SDND121 – System mass-arises with shocks under forced excitation

Summary:

This problem corresponds to a transitory analysis by modal recombination of a nonlinear discrete system to a degree of freedom. Non-linearity consists of a contact with shock on a rigid level. The mass is subjected to a forced excitation. The problem is stiff, i.e. the stiffness is very different between the phases from coasting flight and the phases of contact. This problem makes it possible to test the energy assessment for the various diagrams of temporal integration, as well as kinematics.

1 Problème of reference

1.1 Geometry

The structure is a rigid shoe modelled by only one node. The shoe is provided with a return spring. It is subjected to a harmonic external force $F_{ext}(t)=a\sin(2\pi f t)$. It carries a condition of normal shock against a rigid plan, treated by penalization.

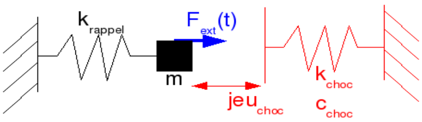

Figure 1.1-a: Geometry

1.2 Properties of materials

Properties of the structure:

$m=156\,kg$

$k_{rappel}=2\cdot10^{6}\,N/m$

Properties of the thrust of shock:

$k_{choc}=10^{10}\,N/m$

$c_{choc}=0\,Ns/m$

$jeu_{choc}=1\,mm$

1.3 Initial conditions, with the limits and loading

The shoe leaves with worthless initial conditions: $x_0=x_{t=0}=0$ and $\dot{x}_0=\dot{x}_{t=0}=0$.
It moves in only one direction.
The external force is sinusoidal: $F_{ext}(t)=Fa\cdot\sin(2\pi f t)$, with $Fa=3\cdot10^{3}N$ and $f=5\,Hz$.

2 Reference solution

2.1 Method of calculating used for the reference solution

At the time of a phase of coasting flight, the equation of the movement is written $m\ddot{x}+k_{rappel}x=F_{ext}(t)$. Taking into account the sinusoidal form of $F_{ext}(t)$, this equation admits an analytical solution. One can calculate numerically, with a precision as large as one wants, the moment t_{in} of entry in the contact, checking $x(t_{\mathrm{in}})=jeu_{choc}$.
One is then in a phase of contact, whose equation is $m\ddot{x}+c_{choc}\dot{x}+(k_{rappel}+k_{choc})x=F_{ext}(t)$. There exists an analytical solution there too. With the boundary conditions resulting from the moment of contact, one can calculate the moment numerically t_{out} of exit of the contact.
While proceeding thus repeatedly, one obtains the total solution of the problem.

Note: the analytical formulas here are not given. The files containing the formulas and making it possible to calculate the total solution are joined with the command file.

2.2 Results of reference

One tests the energy assessment, the adequacy between the forces of contact and kinematics, as well as kinematics.

For the energy assessment, the energies kinetic are calculated $E_i^{cin}=\frac{1}{2}m\dot{x}_i^2$, potential $E_i^{pot}=\frac{1}{2}k_{rappel}x_i^2$, of shock $E_i^{choc}=\frac{1}{2}k_{choc}p_i^2$ (p is the penetration; this expression is valid only if there is no damping of shock), not injected by the external force $E_i^{inj}=\sum_{j=1}^{i} f_j^{ext}\dot{x}_{j+\delta\frac{1}{2}}\Delta t$ (with $\delta=1$ for the diagram of Euler, $\delta=0$ for the diagram of the centered differences). Total energy is obtained $E_i^{tot}=E_i^{cin}+E_i^{pot}+E_i^{choc}$. One calculates finally the total error on the energy assessment by

$$erreur_{globale}^{énergie}=\sqrt{\frac{\sum_i\left(E_i^{tot}-E_i^{inj}\right)^2}{\sum_i\left(E_i^{inj}\right)^2}}$$

, which is worth 0 ideally.

For the adequacy between the forces of contact and kinematics, one calculates the size

$$erreur_{globale}^{force}=\sqrt{\frac{\sum_i\left(F_i^{choc}-k_{choc}p_i\right)^2}{\sum_i\left(k_{choc}p_i\right)^2}}$$

, which is worth 0 ideally.

For kinematics, one compares the calculated moments of entry and exit of contact, at the analytical moments.

2.3 Uncertainty on the solution

The solution is analytical per pieces. The moments of entry and exit of contact are numerically given with $10^{-9}s$ near.

3 Modeling

3.1 Characteristics of modeling

The shoe is modelled by a mesh with a node (of type POI1), located at rest in $O=(0,0,0)$. The obstacle is of type 'PLAN_Z'. Not to have a shock with the symmetrical plan, one shifts sufficiently the center of the two plans: ORIG_OBST= (- 0.5, 0. , 0. ,). As the real game between the shoe and the plan of right-hand side must be of $1\,mm$, an artificial game is used JEU= (0.5+jeu_choc).

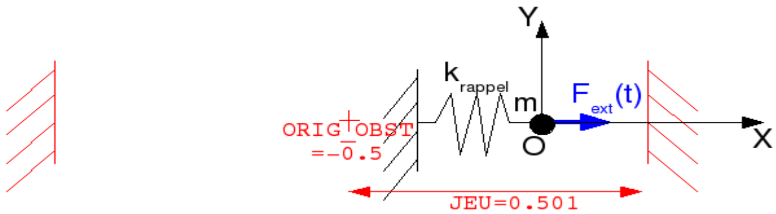

Figure 3.1-a: Modelled geometry

Temporal integration is carried out over one duration $T=4s$ either with the diagram of Euler, or with the diagram of the centered differences ('ADAPT_ORDRE2'that one forces with a step of constant time thanks to the parameters COEF_MULT_PAS=1.0, COEF_DIVI_PAS=1.0). The problem is stiff (the report of the stiffnesses between the phases of coasting flight and contact is about 5000), one is thus brought to use a being worth step of very weak time $\Delta t=4\cdot 10^{-6}s$.

One carries out also a calculation with 'ADAPT_ORDRE2'with indeed variable step, of which the goal is not the precision, but simply the checking of the adequacy between the forces of contact and kinematics.

3.2 Characteristics of the grid

The grid consists of a single node and a single mesh of the type POI1.

3.3 Sizes tested and results

3.3.1 Energy assessment and kinematics

In the following tables, one gives the error values de l' on energy and a few moments of input/output of contact.

- Diagram of Euler:

Values	Reference	Aster	Tolerance
$erreur_{globale}^{énergie}$	0 J	0,092 J	0.1 J
1st entry of contact (S)	2.4867876 E-02 S	2.4868 E-02 S	1.2E-5 S
1st exit of contact (S)	2.5260518 E-02 S	2.5264 E-02 S	1.2E-5 S
last entry of contact (S)	3.886525493 E+00 S	3.886528 E+00 S	1.2E-5 S
last exit of contact (S)	3.886916559 E+00 S	3.886916 E+00 S	1.2E-5 S

Table 4.1-1: Results for the diagram of Euler

- Diagram of the centered differences:

Values	Reference	Aster	Tolerance
$erreur_{globale}^{énergie}$	0 J	0,063 J	0.1 J
1st entry of contact (S)	2.4867876 E-02 S	2.4868 E-02 S	1.2E-5 S
1st exit of contact (S)	2.5260518 E-02 S	2.5264 E-02 S	1.2E-5 S
last entry of contact (S)	3.886525493 E+00 S	3.886528 E+00 S	1.2E-5 S
last exit of contact (S)	3.886916559 E+00 S	3.886916 E+00 S	1.2E-5 S

Table 4.1-2: Results for the diagram of the centered differences

3.3.2 Adequacy between force of contact and kinematics

Value	Reference	Aster	Tolerance
$erreur_{globale}^{force}$	0 NR	2.22 E-10 NR	1.E-8 NR

Table 4.2-1: Results for the adequacy between force of contact and kinematics

4 Summary of the results

One observes as the results are very close to the analytical solution, as well for the diagram of Euler as for the diagram of the centered differences. The moments of change of stiffness obtained by these two diagrams are identical.

The energy assessments present an error lower than $0{,}1\,J$ in both cases. This value is to be relativized taking into account the step of very weak time which tends to reduce the variations, but a step of so weak time is necessary to deal with this stiff problem.

Lastly, there is a perfect adequacy (with the digital precision near) between the forces of contact and kinematics, which means that the treatment of the contact by penalization is correctly carried out.