SSNA103 - Chock of the parameters of the model of Weibull

Summary:

This test validates the order RECA_WEIBULL allowing the identification of the parameters $m$ and $\sigma_u$ model of Weibull.

The identification is carried out using a database made up of 45 tests, all carried out on smooth cylindrical test-tubes at three different temperatures, $-150^\circ C$, $-100^\circ C$ and $-50^\circ C$. This database is obtained by random pulling of a representative sample of the statistical law of Weibull corresponding to values of $m$ and $\sigma_u$ fixed arbitrarily.
1 Problem of reference

1.1 Geometry

Each test is carried out on a smooth cylindrical test-tube. For obvious reasons of symmetries, an axisymmetric modeling 2D of the quarter of the structure is sufficient.

![Diagram showing geometry](image)

Rayon de l'éprouvette : $R = 68$ mm.
Demi-longueur de référence pour la mesure de l'elongation : $L_0 = 203.5$ mm

1.2 Properties of material

One describes the behavior of material studied by an elastoplastic law of Von Mises with linear isotropic work hardening, ‘VMIS\_ISOT\_LINE’. The deformations used in the relation of behavior are the linearized deformations.

![Diagram showing stress-strain curve](image)

The Poisson's ratio does not depend on the temperature, $\nu = 0.3$.
Values of the Young modulus $E$, tangent module $E_t$, and of the elastic limit are given in the following table:

<table>
<thead>
<tr>
<th>Temperature [$°C$]</th>
<th>–150</th>
<th>–100</th>
<th>–50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ [MPa]</td>
<td>200000</td>
<td>200000</td>
<td>200000</td>
</tr>
<tr>
<td>$\sigma_Y$ [MPa]</td>
<td>750</td>
<td>700</td>
<td>650</td>
</tr>
</tbody>
</table>

1.3 Boundary conditions and loadings

By referring to the figure of the §1.1 the boundary conditions and loadings are the following:

On the segment $BC$ ($Y = L_0$), imposed displacement following the direction $OY$:

$T[°C]$ | Displacement ($l-l_0$) with the rupture for a reference length $l_0$ of 203.5 mm [mm]

<table>
<thead>
<tr>
<th>$T[°C]$</th>
<th>10.68</th>
<th>28.78</th>
<th>30.31</th>
<th>31.66</th>
<th>32.53</th>
<th>33.90</th>
<th>34.38</th>
<th>35.82</th>
<th>36.69</th>
<th>37.37</th>
<th>37.49</th>
<th>38.45</th>
<th>39.77</th>
<th>44.39</th>
</tr>
</thead>
<tbody>
<tr>
<td>–50</td>
<td>10.68</td>
<td>28.78</td>
<td>30.31</td>
<td>31.66</td>
<td>32.53</td>
<td>33.90</td>
<td>34.38</td>
<td>35.82</td>
<td>36.69</td>
<td>37.37</td>
<td>37.49</td>
<td>38.45</td>
<td>39.77</td>
<td>44.39</td>
</tr>
<tr>
<td>–100</td>
<td>20.57</td>
<td>21.68</td>
<td>23.32</td>
<td>24.37</td>
<td>24.66</td>
<td>25.59</td>
<td>25.84</td>
<td>27.51</td>
<td>28.44</td>
<td>29.30</td>
<td>29.68</td>
<td>30.16</td>
<td>30.18</td>
<td>30.20</td>
</tr>
</tbody>
</table>

The results for each temperature are classified by ascending order

On the segment $OA$ ($Y = 0$) displacements blocked according to the direction $OY$.

On the segment $OB$ ($X = 0$) displacements blocked according to the direction $OX$.

1.4 Initial conditions

Worthless constraints and deformations.
2 Reference solution

2.1 Method of calculating

No calculation is necessary to obtain the reference solution. Values \( m \) and \( \sigma_u \) (\( M \) and SIGM_REFE in the option WEIBULL of DEFI_MATERIAU) that one seeks to identify with Code_Aster are known and allow to generate the base of the experimental data. Thus, the elongations with rupture are in the following way given:

For each couple \( m \) and \( \sigma_u \) associated with a temperature of test, a sample of 15 values of constraint of Weibull to the rupture were determined by random pulling taking into account the following statistical law:

\[
P_f(\sigma_u) = 1 - \exp\left(-\frac{\sigma_w}{\sigma_u}\frac{m}{w}\right)
\]

The constraint of Weibull is defined by:

\[
\sigma_w = \sqrt{\sum (\sigma'_i)^m \frac{V_i}{V_0}}
\]

The summation relates to volumes of matter \( V_i \) plasticized, \( \sigma'_i \) indicating the maximum principal constraint in each one of these volumes (volume \( V_0 \) (VOLU_REFE in the option WEIBULL of DEFI_MATERIAU) is equal to \( 50 \mu m^3 \)).

In the case of a request in simple traction with the assumption of the small deformations, the constraint of Weibull, \( \sigma_W \), expresses itself according to the elongation with the rupture \( (l-l_0)/l_0 \), according to:

\[
\sigma_W = E \left( \frac{l-l_0}{l_0} \right) + \frac{E}{\nu} \sigma_u \frac{V}{V_0}
\]

One thus deduces from this expression and preceding random pulling the values of the lengthenings with rupture deferred in the table of \( \S 1.3 \).

2.2 Sizes and results of reference

Reference variables of \( m \) and \( \sigma_u \) used to create the bases of experimental tests are the following ones:

<table>
<thead>
<tr>
<th>Temperature [°C]</th>
<th>-50</th>
<th>-100</th>
<th>-150</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>24</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>( \sigma_u ) [MPa]</td>
<td>2800</td>
<td>2700</td>
<td>2600</td>
</tr>
</tbody>
</table>

2.3 Uncertainties on the solution

Uncertainty on the solution cannot be given in a precise way. It can be rather high. Indeed, the values of reference can be found only if one considers experimental populations made up of an infinite number of samples.
3 Modeling A

3.1 Characteristics of the grid

Many nodes: 149
Many meshes and types: 40 elements QUAD8

3.2 Sizes tested and results

Identification of one \( m \) commun run at the three experimental bases and of one \( \sigma_u \) by base.

<table>
<thead>
<tr>
<th>Temperature ( ^\circ \text{C} )</th>
<th>Reference ( m )</th>
<th>( \sigma_u ) [MPa]</th>
<th>Code_Aster ( m )</th>
<th>( \sigma_u ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>– 50</td>
<td>24</td>
<td>2800</td>
<td>26.7</td>
<td>2536</td>
</tr>
<tr>
<td>– 100</td>
<td>24</td>
<td>2700</td>
<td>26.7</td>
<td>2428</td>
</tr>
<tr>
<td>– 150</td>
<td>24</td>
<td>2600</td>
<td>26.7</td>
<td>2372</td>
</tr>
</tbody>
</table>

3.3 Remarks

Although the variation enters the values of \( (m, \sigma_u) \) obtained with RECA_WEIBULL and their values of reference remains considerable, it is in conformity with the result sought taking into account the relatively low number of samples used for retiming (15 per temperature). To obtain the values of reference it would be necessary considerably to increase the number of samples per temperature (Warnings: The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience. Copyright 2020 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)
$N > 1000$). The noted variation remains however reasonable (about 10%). In addition, growth of $\sigma_w$ according to the temperature is respected.

4 **Summary of the results**

Results got by *Code_Aster* show that the automatic procedure of chock of the parameters of the models of Weibull functions and gives coherent results with the expected theoretical results.