SSNP148 - Calculation of the stress intensity factor by the regularization of the constraints with ENDO_HETEROGENE

Summary:

This test calculates the threshold of propagation of a central crack in a plate infinite length requested by a constraint with the infinite one. In this case an analytical solution exists since one can calculate the stress intensity factor. This test gives a comparison between this analytical solution and the value of the factor of intensity of the constraints calculated starting from the regularized constraints (modeling D_PLAN_GRAD_SIGM ). The objective of this approach is to validate the method of calculating (modeling D_PLAN_GRAD_SIGM and law ENDO_HETEROGENE. ) for cases where the characteristic length is low in front of the size of the crack and the structure.
1 Problem of reference

1.1 Geometry

A field length is represented \( l = 6000 \text{mm} \), height \( h = 1000 \text{mm} \) containing a vertical initial crack length \( 2a \). By condition of symmetry, one models only half of the field (Illustration 1).

![Illustration 1: Geometry of the case test](image)

1.2 Properties of materials

Parameters of elasticity:
- Young modulus \( E_1 = 20 \times 10^9 \text{MPa} \)
- Poisson's ratio \( \nu_1 = 0.25 \)

Parameters of the law \textit{ENDO\_HETEROGENE}:
- Yield stress \( \sigma_y = 10^{18} \text{Pa} \)
- Module of Weibull \( m = 2 \)
- Tenacity \( K_c = 1000 \text{MPa.m}^{1/2} \)
- Thickness of the sample \( e_p = 1 \text{m} \)
- Seed \( GR = 121 \)

Parameter of the nonlocal model:
- Characteristic length \( l_c = 0.02 \text{m} \)

1.3 Boundary conditions and loading

One blocks vertical displacements on the lower edge of the model as well as horizontal displacements on the left edge and one imposes on the flat rim a horizontal constraint. The central crack is represented by a vertical band of broken finite elements (i.e., \( d = 1 \)).

The pressure applied to the edge right-hand side varies from 0 with 10 MPa during the i.e., computing time. 1 s.

![Illustration 2: Diagram of the boundary conditions](image)
2 Reference solution

For a central crack length $2a$ in a bar thickness $2b$ and infinite length requested by a constraint $\sigma_\infty$ with infinite, one can express the factor of intensity of the analytical constraints by the following equation:

$$K_{Ia} \approx \sqrt{\frac{\pi a}{\cos\left(\frac{\pi a}{2b}\right)}}$$

Since in the case treated the crack is solicited in mode $I$ one can introduce for a length characteristic given an equivalent to the factor of intensity of the constraints [1]:

$$K_{IIc} = \frac{5\pi}{6\Gamma} \cdot \frac{\sigma_{lp}}{\pi l_c}$$

with $\sigma_{lp}$ the maximum principal constraint regularized at a peak of crack. In order to compare the digital results with the analytical solution the parameter is introduced $RKI = K_{IIc}/K_{Ia}$.

The 2 equations above were introduced into the command file of the case test by the means of handling of tables. One extracts the value from the constraint regularized to the forefront of the crack. Values of factor of intensity of the constraints analytical and digital are calculated in the command file. One calculates then the relationship between the two values ($RKI$). This report was tested via the order TEST_TABLE.

3 Bibliographical references


4 Modeling A

4.1 Characteristics of modeling

The higher half of the field is with a grid in triangular elements with 6 nodes. The grid comprises 16602 triangles TRIA6 and 223 SEG2.

The size of the central crack, has, is equal $0,3571 \, m$. 1 time of $1 \, s$ is modelled.

4.2 Results

One traces on the figures 3 and 4 respectively horizontal displacements $DX$ and the criterion of damage (variable internal $V_1$) at the end of $1 \, s$.

Illustration 3: Horizontal displacements $DX$, $t=1\,s$
4.3 Values tested

One tests the value of intensity of constraint via TEST_FUNCTION.

<table>
<thead>
<tr>
<th>Function</th>
<th>Moment</th>
<th>Value of reference</th>
<th>Tolerance (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$RKI = K_{II} / K_{Ia}$</td>
<td>1</td>
<td>1</td>
<td>1.E-2</td>
</tr>
</tbody>
</table>
5 Summary of the results

This test allows to calculate the threshold of propagation of a central crack in a plate infinite length requested by a constraint with the infinite one. We can compare the results got with an analytical solution: the results correspond. This test thus makes it possible to validate the model D_PLAN_GRAD_SIGM and the law ENDO_HETEROGENE.