Summary:
The objective of this test is to validate the calculation of the frequencies and modes of vibrations of a auto--constrained framework. The efforts in the structure are only generated by the setting in tension of the cables. The principle of the case test is of:

- to carry out a non-linear calculation,
- to recover the various fields at a given moment,
- to calculate the matrices of stiffness and mass,
- to calculate the frequencies and modes of vibrations.

Several modeling are carried out:

- calculation with STAT_NON_LINE, use of the operators CALC_MATR_ELEM, ASSE_MATRICE and COMB_MATR_ASSE for the assembly of the matrices. Then calculation of the frequencies and modes of vibrations with the operator CALC_MODES.
- calculation with DYNA_NON_LINE and the option MODE_VIBR.
- calculation with STAT_NON_LINE, determination of the matrix of stiffness using the operator CALCULATION. Then calculation of the frequencies and modes of vibrations with the operator CALC_MODES.

The results are compared with a theoretical solution.
1 Problem of reference

1.1 Geometry

One considers a framework made up of 2 bars and 4 cables.

1.2 Properties of material

The materials are elastic, their properties are:

- For the bars:
  \[ E = 2.1 \times 10^{11} \quad \alpha = 0.0 \quad \rho = 7800.0 \]
- For the cables:
  \[ E = 2.1 \times 10^{11} \quad \alpha = 1.0 \times 10^{-5} \quad \rho = 7800.0 \]

1.3 Mechanical characteristics

The only characteristics necessary are the sections of the various elements.

- for the bars: \[ A = 1.0 \times 10^{-4} \]
- for the cables: \[ A = 5.026 \times 10^{-5} \]

1.4 Boundary conditions and loadings

In displacement:

- node \textit{N3}, all displacements are blocked: \[ DX = DY = DZ = 0.0 \]
- node \textit{N4}, displacements blocked according to \( Y \) and \( Z \): \[ DY = DZ = 0.0 \]
- nodes \textit{N1}, \textit{N2}: displacements blocked according to \( Z \): \[ DZ = 0.0 \]

The loading is applied by the setting in tension of the cables. A fictitious temperature of -200°C, is imposed on the elements of cables in 20 pas de loading.

1.5 Initial conditions

Without object.
2 Reference solution

2.1 Method of calculating

The adopted reference solution is that obtained by solving the equation of the vibrating cords:

\[
\frac{\partial^2 y}{\partial t^2} = \frac{F_0}{\rho S} \frac{\partial^2 y}{\partial x^2}
\]

with:

- \( F_0 \): tension in the cord,
- \( \rho \): density,
- \( S \): section of the cord.

For a cord length \( L \), fixed at these 2 ends, \( N \text{ème} \) Eigen frequency of vibration is:

\[
f = \frac{n}{2L} \sqrt{\frac{F_0}{\rho S}}
\]

2.2 Sizes and results of reference

The sizes tested are the Eigen frequencies of vibrations of the cables for 2 moments of calculation, which correspond to 2 different tensions.

<table>
<thead>
<tr>
<th>Not</th>
<th>Tension of the cables</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>6171.050459855 N</td>
<td>62.73227096292042 Hz</td>
</tr>
<tr>
<td>20</td>
<td>12345.2954376 N</td>
<td>88.72830898253936 Hz</td>
</tr>
</tbody>
</table>

2.3 Uncertainties on the solution

The solution of the equation of the vibrating cords is obtained under the following assumptions:

- the movements of the cord around its position of balance remain small.
- variations of the angle \( \alpha \) remain small.
- the variation of the tension of the cord moving remains small.

Under these assumptions, the reference solution is without uncertainties.
3 Modeling A

3.1 Characteristics of modeling

Modeling with elements of BAR and of CABLE.

3.2 Characteristics of the grid

The grid contains:

- 2 elements of the type BAR.
- 60 elements of the type CABLE (15 elements by cables).

3.3 Sizes tested and results

One tests the value of the 1st Eigen frequency of vibration of the cables. As the 4 cables have the same tension, there thus exist multiple modes of vibration. Six multiple modes are tested, for each step.

<table>
<thead>
<tr>
<th>Not</th>
<th>Type of reference</th>
<th>Mode</th>
<th>Value of reference</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>'ANALYTICAL'</td>
<td>Identical value for the first 6 multiple modes</td>
<td>62.732270963 Hz</td>
<td>0.5 %</td>
</tr>
<tr>
<td></td>
<td>'NON_REGRESSION'</td>
<td>1</td>
<td>62.7962 Hz</td>
<td>defect</td>
</tr>
<tr>
<td></td>
<td>'NON_REGRESSION'</td>
<td>2</td>
<td>62.9083 Hz</td>
<td>defect</td>
</tr>
<tr>
<td></td>
<td>'NON_REGRESSION'</td>
<td>3</td>
<td>62.9458 Hz</td>
<td>defect</td>
</tr>
<tr>
<td></td>
<td>'NON_REGRESSION'</td>
<td>4</td>
<td>62.9626 Hz</td>
<td>defect</td>
</tr>
<tr>
<td></td>
<td>'NON_REGRESSION'</td>
<td>5</td>
<td>62.9626 Hz</td>
<td>defect</td>
</tr>
<tr>
<td></td>
<td>'NON_REGRESSION'</td>
<td>6</td>
<td>62.9626 Hz</td>
<td>defect</td>
</tr>
<tr>
<td>20</td>
<td>'ANALYTICAL'</td>
<td>Identical value for the first 6 multiple modes</td>
<td>88.728308982 Hz</td>
<td>0.5 %</td>
</tr>
<tr>
<td></td>
<td>'NON_REGRESSION'</td>
<td>1</td>
<td>88.5701 Hz</td>
<td>defect</td>
</tr>
<tr>
<td></td>
<td>'NON_REGRESSION'</td>
<td>2</td>
<td>88.8995 Hz</td>
<td>defect</td>
</tr>
<tr>
<td></td>
<td>'NON_REGRESSION'</td>
<td>3</td>
<td>89.0065 Hz</td>
<td>defect</td>
</tr>
<tr>
<td></td>
<td>'NON_REGRESSION'</td>
<td>4</td>
<td>89.0541 Hz</td>
<td>defect</td>
</tr>
<tr>
<td></td>
<td>'NON_REGRESSION'</td>
<td>5</td>
<td>89.0541 Hz</td>
<td>defect</td>
</tr>
<tr>
<td></td>
<td>'NON_REGRESSION'</td>
<td>6</td>
<td>89.0541 Hz</td>
<td>defect</td>
</tr>
</tbody>
</table>

3.4 Remarks

Calculation is carried out with the operator STAT NON LINE, then operators CALC MATR_ELEM, ASSE_MATRICE and COMB_MATR_ASSE for the assembly of the matrices. The calculation of the frequencies and modes of vibration is carried out with the operator CALC_MODES.
4 Modeling B

4.1 Characteristics of modeling

Modeling with elements of BAR and of CABLE.

4.2 Characteristics of the grid

The grid contains:
- 2 elements of the type BAR.
- 60 elements of the type CABLE. (15 elements by cables).

4.3 Sizes tested and results

One tests the 1st Eigen frequency of vibration of the cables. The operator DYNA_NON_LINE with the option MODE_VIBR does not allow to know the following ones. It is a test of nonregression.

<table>
<thead>
<tr>
<th>Not</th>
<th>Type of reference</th>
<th>Computed value</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>'NON_REGRESSION'</td>
<td>62.7962 Hz</td>
<td>defect</td>
</tr>
<tr>
<td>20</td>
<td>'NON_REGRESSION'</td>
<td>88.5701 Hz</td>
<td>defect</td>
</tr>
</tbody>
</table>

Note: For memory, the frequencies of vibrations of references are: 62.732270963 Hz and 88.728308982 Hz

4.4 Remarks

Calculation is carried out with the operator DYNA NON LINE, with the key word factor MODE_VIBR. The operator allows to know only one frequency which is displayed in the file message and memorized in the parameter FREQ structure of data produced by the operator. This is why it is a test of nonregression.
5 Modeling C

5.1 Characteristics of modeling

Modeling with elements of BAR and of CABLE.

5.2 Characteristics of the grid

The grid contains:
- 2 elements of the type BAR.
- 60 elements of the type CABLE. (15 elements by cables).

5.3 Sizes tested and results

One tests the 1st Eigen frequency of vibration of the cables to the last step of calculation, NOT = 20. As the 4 cables have the same tension, there thus exist multiple modes of vibration. Six multiple modes are tested. It is a test of nonregression.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Type of reference</th>
<th>Computed value</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>'NON_REGRESSION'</td>
<td>88.5312</td>
<td>defect</td>
</tr>
<tr>
<td>2</td>
<td>'NON_REGRESSION'</td>
<td>88.8603</td>
<td>defect</td>
</tr>
<tr>
<td>3</td>
<td>'NON_REGRESSION'</td>
<td>88.9670</td>
<td>defect</td>
</tr>
<tr>
<td>4</td>
<td>'NON_REGRESSION'</td>
<td>89.0146</td>
<td>defect</td>
</tr>
<tr>
<td>5</td>
<td>'NON_REGRESSION'</td>
<td>89.0146</td>
<td>defect</td>
</tr>
<tr>
<td>6</td>
<td>'NON_REGRESSION'</td>
<td>89.0146</td>
<td>defect</td>
</tr>
</tbody>
</table>

Note:

For memory the frequency of reference is 88.728308982 Hz.

5.4 Remarks

The 1st calculation is carried out with the operator STAT_NON_LINE. The operator CALCULATION who has like entries the fields of displacement, constraint, internal variables as well as an increment of displacements makes it possible to know the matrix of stiffness of the system. calculation of the frequencies and modes of vibrations is carried out with the operator CALC_MODES by using the matrix of stiffness which comes from the operator CALCULATION.

The operator CALCULATION fact the assumption of linearity of the fields according to the increment of displacements. However, in the case as of cables this assumption is not completely "just". This increment of displacement must make it possible to approximate the matrix the tangent, it must thus be "small". It is determined by a study of sensitivity. This is why this modeling is a test of nonregression.
6 Summary of the results

The results of modelings A, B and C are in good adequacy with the reference solution.

The results of modeling C (with only teaching goal) requires a study of sensitivity to find the increment of displacement which makes it possible to approximate the tangent matrix and to approach the reference solution. That is due to the assumption of linearity of the operator \textit{CALCULATION}. 