SSNA106 - Subjected hollow roll with a behavior thermoviscoelastic

Summary:

This CAS-test makes it possible to validate the law of LEMAÎTRE established in Code_Aster in the case of linear behavior thermoviscoelastic. The found results are compared with an analytical solution.
1 Problem of reference

1.1 Geometry

![Diagram of cylinder](image)

\[ R_0 \quad 1 \text{ m} \]
\[ R_1 \quad 2 \text{ m} \]

1.2 Properties of materials

Young modulus: \( E = 1 \text{ MPa} \)

Poisson's ratio: \( \nu = 0.3 \)

Dilation coefficient: \( \alpha = 0.7 \)

Law of LEMAITRE:

\[
g(\sigma, \lambda, T) = \left( \frac{1}{K} \frac{\sigma}{\lambda^m} \right)^n \quad \text{with} \quad \frac{1}{K} = 1, \quad \frac{1}{m} = 0, \quad n = 1
\]

1.3 Boundary conditions and loading

Boundary conditions:

The cylinder is blocked in \( DY \) on the sides \([AB]\) and \([CD]\).

Loading:

The cylinder is subjected to a field of temperature \( T(r, t) = t r^2 \).
2 Reference solutions

2.1 Method of calculating used for the reference solutions

The whole of this demonstration can be read with more details in the document [bib1].

In the case of a linear viscoelastic isotropic material, one can describe the behavior in the course of time using two functions \( I(t) \) and \( K(t) \) so that strains and stresses can be written:

\[
\varepsilon(t) = (I + K) * \frac{d\sigma(t)}{dt} - K * \frac{d(Tr(\sigma(t)))}{dt} I_3 + \alpha dT(r,t) I_3
\]

where \( I_3 \) indicate the matrix identity of row 3

and \( * \) the product of convolution:

\[
(f * g)(t) = \int f(t - \tau) g(\tau) d\tau
\]

The thermoelastic problem are equivalent, via the transform of Laplace is:

\[
\begin{align*}
\varepsilon^+ & = (I^+ + K^+) \sigma^+ - K^+ Tr(\sigma^+) I_3 + \frac{\alpha r^2}{p} I_3 \\
\sigma^+ & = \frac{d\sigma^+}{dr} = \frac{1}{r} \left( \sigma^+ - \sigma_r^+ \right) \\
\varepsilon_r^+ & = 0 \\
\varepsilon_\theta^+ & = 0
\end{align*}
\]

By eliminating the sign "+":

\[
\begin{align*}
(I + K) \sigma_r - K(\sigma_r + \sigma_\theta + \sigma_z) + \frac{\alpha r^2}{p} & = 0 \\
(I + K) \sigma_\theta - K(\sigma_r + \sigma_\theta + \sigma_z) + \frac{\alpha r^2}{p} & = (I + K) \sigma_r - \frac{(I + K) K}{I} (\sigma_r + \sigma_\theta) + \frac{(I + K) \alpha r^2}{p}
\end{align*}
\]

maybe,

\[
\begin{align*}
\sigma_r & = \frac{K}{I} (\sigma_r + \sigma_\theta) - \frac{\alpha r^2}{p I} \\
(I + K) \sigma_\theta & = (I + K) \sigma_r - \frac{(I + K) K}{I} (\sigma_r + \sigma_\theta) + \frac{(I + K) \alpha r^2}{p}
\end{align*}
\]
\[(I + K)\sigma_0 + r_0^2 (I + K)\sigma_0 - \frac{(I + K)K}{I} (\sigma_r + \sigma_0) + \frac{(I + K) \alpha r^2}{I} \frac{I}{p} = (I + K)\sigma_r,\]

According to the equilibrium equation, one has \(\sigma_0 = r\sigma_r^2 + \sigma_r\), one obtains:

\[(I + K)\sigma_r^2 + r_0^2 (I + K)(r \sigma_r + \sigma_r) - \frac{(I + K)K}{I} (2\sigma_r + r \sigma_r^2) = 0,\]

\[2\sigma_r + r \sigma_r^2 = A + \frac{\alpha r^2}{4p(K - I)} \frac{I}{K - I},\]

what while integrating compared to R gives:

\[\sigma_r = \frac{A}{2} + \frac{B}{r^2} + \frac{\alpha r^2}{4p(K - I)},\]

boundary conditions \(\sigma_r(r_0) = \sigma_r(r_1) = 0\) give:

\[A = -\frac{\alpha}{2p(K - I)} (r_0^2 + r_1^2),\]

\[B = \frac{\alpha r_0^2}{4p(K - I)} (r_1 - r_0).\]

One thus has by taking again the initial notations:

\[\sigma_r^2 = \frac{\alpha}{4p(I^+ - K^+)} (r_0^2 + r_1^2 - r^2 - \frac{r_0^2 r_1^2}{r^2}),\]

\[\sigma_0^2 = \frac{\alpha}{4p(I^+ - K^+)} (r_0^2 + r_1^2 - 3r^2 + \frac{r_0^2 r_1^2}{r^2}),\]

\[\sigma_r^2 = \frac{\alpha}{p(I^+ - K^+)} \frac{K^+ (r_0^2 + r_1^2)}{I^+} \frac{2}{2} - r^2.\]

Maybe, by taking the opposite transform,

\[\alpha \left(1 - e^{-r} \right) \frac{r_0^2}{r_1^2} + r_1^2 - r^2 - \frac{r_0^2 r_1^2}{r^2} \]

\[= 0 \]

\[\frac{\alpha}{2k} \left(1 - e^{-r} \right) \frac{r_0^2}{r_1^2} + r_1^2 - 3r^2 + \frac{r_0^2 r_1^2}{r^2} \]

\[= 0 \]

\[\frac{\alpha}{k} \left(1 - e^{-r} \right) \frac{r_0^2}{r_1^2} + r_1^2 - 2r^2 + \frac{r_0^2 r_1^2}{r^2} \]

\[= 0 \]

One from of deduced \(\varepsilon_r\) and \(w\):

\[w(r, t) = \frac{1 - 2\nu}{E} \frac{\alpha r^2}{4} \left[ \left(1 - e^{-r} \right) \frac{r_0^2}{r_1^2} + r_1^2 - \frac{r_0^2 r_1^2}{r^2} \right] + \left(1 - e^{-Ekt} \right) \frac{r_0^2 + r_1^2}{4} \frac{3Ekt}{(1 - 2\nu)} \frac{r_0^2 r_1^2}{r^2} + r_1^2 \]
2.2 Results of reference

Displacement $DX$ on the node $B$

2.3 Uncertainty on the solution

0% : analytical solution

2.4 Bibliographical references

PH. BONNIERES, two analytical solutions of axisymmetric problems in linear viscoelasticity and with unilateral contact, Note H1-71/8301
3 Modeling A

3.1 Characteristics of modeling

The problem is modelled in axisymetry

3.2 Characteristics of the grid

120 meshes QUAD4

3.3 Sizes tested and results

<table>
<thead>
<tr>
<th>Identification</th>
<th>Moments</th>
<th>Reference</th>
<th>Tolerance %</th>
</tr>
</thead>
<tbody>
<tr>
<td>DX (B)</td>
<td>0.24</td>
<td>1,110</td>
<td>0.1%</td>
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4 Summary of the results

Results calculated by Code_Aster are in agreement with the analytical solutions but very strongly depend on the refinement of the grid.