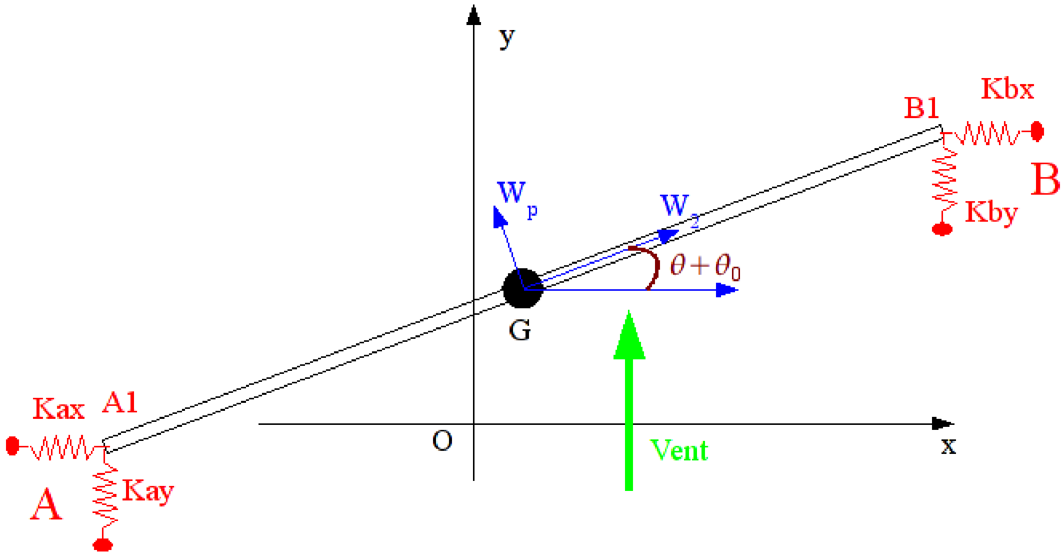
SDNL102 - Beam subjected to a field speed of wind

Summary:

This test relates to the validation of the application of the loadings of wind on the linear elements. The loading is described by fields speeds of wind.
This problem makes it possible to test:
- linear finite elements [bars, cables, beams (except the curved beams)] with loadings follower of natural “wind”,
- loadings using speeds of wind:
  - reading of the data of the fields of wind,
  - projection of the fields of wind attached to the group of dots on the deformed grid of the structure,
  - calculation relative speed,
- the taking into account of the function giving the force distributed according to the relative speed of the structure,
- the reactualization of the geometry to take account of great displacements and great rotations.

1 Problem of reference

1.1 Geometry



Length of the beam: 1.5m
Stiffnesses of the discrete ones: kax , kay , kbx , kby

1.2 Properties of material

Material for the linear element: $E=2.0\text{E}+10$, $\rho=1000.0$
Characteristics mechanics of the beam: $section='CERCLE'$, $rayon=0.1$, $ep=0.1$
Stiffness of the springs:

Kxa	Kya	Kxb	Kyb
$10\,N/m$	$20\,N/m$	$25\,N/m$	$22\,N/m$

1.3 Boundary conditions and loadings

At the points A and B : blockings of the degrees of freedom: DX, DY, DZ
At the points $A1$ and $B1$: blockings of the degrees of freedom : DZ, DRX, DRY
The springs are modelled by the discrete ones without dimensions. Nodes A and $A1$, B and $B1$ are geometrically confused.
Characteristics of the field speed of wind, along the axis y :

$$Vy=20.\sin(\omega.t) \text{ , with } \omega=2.\pi.f \text{ and } f=0.2\,Hz$$

1.4 Initial conditions

The beam forms an angle of $30°$ ($\theta_0=30°$) compared to the axis x .

2 Reference solution

2.1 Equilibrium equations

The study is carried out around the initial position of the structure in the plan xy. The equations are written in the centre of gravity of the beam.
Effort of inertia:

$$M.\gamma_g=\begin{Bmatrix} Mx'' \\ My'' \\ \dfrac{M L^2}{12}.\theta'' \end{Bmatrix}$$

Effort at the point $A1$

$$Fa=\begin{Bmatrix} -kxa.\delta xa \\ -kya.\delta ya \\ L.(\delta ya.kya.\cos(\theta_0+\theta)-\delta xa.kxa.\sin(\theta_0+\theta))/2 \end{Bmatrix} \begin{matrix} \text{avec les déplacements du point } A1 \\ \delta xa=L.cos(\theta_0)/2-L.cos(\theta_0+\theta)/2+x \\ \delta ya=L.sin(\theta_0)/2-L.sin(\theta_0+\theta)/2+y \end{matrix}$$

Effort at the point $B1$

$$Fb=\begin{Bmatrix} -kxb.\delta xb \\ -kyb.\delta yb \\ L.(-\delta yb.kyb.\cos(\theta_0+\theta)+\delta xb.kxb.\sin(\theta_0+\theta))/2 \end{Bmatrix} \begin{matrix} \text{avec les déplacements du point } B1 \\ \delta xb=-L.cos(\theta_0)/2+L.cos(\theta_0+\theta)/2+x \\ \delta yb=-L.sin(\theta_0)/2+L.sin(\theta_0+\theta)/2+y \end{matrix}$$

Effort due to the wind

- Relative speed of a point M

$$V_r=\begin{Bmatrix} Vvx+s.\sin(\theta_0+\theta).\theta'-x' \\ Vvy-s.\cos(\theta_0+\theta).\theta'-y' \\ 0 \end{Bmatrix}$$

with s : the curvilinear X-coordinate of the point M on the beam $s\in[-L/2,L/2]$
Vvx , Vvy : speed of the wind following axis X and centers it y .

- Speed relative perpendicular to the bar to the point M:

$$V_p=\begin{Bmatrix} \sin(\theta_0+\theta).(-Vvy.\cos(\theta_0+\theta)+Vvx.\sin(\theta_0+\theta)+s.\theta'-\sin(\theta_0+\theta).x'+\cos(\theta_0+\theta).y') \\ \cos(\theta_0+\theta).(Vvy.\cos(\theta_0+\theta)-Vvx.\sin(\theta_0+\theta)-s.\theta'+\sin(\theta_0+\theta).x'-\cos(\theta_0+\theta).y') \\ 0 \end{Bmatrix}$$

Force due to the wind in a point M

$Fvent_{(M)}=Fcx_{(M)}.\dfrac{V_p}{\|V_p\|}$ in our case one chooses $Fcx_{(M)}=\|V_p\|$

one thus obtains $Fvent_{(M)}=V_p$

- Resultant of the force due to the wind on the bar

$$Fvent=\begin{Bmatrix} L.\sin(\theta_0+\theta).((-Vvy+y').\cos(\theta_0+\theta)+(Vvx-x').\sin(\theta_0+\theta)) \\ L.\cos(\theta_0+\theta).((Vvy-y').\cos(\theta_0+\theta)+(-Vvx+x').\sin(\theta_0+\theta)) \\ -L^3.\theta'/12 \end{Bmatrix}$$

Final equation of dynamics

$$M.\gamma_g=Fa+Fb+Fvent$$

2.2 Sizes and results of reference

Displacements and rotation of the point G at the moments: 2.0sec , 3.0sec , 4.0sec , 5.0sec and 6.0sec .

2.3 Uncertainties on the solution

None. The resolution of the equilibrium equation is done by a method of integration of Runge Kutta of order 4.

3 Modeling A

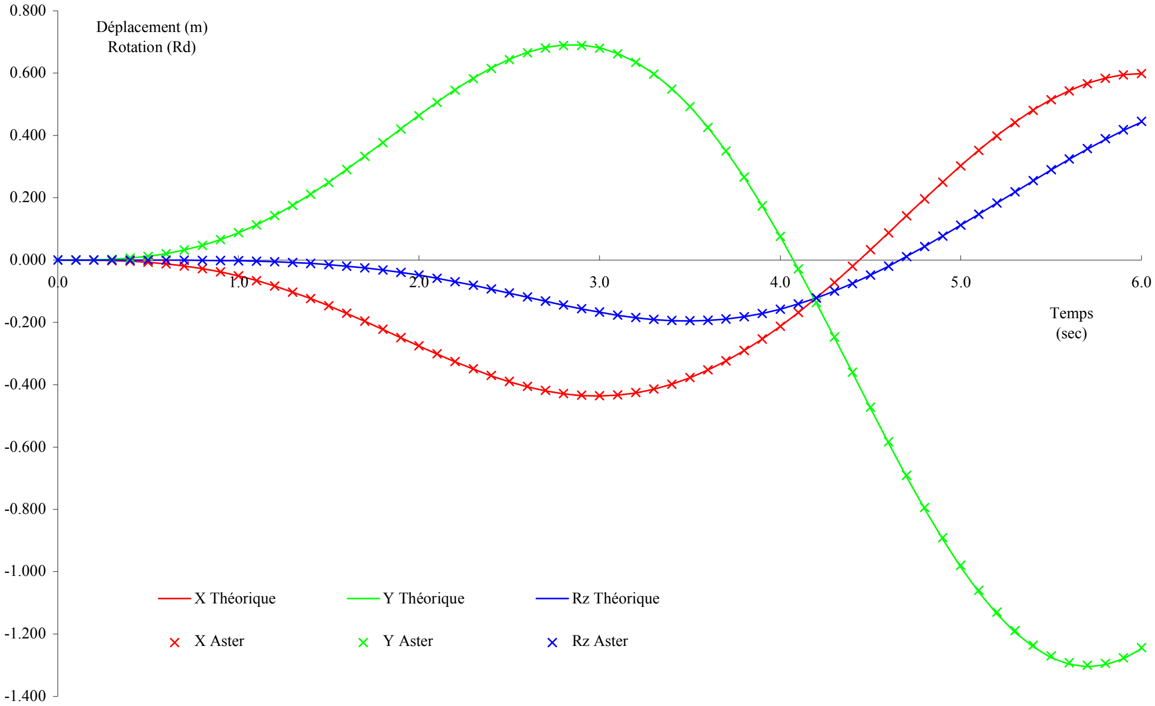
3.1 Characteristics of modeling and the grid

The linear element: 'beam' cut out in 12 meshs.
The discrete ones: `'DIS_T'`

3.2 Sizes tested and results

Time 2.0sec	Analytical	Absolute error	Relative error
$x(m)$	– 0.27571	0.00070	0.00255
$y(m)$	0.46478	0.00120	0.00259
$Rz(rd)$	– 0.04851	0.00001	0.00027
Time 3.0sec	**Analytical**	**Absolute error**	**Relative error**
$x(m)$	– 0.43640	0.00118	0.00271
$y(m)$	0.68149	0.00190	0.00279
$Rz(rd)$	– 0.16767	0.00079	0.00472
Time 4.0sec	**Analytical**	**Absolute error**	**Relative error**
$x(m)$	– 0.21266	0.00043	0.00201
$y(m)$	0.07494	0.00111	0.01476
$Rz(rd)$	– 0.15769	0.00026	0.00163
Time 5.0sec	**Analytical**	**Absolute error**	**Relative error**
$x(m)$	0.30290	0.00108	0.00357
$y(m)$	– 0.98487	0.00536	0.00544
$Rz(rd)$	0.11188	0.00027	0.00241
Temps6.0sec	**Analytical**	**Absolute error**	**Relative error**
$x(m)$	0.59847	0.00032	0.00054
$y(m)$	– 1.24735	0.00322	0.00258
$Rz(rd)$	0.44284	0.00251	0.00566

4 Summary of the results



Comparison enters the theoretical results and those of *Code_Aster.*