac{\mu_D F_N}{m}$$

$$\ddot{Y}+\Omega_0^2 Y=-\frac{\mu_D F_N}{m}$$

For $t>t_2$, general solution of X is form

$$X=-\frac{\mu_D F_N}{2m}(t-t_2)^2+\tilde{G}(t-t_2)+\tilde{H}$$

The general solution of Y is form

$$Y=\tilde{I}\cos(\Omega_0(t-t_2))+\tilde{J}\sin(\Omega_0(t-t_2))-\frac{\mu_D F_N}{2k}$$

By taking account of the initial conditions, expressions of the constants $\tilde{G}$, $\tilde{H}$, $\tilde{I}$ and $\tilde{J}$ are worth:

$$\tilde{G}=\dot{x}_1(t_2)+\dot{x}_2(t_2)$$
$$\tilde{H}=x_1(t_2)+x_2(t_2)$$
$$\tilde{J}=\frac{\mu_D F_N}{2k}+x_1(t_2)-x_2(t_2)$$
$$\tilde{J}=\frac{\dot{x}_1(t_2)-\dot{x}_2(t_2)}{\Omega_0}$$

Displacements x_1 and x_2 are deduced from the preceding expressions.

2.2 Sizes and results of reference

The sizes tested are the kinematics of the masses m_1 and m_2 at various moments in the various phases of modeling. Are also tested the following moments of transition:

- passage de la phase of adherence to the phase of slip (urgent t_1) ;
- passage de la phase of slip to the phase of adherence (moment t_3).

2.3 Uncertainties on the solution

Exact analytical solution.

2.4 Bibliographical references

[1] E. BOYERE: Modeling of the shocks and friction in transitory analysis by modal recombination. Reference material of *Code_Aster* R5.06.03. September 2009.

3 Modeling A

3.1 Characteristics of modeling

In this modeling, the force exerted on the mass m_2 cancel yourself in $t_2=0,2\,s$.

3.2 Characteristics of the grid

The grid contains 2 discrete meshs to a node (for each mass) and 1 discrete mesh for the spring.

3.3 Sizes tested and results

Type	Moment (s)	Size	Reference	Aster	Difference (%)
Analytical	0.02	x_1	0.	2.0621E-05	
Analytical	0.02	x_2	0.11221	0.11221	0
Analytical	0.15	x_1	1.35332	1.35329	-0,002
Analytical	0.15	x_2	1.80751	1.80751	0
Analytical	0.34	$\dot{x}_1$	0.	-3.3802E-5	
Analytical	0.34	x_2	3.96813	3.97012	0.05

Type	Size (S)	Reference	Aster	% difference
Analytical	t_1	0.03512	0.03520	0.23
Analytical	t_3	0.31492	0.31500	0.03

3.4 Remarks

The difference in $\%$ is not given for the cases where the value of reference is worth 0, but it is observed that the absolute difference is about 10^{-5} .

4 Summary of the results

This CAS-test shows that the transitions adherence – slip and slip – adherence are well collected.
It validates the capacity of the operator in addition DYNA_VIBRA to integrate problems in great displacements in translation.