

SDNV106 – Analysis with the eigenvalues in DYNA_NON_LINE (stability and oscillatory modes)

Summary:

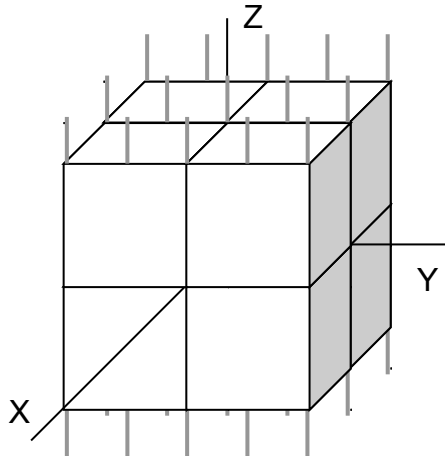
This case test makes it possible to validate the analysis of buckling, as well as the vibratory modal analysis in DYNA_NON_LINE.

Only one modeling is used: Massive modeling A 3D composed of meshes HEXA8.

1 Problem of reference

1.1 Geometry

One considers a cube on side length $2m$ subjected to a uniform traction according to the vertical direction Z :



For reasons of symmetry, one will consider only one eighth of the structure, which will be with a grid by only one cubic linear voluminal element.

1.2 Properties of material

The structure is supposed to be homogeneous, composed of an isotropic elastoplastic material, with linear isotropic work hardening:

- $E = 2.10^4 \text{ MPa}$
- $\nu = 0.49999$
- $\rho = 7900 \text{ kg/m}^3$
- $\sigma_y = 0,1 \text{ Mpa}$ (elastic threshold S_Y)
- $E_T = 200 \text{ Mpa}$ (plastic tangent module D_SIGM_EPSI)

One thus chooses a material which remains always almost incompressible, that one is in elastic or plastic mode. Moreover, one imposes a relationship 100 between the elastic stiffness and the plastic tangent stiffness.

1.3 Boundary conditions

One imposes a uniform loading of standard traction imposed according to Z on the higher face of the cube. This force imposed, initially worthless, grows linearly with time.

The other boundary conditions are of Dirichlet type and translate the conditions of symmetries of the problem (according to the 3 orthogonal plans (xOy) , (xOz) and (yOz)).

These boundary conditions are sufficient to block all the movements of rigid body of the system.

1.4 Initial conditions

The first calculation being quasistatic, one imposes just an initial displacement no one.

2 Reference solution

2.1 Method of calculating

One wants to check two types of quantities:

- the first critical load of buckling,
- the first Eigen frequency of the system in vibration.

The value of reference of the required critical load is obtained by a quasistatic calculation (keyword `CRIT_STAB` of `STAT_NON_LINE`). One takes this value obtained with the last step of quasistatic calculation, which corresponds to the moment $t = 1 s$.

The number stored under `CHAR_CRIT` in the structure of data result (it is the minimal coefficient multiplying of the loading forced to obtain the buckling load) being proportional to the imposed loading which is monotonous growing linearly with time, one corrects it to have the true value at the first moment of transitory dynamic calculation, that is to say $1,001 s$.

One has, by definition of the multiplying coefficient `CHAR_CRIT` :

$$F_{critique} = \text{CHAR_CRIT}(t_i) \cdot F_{ext}(t_i)$$

The external force is proportional to time: $F_{ext}(t_i) = F_{ext} \cdot t_i$, therefore $F_{critique} = \text{CHAR_CRIT}(t_i) \cdot F_{ext} \cdot t_i$.

The assumption is made that on a step, the loading evolves very slowly and thus that one can compare the result of dynamic calculation to a quasistatic evolution during this step. One can then write, for the first dynamic step, which follows quasistatic calculation:

$$F_{critique} = \text{CHAR_CRIT}_{\text{STAT_NON_LINE}}(t_i) \cdot F_{ext} \cdot t_i \approx F_{critique} = \text{CHAR_CRIT}_{\text{DYNA_NON_LINE}}(t_i) \cdot F_{ext} \cdot t_i$$

\Rightarrow

$$\text{CHAR_CRIT}_{\text{DYNA_NON_LINE}}(t_{i+1}) \approx \text{CHAR_CRIT}_{\text{STAT_NON_LINE}}(t_i) \cdot \frac{t_i}{t_{i+1}} = \text{CHAR_CRIT}_{\text{STAT_NON_LINE}}(t_i) \cdot \frac{1}{1,001}$$

For the vibratory analysis, one will make two tests:

- by using the elastic matrix of stiffness,
- by using the matrix of tangent stiffness plastic.

The two values of reference are obtained by two linear modal calculations carried out with the operator `CALC_MODES`.

To obtain the first Eigen frequency corresponding to the elastic case, one does a linear elastic design with `CALC_MODES` and the definite initial material above (of being worth modulus Young 2.10^4 Mpa).

To obtain the first Eigen frequency corresponding to the tangent plastic case, one does a linear elastic design with `CALC_MODES` and a fictitious elastic material whose Young modulus is worth the definite plastic tangent module above: 200 Mpa , that is to say 100 times less than the real elastic module. One will thus have an Eigen frequency 10 times weaker than the preceding one.

One knows also the analytical solution of our problem (cubic length 1 constituted of only one linear finite element) which is brought back to a case 1D of traction compression:

$$\omega = \sqrt{2 E / \rho} \approx \begin{cases} 0,358128 \text{ rad/s} : \text{matériau élastique} \\ 0,0358128 \text{ rad/s} : \text{matériau plastique} \end{cases}$$

2.2 Sizes and results of reference

Sizes	Values	Unit
Multiplying coefficient of the first critical load of buckling	-2.85714E+01/1,001	
First elastic Eigen frequency	3.58128E-01	Hz
First plastic Eigen frequency	3.58128E-02	Hz

3 Modeling A

3.1 Characteristics of the grid

Many meshes: 1 HEXA8
Many nodes: 8

3.2 Sizes tested and results

Identification		Reference	Aster	% difference
Plastic vibratory Eigen frequency	$Tps = 1.01$	3.58128E-02	3.5812661359567D-02	-3.87E-04
	$Tps = 1.06$	3.58128E-02	3.5812661359997D-02	-3.87E-04
	$Tps = 1.25$	3.58128E-02	3.5812661358541D-02	-3.87E-04
	$Tps = 1.49$	3.58128E-02	3.5812661355801D-02	-3.87E-04
Elastic vibratory Eigen frequency	$Tps = 1.51$	3.58128E-01	3.5812779545194D-01	-5.71E-05
	$Tps = 1.52$	3.58128E-01	3.5812779545194D-01	-5.71E-05
	$Tps = 1.56$	3.58128E-01	3.5812779545194D-01	-5.71E-05
	$Tps = 1.75$	3.58128E-01	3.5812779545194D-01	-5.71E-05
	$Tps = 1.99$	3.58128E-01	3.5812779545194D-01	-5.71E-05
Coefficient of the first critical load	$Tps = 1,001$	-2.854285714E+01	-2.85701899729E+01	0,096

One supplements these tests by two tests on the oscillatory mode `DEPL_VIBR` calculated with `MODE_VIBR`. More precisely, one will test the value of this field in two nodes:

- `GROUP_NO=' A '` (node in (0,0,0): who is embedded, one must thus find a displacement identically no one,
- `GROUP_NO=' H '` (node in (0,1,1) : one makes a test of nonregression according to the direction `DY` .

Identification		Reference	Aster	% difference
DEPL_VIBR in 'With' according to $D\bar{X}$	$Tps = 1.2$	0.	0.	0.
DEPL_VIBR in 'H' according to DY	$Tps = 1.2$	-0.49999288483407	-0.49999288483077	6.61E-10

This test makes it possible to validate calculations of critical loads of buckling, of vibratory frequencies and clean modes in `DYNA_NON_LINE`.

4 Summary of the results

This test makes it possible to validate calculations of critical loads of buckling, of vibratory frequencies and clean modes in DYNA_NON_LINE.