SSNP307 - Validation of modeling GVNO and of the law of behavior ENDO_CARRE in D_PLAN

Summary:
This test allows the validation of modeling GVNO in 2D, for modeling D_PLAN_GVNO, which makes it possible to carry out the calculations of damage regularized by the gradient of the damage, by taking into account only degrees of freedom of displacement and damage to the nodes. The resolution of the criterion is total, unlike modeling GRAD_VARI who carries out a local resolution, points of Gauss by points of Gauss. One validates simultaneously the law of behavior ENDO_CARRE, of quadratic formulation in damage, which is for the moment the law that one can use with modeling GVNO.
1 Problem of reference

1.1 Tally theoretical

The unknown factors of the problem are the degrees of freedom of nodal displacement and damage. It is then a question of minimizing an energy of the form:

$$\phi[u, \alpha] = \frac{1}{2} A[d] E c^2 + \psi[d] + \frac{c}{2} \nabla \alpha \cdot \nabla \alpha$$

Where $E$ is the Young modulus of material, $A[d]$ the function of rigidity, $\psi[d]$ dissipation and $c$ the nonlocal coefficient.

In the case of the law ENDO_CARRE:

$$A[d] = (1 - d)^2 \quad \text{and} \quad \psi[d] = \frac{\sigma^2}{E} d$$

The criterion corresponding to the law ENDO_CARRE, for a homogeneous solution ($\nabla \alpha = 0$), is thus written:

$$d = 1 - \left( \frac{W_y}{W_{el}} \right)$$

Where $W_{el}$ is the elastic deformation energy and:

$$W_y = \frac{\sigma_y^2}{2E}$$

1.2 Geometry

A square on side is considered $L = 1 \text{ m}$.

![Figure 1: Representation of the problem](image-url)
1.3 Properties of material

1.3.1 Law of damage: material ENDO_CARRE

Characteristics rubber bands:
\[ E = 1 \text{ Pa} \]
\[ \nu = 0. \]
Characteristics related to the law of damage:
Elastic limit:
\[ SY = 0.01 \text{ Pa} \]
Not-local characteristics:
\[ c = 1.0 \text{ N} \]

1.4 Boundary conditions and loadings

**Embedding**: Imposed displacements no one \( Dy = 0 \text{ m} \) on stop horizontal bottom and top ( \( y = 0. \) and \( y = 1. \)) and \( Dx = 0 \text{ m} \) on stops left ( \( x = 0. \)). See Figure 1.

**Loading 1**: Imposed displacement \( U_1 \) on vertical right-hand side stops:
At the moment \( t_1 \): \( Dx = 0.01 \text{ m} \)
At the moment \( t_2 \): \( Dx = 0.0125 \text{ m} \)
At the moment \( t_3 \): \( Dx = 0.02 \text{ m} \)

2 Reference solution

The imposed loading enables us to obtain a homogeneous solution, equivalent to a bar length \( L \) subjected to a uniaxial loading of traction. One can then express elastic energy in the following way:

\[ W_{el} = \frac{E}{2} \left( \frac{dx}{L} \right)^2 \]

One from of deduced analytically the values of damage associated with the moments \( t_1, t_2 \) and \( t_3 \) starting from the formula:

\[ d = 1 - \left( \frac{\sigma_y L}{E dx} \right)^2 \]

That is to say: \( d_1 = 0. \), \( d_2 = 0.36 \) and \( d_3 = 0.75 \). It is considered whereas the test is checked if Newton us reference well the same values of damage, with a precision of \( 10^{-6} \).
3. Modeling

3.1 Characteristics of modeling

Modeling is used D_PLAN_GVNO.

3.2 Characteristics of the grid

The grid contains 25 elements QUAD8.

3.3 Sizes tested and Results

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<th>VALE_REF</th>
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<td>0.75</td>
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</tr>
</tbody>
</table>

Table 1: Comparison of eigenvalues in room and not-room
4 Summary of the results

Convergence was checked starting from the criterion `RESI_REFE_RELA`. This case test thus allows a simultaneous validation of `D_PLAN_GVNO` and of the developments related to `RESI_REFE_RELA`, which adimensionne the residues starting from values of reference declared in the command file (one declares `SIGM_REFE = SY`).

We find the results of references well, which validates modeling `GVNO` and the law of behavior `ENDO_CARRE` in 2D, for modeling `D_PLAN_GVNO` and what also validates `RESI_REFE_RELA` for `GVNO`.