SSNP306 - Validation of the criterion of buckling per selective research of the eigenvalues

Summary:
This test allows the validation of the generalization of the criterion of buckling (loss of unicity of the solution, presence of junction) for the models in mixed formulations. The new criterion which is written as a problem with the eigenvalues under-constraints is validated for the modeling of damage not-room GRAD_VARI. The smallest modes of the formulation are compared GRAD_VARI and those of local modeling in the case of a homogeneous study.
1 Problem of reference

1.1 Tally theoretical

The criterion of usual buckling results in a checking of sign of the smallest eigenvalue of the tangent matrix \( K \delta u = \mu \delta u \). If \( \mu \leq 0 \) the solution of the mechanical problem is not single any more.

For the mixed formulations, the clean modes associated with the presence of coefficients of Lagrange must be eliminated. This brings back for us towards a problem to the clean modes under constraint.

\[
K \begin{pmatrix} \delta u \\ \delta \lambda \end{pmatrix} = \mu \begin{pmatrix} \delta u \\ 0 \end{pmatrix}
\]

is the new criterion of buckling for a mixed modeling.

In the case GRAD_VARI the criterion is even more restrictive, because it is also necessary to take into account the law of behavior to obtain physical modes. This must be able to spread with other types of modelings. The criterion is written then like a problem with the eigenvalues generalized:

\[
K \begin{pmatrix} \delta u \\ \delta \alpha \\ \delta \lambda \end{pmatrix} = \mu \begin{pmatrix} \delta u \\ 0 \\ 0 \end{pmatrix}
\]

new criterion of buckling for a modeling GRAD_VARI.

That provides us two constraints of equalities in the research of the mode in displacement. The first enables us to check the law of behavior of material and the second the equality of the disturbances of the damage to the nodes with that calculated locally at the points of Gauss.

We validate the new criterion of buckling by making a comparison between the smallest modes of the formulation GRAD_VARI and those of local modeling in the case of a homogeneous study.

Notice:
Technically in order to arrive the new orders there RIGI_GEOM and DDL_EXCLUS were added in the operator CRIT_STAB. One uses them in this precise case in the following way:

\[
\text{CRIT\_STAB} = \_F \left( \text{RIGI\_GEOM} = \text{‘NOT’}, \# \text{ Necessary to call then} \text{ DDL\_EXCLUS} = \left( \text{‘VARI’}, \text{‘LAG\_GV’} \right) \right)
\]

1.2 Geometry

Figure 1: Representation of the problem in two quadratic meshes
1.3 Properties of material

Law of damage: material ENDO_FRAGILE

Characteristics rubber band:
- \( E = 3 \times 10^4 \text{ Pa} \)
- \( \nu = 0.25 \)

Characteristics related to the law of damage:
- Elastic limit: \( \sigma^Y = 3.0 \text{ Pa} \)
- Slope of softening: \( E^T = -1.95 \times 10^3 \text{ Pa} \)
- Not-local characteristics: \( L_c = 1.0 \); \( r = 100. \)

1.4 Boundary conditions and loadings

Embedding: Worthless imposed displacements \( DY = 0. \) on horizontal bottom stops ( \( y = 0. \) ) and \( DX = 0. \) on the node far left ( \( x = y = 0. \) ). See figure1.

Loading 1: Imposed displacement \( U_1 \) on horizontal top stops ( \( y = 1. \) ):
\[
DY = 2.10^{-6} t
\]

Loading 2: Imposed displacement \( U_2 \) on all the nodes of coordinates \( y = 0.5 \):
\[
DY = 1.10^{-6} t
\]

2 Reference solution

The same study is made in room and nonroom. One compares the smallest eigenvalues of the tangent operator calculated with the criterion of standard buckling in room (which one will take as reference) and with the new criterion in nonroom. Modeling D_PLAN is used locally, modeling D_PLAN_GRAD_VARI is used in nonroom. The loading is carried out over one duration of 100 pas de time, of kind to visualize the passage by a state of damage not-no one of the structure. The successful test is considered, if the relative variation does not exceed 0.01%.
3 Modeling A

3.1 Characteristics of the grid

The grid consists of 2 TRIA6 as presented of Figure 1.

3.2 Sizes tested and results

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Table 1: Comparison of eigenvalues in room and not-room
4 Summaries of the results

One finds identical results in modeling local and nonlocal, which validates the introduction of the new criterion of buckling for the mixed problems.