SSNA107 – Hollow roll in nonlinear viscoelasticity

Summary:

This CAS-test makes it possible to validate the law of LEMAITRE established in nonlinear Code_Aster in the case of viscoelastic behavior. The found results are compared with an analytical solution.
1 Problem of reference

1.1 Geometry

![Diagram of a cylinder with dimensions labeled](image)

Geometry of the cylinder \( m \):

\[
\begin{align*}
R_0 & = 1 \\
R_1 & = 1.02 \\
H & = 1
\end{align*}
\]

1.2 Properties of material

Rubber band

- Young modulus: \( E = 1.0 \times 10^6 \) Pa
- Poisson's ratio: \( \nu = 0.3 \)

LEMAITRE

\[
g(\sigma, \lambda, T) = \left( \frac{1}{K} \frac{\sigma}{1 - \lambda^m} \right)^n \quad \text{with} \quad n = 2; \quad \frac{1}{K} = 1; \quad \frac{1}{m} = 0
\]

CIN1_CHAB

\[
\begin{align*}
R_0 & = 0. \\
R_1 & = 0. \\
B & = 0. \\
C_0 & = 0. \\
K & = 0. \\
W & = 0. \\
G_0 & = 0.
\end{align*}
\]
\[ A_I = 0. \]
1.3 Boundary conditions and loadings

Imposed displacement (m):

Dimensioned $CD$

![Diagram of Dimensioned CD](image1)

Dimensioned $AB$

![Diagram of Dimensioned AB](image2)
2 Reference solution

2.1 Method of calculating used for the reference solutions

The whole of this demonstration can be read with more details in the document [1]

The tensor of constraints is written:

\[
\sigma = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & \sigma_z
\end{pmatrix}
\]

Because of the loading, one a:

\[
\begin{align*}
\varepsilon_z - \varepsilon_{vz} &= \frac{\sigma_z}{E} \\
\varepsilon_0 - \varepsilon_{v0} &= -\frac{\nu}{E} \sigma_z \\
\varepsilon_r - \varepsilon_{vr} &= -\frac{\nu}{E} \sigma_z \\
\end{align*}
\]

Thus

\[
\begin{align*}
\dot{\varepsilon}_z &= g(\sigma_z) \\
\dot{\varepsilon}_0 &= -\frac{1}{2} g(\sigma_z) \\
\dot{\varepsilon}_r &= -\frac{1}{2} g(\sigma_z)
\end{align*}
\]

If \( t \leq t_0 \), one has \( \varepsilon_z = \frac{\varepsilon_0}{t_0} t \),

That is to say \( a = \frac{\varepsilon_0}{t_0} \)

One obtains \( \varepsilon_z = a^2 t \)

While replacing, one finds:

\[
\dot{\varepsilon}_z = g(\left(a^2 t - \varepsilon_{vz}\right) E)
\]

One poses \( E = 1 \) and \( z = a^2 t - \varepsilon_{vz} \), one obtains: \( \dot{z} = a^2 - z^2 \)

While integrating with \( z(0) = 0 \) one obtains:
\[ z = a \tanh(at) \]

For \( t \leq t_0 \)
\[
\begin{align*}
\sigma_r &= \sigma_0 = 0 \\
\sigma_z &= a \tanh(at) \\
\varepsilon_r &= \varepsilon_0 = a \left( \left( \frac{1}{2} - \nu \right) \tanh(at) - \frac{1}{2} at \right) \\
\varepsilon_z &= a^2 t \\
w &= ar \left( \left( \frac{1}{2} - \nu \right) \tanh(at) - \frac{1}{2} at \right)
\end{align*}
\]

If \( t \geq t_0 \)
\[
\begin{align*}
\varepsilon_z &= a^2 t_0 \\
\varepsilon_{\nu z} &= g (a^2 t_0 - \varepsilon_{\nu z}) = (a^2 t_0 - \varepsilon_{\nu z})^2
\end{align*}
\]

What gives while integrating:
\[
\varepsilon_{\nu z} = a^2 t_0 - \frac{1}{\left( \frac{1}{2} - \nu \right) \tanh(at_0) + t - t_0} + \frac{1}{a \tanh(at_0) + t - t_0}
\]

One thus has with the final one
\[
\begin{align*}
\sigma_r &= \sigma_0 = 0 \\
\sigma_z &= \frac{1}{a \tanh(at_0) + t - t_0} \\
\varepsilon_r &= \varepsilon_0 = \left( \left( \frac{1}{2} - \nu \right) \frac{1}{a \tanh(at_0) + t - t_0} - \frac{1}{2} a^2 t_0 \right) \\
\varepsilon_z &= a^2 t_0 \\
w &= ar \left[ \left( \frac{1}{2} - \nu \right) \frac{1}{a \tanh(at_0) + t - t_0} - \frac{1}{2} a^2 t_0 \right]
\end{align*}
\]

### 2.2 Reference variables

- Displacement \( DX \) with the node \( B \)
• Constraints SIXX, SIYY and SIZZ with the node B

2.3 Results of reference

<table>
<thead>
<tr>
<th>Size</th>
<th>Not</th>
<th>Moments</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>DX</td>
<td>B</td>
<td>4</td>
<td>$-0.2109$</td>
</tr>
<tr>
<td>SIXX</td>
<td>B</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>SIYY</td>
<td>B</td>
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</tr>
<tr>
<td>SIZZ</td>
<td>B</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

2.4 Uncertainty on the solution

Analytical solution

2.5 Bibliographical references

[1] PH. BONNIERES, Mr. ZIDI: Introduction of viscoplasticity into the module of thermomechanics of Cyrano3: Principle, description and validation, Note HI-71/8334.
3 Modeling A

3.1 Characteristics of modeling A

Modeling AXIS
Viscoelastic relation of behavior of LEMAITRE

3.2 Characteristics of the grid

Many nodes 12
Many meshes 17
That is to say:

\[
\begin{align*}
\text{SEG2} & : 12 \\
\text{QUAD4} & : 5
\end{align*}
\]

Groups of nodes:
\[A, B, C, D\]

Groups of meshes:
\[
\begin{align*}
\text{MAIL} & : \text{surface} \quad ABCD \\
\text{DAB} & : \text{segment} \quad AB \\
\text{DBC} & : \text{segment} \quad BC \\
\text{DCD} & : \text{segment} \quad CD \\
\text{DDA} & : \text{segment} \quad DA
\end{align*}
\]

3.3 Sizes tested and results

<table>
<thead>
<tr>
<th>Size</th>
<th>Not</th>
<th>Moments</th>
<th>Reference</th>
<th>Aster</th>
<th>Variation %</th>
</tr>
</thead>
<tbody>
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<td>−0.2109</td>
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<tr>
<td>SIXX</td>
<td>B</td>
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<td>B</td>
<td>4</td>
<td>0.2616</td>
<td>0.2616</td>
<td>0.002%</td>
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<table>
<thead>
<tr>
<th>SIZZ</th>
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<th>4</th>
<th>0.</th>
<th>4,82E-9</th>
<th>-</th>
</tr>
</thead>
</table>

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4 Modeling B

4.1 Characteristics of modeling B

Modeling `AXIS`

Viscoelastic relation of behavior of `VISC_CIN1_CHAB`

![Diagram showing the modeling of a cylinder with nodes and meshes]

4.2 Characteristics of the grid

- Many nodes: 12
- Many meshes: 17

That is to say:

- `SEG2`: 12
- `QUAD4`: 5

Groups of nodes:

- `A`, `B`, `C`, `D`

Groups of meshes:

- `MAIL`: surface `ABCD`
- `DAB`: segment `AB`
- `DBC`: segment `BC`
- `DCD`: segment `CD`
- `DDA`: segment `DA`

4.3 Sizes tested and results

<table>
<thead>
<tr>
<th>Size</th>
<th>Not</th>
<th>Moments</th>
<th>Reference</th>
<th>Aster</th>
<th>Variation %</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-0.2109</td>
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<td>4</td>
<td>0</td>
<td>1.55E-10</td>
<td>-</td>
</tr>
<tr>
<td><code>SIYY</code></td>
<td><code>B</code></td>
<td>4</td>
<td>0.2616</td>
<td>0.2616</td>
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</tr>
</tbody>
</table>

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| SIZZ | B  | 4  | 0.  | 1.55E-10 | -   |
5 Modeling C

5.1 Characteristics of modeling A

Modeling COQUE_AXIS
Viscoelastic relation of behavior of LEMAITRE

5.2 Characteristics of the grid

Many nodes 5
Many meshes 2 SEG3

Groups of nodes: B, H

5.3 Sizes tested and results

<table>
<thead>
<tr>
<th>Size</th>
<th>Not</th>
<th>Moments</th>
<th>Reference</th>
<th>Aster</th>
<th>Variation %</th>
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<td>B</td>
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<tr>
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<td>B</td>
<td>4</td>
<td>0.362E-9</td>
<td>4.82E-9</td>
<td>-</td>
</tr>
</tbody>
</table>
6 Summary of the results

The results calculated by Code_Aster are in excellent agreement with the analytical solutions.