SSNV135 - Triaxial compression test drained with model CJS (level 1)

Summary

This test makes it possible to validate level 1 of model CJS. It is about a triaxial compression test in drained condition. Three levels of containment are simulated: 100, 200, then 400 kPa.

By reason of symmetry, one is interested only in the eighth of a sample subjected to a triaxial compression test.

The results got with model CJS1 are compared with the analytical solution.
# Problem of reference

## 1.1 Geometry

![Geometry Diagram]

Coordinates of the points (in meters):

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>0.0</td>
<td>0.0</td>
<td>0.5</td>
</tr>
<tr>
<td>y</td>
<td>0.0</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>z</td>
<td>0.0</td>
<td>0.0</td>
<td>0.5</td>
</tr>
</tbody>
</table>

## 1.2 Material property

\( E = 22,4 \times 10^3 \text{kPa} \)

\( \nu = 0.3 \)

Parameters CJS1: \( \beta = -0.03 \quad \gamma = 0.82 \quad R_m = 0.289 \quad P_a = -100 \text{kPa} \)

## 1.3 Initial conditions, boundary conditions, and loading

**Phase 1:**

One brings the sample in a homogeneous state: \( \sigma_{xx}^0 = \sigma_{yy}^0 = \sigma_{zz}^0 \), by imposing the corresponding confining pressure on the front, side right-hand side and higher faces. Displacements are blocked on the faces postpones \( u_x = 0 \), side left \( u_y = 0 \) and lower \( u_z = 0 \).

**Phase 2:**

One maintains displacements blocked on the faces postpones \( u_x = 0 \), side left \( u_y = 0 \) and lower \( u_z = 0 \), as well as the confining pressure on the front faces and side right-hand side. One applies a displacement imposed to the higher face: \( u_z(t) \), in order to obtain a deformation \( \varepsilon_{zz} = -20\% \) (counted starting from the beginning of phase 2).
2 Reference solution

2.1 Development of the analytical solution for CJS1

One has permanently:

\[ \sigma_{xx} = \sigma_{yy} = \sigma_{xx}^0 \]

where \( \sigma_{xx}^0 = C_{xx}^{\text{te}} \) represent the confining pressure.

Remain to determine \( \sigma_{zz} \).

Elastic phase:

By writing the elastic law simply, one a:

\[
\begin{align*}
\sigma_{xx}^0 &= \sigma_{xx}^0 + \lambda \varepsilon_{zz} + (\lambda + 2\mu) \varepsilon_{xx} + \lambda \varepsilon_{xx} \\
\sigma_{zz}^0 &= \sigma_{zz}^0 + (\lambda + 2\mu) \varepsilon_{zz} + 2\lambda \varepsilon_{xx}
\end{align*}
\]

where here \( \lambda \) and \( \mu \) are the coefficients of Lamé.

While eliminating \( \varepsilon_{xx} \) between these two equations, one finds:

\[ \sigma_{zz} = \sigma_{zz}^0 + \frac{\mu (3\lambda + 2\mu)}{(\lambda + \mu)} \varepsilon_{zz} \]

Plastic phase:

One a:

\[ I_1 = \sigma_{zz}^0 + 2 \sigma_{xx}^0 \] where \( \sigma_{xx}^0 = C_{xx}^{\text{te}} \) represent the confining pressure.

One from of deduced for the components from the diverter \( S \):

\[
\begin{align*}
S_{zz} &= 2 \left[ \frac{1}{3} I_1 - \sigma_{xx}^0 \right] \quad \text{and} \quad S_{xx} = \sigma_{xx}^0 - \frac{1}{3} I_1 \\
S_{zz} &= \sqrt{6} \left( \sigma_{xx}^0 - \frac{1}{3} I_1 \right) \quad \text{and} \quad \det(S) = 2 \left[ \frac{1}{3} I_1 - \sigma_{xx}^0 \right]^3
\end{align*}
\]

that is to say:

\[ S_{zz} = \sqrt{6} \left( \sigma_{xx}^0 - \frac{1}{3} I_1 \right) \] and \[ \det(S) = 2 \left[ \frac{1}{3} I_1 - \sigma_{xx}^0 \right]^3 \]

Consequently:

\[ h(\theta_S) = (1 - \gamma)^{1/6} \]

In addition, when one reaches the criterion of the mechanism déviatoire:

\[ S_{zz} h(\theta_S) + R_m I_1 = 0 \]

from where the relation:

\[ I_1 = \frac{\sqrt{6} \sigma_{xx}^0}{\frac{2}{3} \frac{R_m}{(1 - \gamma)^{1/6}}} \]
and finally, one has for the vertical constraint:

\[ \sigma_{zz} = \frac{\sqrt{6} \sigma_{xx}^0 - 2 \sigma_{xx}^0}{\frac{2}{3} - \frac{R_m}{(1 - \gamma)^{1/6}}} \]

Moreover, one can calculate that the transition enters the states rubber band and perfectly plastic is done for an axial deformation equalizes with:

\[ \varepsilon_{zz} = \frac{\mu (3\lambda + 2\mu)}{\lambda + \mu} \left[ \frac{\sqrt{6} \sigma_{xx}^0 - 2 \sigma_{xx}^0}{\frac{2}{3} - \frac{R_m}{(1 - \gamma)^{1/6}}} \right] \]

### 2.2 Results of reference

Constraints \( \sigma_{xx}, \sigma_{yy}, \text{ and } \sigma_{zz} \) at the points \( A, B \text{ and } C \).

### 2.3 Uncertainty on the solution

Analytical solution for CJS1.
3 Modeling A

3.1 Characteristics of modeling

3D:

Cutting: 2 in height, in width and thickness.

Loading of phase 1:
Confining pressure: \( \sigma_{xx}^0 = \sigma_{yy}^0 = \sigma_{zz}^0 \) : successively \( -100 \text{kPa}, -200 \text{kPa} \) and \( -400 \text{kPa} \).

Level 1 of model CJS

3.2 Characteristic of the grid

Many nodes: 27
Many meshes and types: 8 HEXA8 and 24 QUA4

3.3 Values tested

For \( \sigma_{xx}^0 = \sigma_{yy}^0 = \sigma_{zz}^0 : -100 \text{kPa} \)

<table>
<thead>
<tr>
<th>Localization</th>
<th>Sequences number</th>
<th>axial deformation ( \epsilon_{zz} ) (%)</th>
<th>constraint (kPa)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not A, B and C</td>
<td>10</td>
<td>-0.8%</td>
<td>( \sigma_{zz} )</td>
<td>-100.0</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>-20.0%</td>
<td>( \sigma_{xx} )</td>
<td>-100.0</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>-0.8%</td>
<td>( \sigma_{yy} )</td>
<td>-100.0</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>-20.0%</td>
<td>( \sigma_{yy} )</td>
<td>-100.0</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>-0.8%</td>
<td>( \sigma_{zz} )</td>
<td>-279.2</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>-1.6%</td>
<td>( \sigma_{zz} )</td>
<td>367,159</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>-3.2%</td>
<td>( \sigma_{zz} )</td>
<td>367,159</td>
</tr>
</tbody>
</table>
Summary of the results

Values of Code_Aster are in triad with the values of the analytical solution of reference.