SSNV102 - Tensile test shearing with the model of TAHERI

Summary:

The problem is quasi-static nonlinear in mechanics of the structures.

One analyzes the response of an element of volume to a loading in traction-shearing, carried out in such way that imposes a state of uniform stress-strain in the element.

There are 2 modelings: one in 3D voluminal and another in plane constraints 2D.

One validates by this test the digital integration of the elastoplastic model of behavior of Saïd Taheri.
1 Problem of reference

1.1 Geometry

Face $YZ: (1,3,5,7)$
Face $XZ: (3,4,7,8)$
Face $1YZ: (2,4,6,8)$
Face $1XZ: (1,2,5,6)$

\[\beta(t) \tau_0 \text{ imposed shearing}\]
\[\beta(t) \sigma_0 \text{ imposed pressure}\]
\[\beta(t) \text{ function of effort}\]

1.2 Material properties

isotropic elasticity
\[E = 200000 \text{ MPa}\]
\[\nu = 0.3\]
Taheri plasticity
\[C_{inf} = 0.065 \text{ MPa}\]
\[C_1 = -0.012 \text{ MPa}\]
\[m = 0.1\]
\[s = 450\]
\[b = 30\]
\[a = 312\]
\[\alpha = 0.3\]
\[R_o = 72\]

1.3 Boundary conditions and loadings

\[N04\]
\[dx = dy = 0\]
\[Face YZ:\] \[FX = FY = -F(t)\]
\[N08\]
\[dx = dy = dz = 0\]
\[Face XZ:\] \[FX = -F(t)\]
\[N02, N06\]
\[dx = 0\]
\[Face 1YZ:\] \[FY = F(t)\]
\[Face 1XZ:\] \[FX = F(t)\]

\[F(N)\]
88.
0.
1.
\[t(s)\]

1.4 Initial conditions

Worthless constraints and deformations with $t = 0$.
2 Reference solution

2.1 Method of calculating used for the reference solution

One integrates the following system numerically enters \( t = 0 \) and \( t = 1 \).

(Nonlinear ordinary Differential connection of 6 equations to 6 unknown factors solved using library NAG by a 'Backward difference method')

\[
\begin{align*}
\dot{\varepsilon} - \dot{\varepsilon} p &= \dot{\beta} \frac{\sigma_p}{E} \\
\dot{\gamma} - \dot{\gamma} p &= \dot{\beta} \frac{\gamma_o}{2 \mu} \\
\dot{\varepsilon} p - \dot{p} \frac{\partial F}{\partial \varepsilon} &= 0 \\
\dot{\gamma} p - \dot{p} \frac{\partial F}{\partial \gamma} &= 0 \\
\frac{3}{2} \frac{\partial F}{\partial \varepsilon} \left( Kx + C S \frac{\partial F}{\partial \varepsilon} \right) - 2 \frac{\partial F}{\partial \gamma} \left( Kx + C S \frac{\partial F}{\partial \varepsilon} \right) - HR - a D \alpha Z^{(\alpha-2)} \\
\left( \varepsilon_p - \varepsilon_p^n \right) \frac{\partial F}{\partial \varepsilon} + \frac{4}{3} \left( \gamma_p - \gamma_p^n \right) \frac{\partial F}{\partial \gamma} \left( \varepsilon_p - \varepsilon_p^n \right) + \frac{3}{2} \frac{\partial F}{\partial \varepsilon} \left( Qx + C_S \varepsilon_p^n \right) + 2 \frac{\partial F}{\partial \gamma} \left( Qx + C_S \varepsilon_p^n \right) + JR \dot{\gamma} p
\end{align*}
\]

\( q \) et \( q \) de la référence

avec

\[
\begin{align*}
u &= b \frac{1 - \sigma_p}{S} \\
v &= C \infty - C \\
w &= 1 - \frac{D}{D} \\
 X &= C \left( \varepsilon_p - \sigma_p \varepsilon_p^n \right) \\
 Y &= C \left( \gamma_p - \sigma_p \gamma_p^n \right) \\
 D &= 1 - me^{\gamma_p^n} \\
 C &= C \infty - C_1 e^{\gamma_p^n} \\
 K &= v u \\
 Q &= \frac{b p}{s} \\
 H &= w u \\
 J &= \frac{b p}{s}
\end{align*}
\]

et

\[
\begin{align*}
 U &= \left( \frac{9}{4} x^2 + 3 y^2 \right)^{1/2} \\
 R &= D \left( a Z^{\alpha} + r \right) \\
 Z &= \left( \varepsilon_p - \varepsilon_p^n \right)^2 + \frac{4}{3} \left( \gamma_p - \gamma_p^n \right)^2 \right)^{1/2}
\end{align*}
\]
with the initial conditions:

\[
\begin{align*}
\beta(0) &= \frac{R(0)}{\left(\sigma_o^2 + 3 \tau_o^2\right)^{1/2}} \\
\epsilon(0) &= \beta(0) \frac{\sigma_o}{E} \\
\mu(0) &= \beta(0) \frac{\tau_o}{2\mu} \\
p(0) &= \epsilon_p(0) = \epsilon^n = 0 \\
R(0) &= (1 - m) r_o = \sigma_p(0)
\end{align*}
\]

d'où \( \sigma(t = 1) = \begin{bmatrix} 88 & 88 & 0 \\ 88 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \)

2.2 Results of reference

Values of \( \epsilon, Y, \epsilon_p, Y_p, \mu \) and \( \sigma_p \) with the nodes with \( t = 1 \) s.

2.3 Uncertainty on the solution

Uncertainty of library NAG.

2.4 Bibliographical references

1) User's manual library NAG on CRAY.

2) S. ANDRIEUX - P. SCHOENBERGER - S. TAHERI: With three dimensional cyclic constitutive law for metals with has semi-discrete memory variable - HI - 71/8147 (1992)

3) P. GEYER - J.M. PROIX - P. SCHOENBERGER - S. TAHERI: Modeling of the phenomena of progressive deformation - Collection of the internal notes of DER 93NB00153
3 Modeling A

3.1 Characteristics of modeling

Modeling in plane constraints $2D, C_{\text{PLAN}}$

3.2 Characteristics of the grid

Square quadrangle with 4 nodes in plane constraints with:

- $\text{largeur} = 1\,\text{mm}$,
- $\text{épaisseur} = 1\,\text{mm}$.

3.3 Sizes tested and results

<table>
<thead>
<tr>
<th>Identification</th>
<th>Reference</th>
<th>Test</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon$ on NO4 with $t=1s$</td>
<td>0.01721</td>
<td>ANALYTICAL</td>
<td>2.5%</td>
</tr>
<tr>
<td>$\gamma$ on NO4 with $t=1s$</td>
<td>0.02573</td>
<td>ANALYTICAL</td>
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</tr>
<tr>
<td>$\varepsilon_p$ on MA1, not of Gauss 4 with $t=1s$</td>
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<td>$\gamma_p$ on MA1, not of Gauss 4 with $t=1s$</td>
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4 Modeling B

4.1 Characteristics of modeling

Modeling 3D:

Cubic elementary with a grid using a hexahedron with 8 nodes.

4.2 Characteristics of the grid

1 mesh HEXA8, width side \( a = 1 \text{ mm} \).

4.3 Sizes tested and results

4.3.1 Values tested

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4.3.2 Remarks

The reduction in the tolerance on total convergence in displacement does not bring significant profit in precision.

The number of increments of load (6) led to a satisfactory precision of the result.
5 Summary of the results

Good precision at the time of the comparison with NAG in spite of some difficulties of convergence with this mathematical library.