SSNV148 - Models of Weibull and Rice-Tracey in 3D and discharge

Summary:

This test of nonlinear quasi-static mechanics makes it possible to validate the models of Weibull and Rice and Tracey in 3D for nonmonotonous cases of mechanical loadings (cf. POST_ELEM).

At the temperature of $–50 \, ^\circ C$, a cylindrical test-tube smoothes is first of all deformed up to 10%. After having slightly discharged it, one maintains constant the level of deformation reaches while decreasing in a homogeneous way the temperature of the test-tube until $–150 \, ^\circ C$. With this new temperature, one applies an additional deformation to reach 15% on the whole. The probability of rupture per cleavage as well as the growth rate of the cavities of the test-tube are calculated for the whole of the way of loading.

The modeling of the test-tube is carried out with elements 3D (HEXA20, PENTA15).
1 Problem of reference

1.1 Geometry

One considers a half-cylindrical test-tube smooth.

Rayon de l'éprouvette : \( R = 68 \text{ mm} \).
Demi-longueur de référence pour la mesure de l'elongation : \( L_0 = 203.5 \text{ mm} \)

1.2 Properties of material

One adopts an elastoplastic law of behavior of Von Mises with linear isotropic work hardening 'VMIS_ISOT_LINE'. The deformations used in the relation of behavior are the linearized deformations.

The Young modulus \( E \), the tangent module \( E_t \) as well as the Poisson's ratio do not depend on the temperature. One takes: \( E = 200 \text{ GPa} \), \( E_t = 2000 \text{ MPa} \) and \( \nu = 0.3 \).

The evolution of the elastic limit with the temperature is given in the following table:

<table>
<thead>
<tr>
<th>Temperature (^{\circ} \text{C} )</th>
<th>( \sigma_f ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>-150</td>
<td>750</td>
</tr>
<tr>
<td>-100</td>
<td>700</td>
</tr>
<tr>
<td>-50</td>
<td>650</td>
</tr>
</tbody>
</table>

Warning: The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

Copyright 2020 EDF R&D - Licensed under the terms of the GNU FDL (http://www.gnu.org/copyleft/fdl.html)
Lastly, thermal dilation is neglected (thermal dilation coefficient taken equal to 0).
1.3 Boundary conditions and loadings

While referring to the figure of [§1.1] the boundary conditions are the following ones:

- on surface $SSUP \ BC \ (Y = L_0)$ displacement $l$ imposed according to the direction $OY$,
- on surface $SINF \ OA \ (Y = 0)$ displacements blocked according to the direction $OY$,
- displacements of $A$ blocked according to $X$ and $Z$,
- displacements of $B$ blocked according to $Z$.

Evolution temporal of the temperature (presumedly homogeneous in the test-tube) and of lengthening $l$ are deferred in the following table:

<table>
<thead>
<tr>
<th>Time [s]</th>
<th>Temperature [°C]</th>
<th>Displacement $l - L_0$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>– 50</td>
<td>20.35</td>
</tr>
<tr>
<td>20</td>
<td>– 50</td>
<td>20.30</td>
</tr>
<tr>
<td>30</td>
<td>– 150</td>
<td>20.30</td>
</tr>
<tr>
<td>40</td>
<td>– 150</td>
<td>32.525</td>
</tr>
</tbody>
</table>

1.4 Initial conditions

Worthless constraints and deformations.

2 Reference solutions

2.1 Method of calculating

In simple traction and with the assumption of the small deformations, the tensile stress $\sigma(u)$ as well as the plastic multiplier $\dot{p}(u)$ at the moment $u$ are given in the case considered by:

- if $0 \leq u \leq t^p_1$:
  $$\sigma(u) = E \left( \frac{l(u) - L_0}{L_0} \right) \dot{p}(u) = 0 \left( \frac{l(t^p_1) - L_0}{L_0} \right) = L_0 \cdot 1 + \sigma_Y (-50°C)$$
- if $t^p_1 \leq u \leq 10$:
  $$\sigma(u) = E_t \left( \frac{l(u) - L_0}{L_0} \right) + E - E_t \sigma_Y (-50°C) \dot{p}(u) = 1 - \frac{E_t}{E} \frac{l(u)}{L_0}$$
- if $10 \leq u \leq 20$:
  $$\sigma(u) = \sigma(u = 10) - E \left( \frac{l(u = 10) - L_0}{L_0} \right) \dot{p}(u) = 0$$
- if $20 \leq u \leq 30$:
  $$\sigma(u) = \sigma(u = 20) - E_t \left( \frac{l(u = 20) - L_0}{L_0} \right) \dot{p}(u) = 1 - \frac{E_t}{E} \frac{l(u)}{L_0}$$
- if $30 \leq u \leq 40$:
  $$\sigma(u) = \sigma(u = 20) + E_t \left( \frac{l(u) - L_0}{L_0} \right) \dot{p}(u) = 1 - \frac{E_t}{E} \frac{l(u)}{L_0}$$

Warning: The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.
2.2 Weibull

Probability of cumulated rupture $P_f$ at the moment $t$ is given by (cf. POST_ELEM):

$$P_f(t) = 1 - \exp \left( - \sum_{V_i} \max_{t^p \leq \Delta u \leq t} \left( \frac{\sigma_f(u)}{\sigma_u(\theta(u))} \right)^m \frac{dV_i}{V_0} \right).$$

The summation relates to volumes of matter $V_i$ plasticized (as from the moment $t_p$), $\sigma_f(u)$ and $\theta(u)$ indicating the maximum principal constraint and the temperature in each one of these volumes at the various moments ($u$). Here, volume $V_0$ of reference is equal to $50 \mu m^3$. The module of Weibull $m$ is equal to 24 while the constraint of cleavage $\sigma_u$ depends on the temperature according to:

<table>
<thead>
<tr>
<th>Temperature [$^\circ C$]</th>
<th>50</th>
<th>100</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_u [MPa]$</td>
<td>2800</td>
<td>2700</td>
<td>2600</td>
</tr>
</tbody>
</table>

The probability of cumulated rupture varies according to $(\theta(t), l(t))$ according to:

$$P_f(t) = 1 - \exp \left( - \sum_{V_i} \max_{t^p \leq \Delta u \leq t} \left( \frac{\sigma(u)}{\sigma_u(\theta(u))} \right)^m \frac{V_i}{V_0} \right).$$

2.3 Rice and Tracey

In simple traction, the Napierian logarithm of the growth rate of the cavities at the moment $t$ is given by (cf. POST_ELEM):

$$\log \left( \frac{R(t)}{R_0} \right) = 0.283 \times \exp(0.5) \times \int_0^t \gamma(u) \, du$$

2.4 Sizes and results of reference

$P_f$ and $\frac{R}{R_0}$ for the couples (temperature, displacements = $(l-l_0)$) following:

$(–50,0 ^\circ C , 20,35 mm)$;
$(–50,0 ^\circ C , 20,30 mm)$; $(–150,0 ^\circ C , 20,30 mm)$ and $(–150,0 ^\circ C , 32,53 mm)$.

2.5 Uncertainties on the solution

Analytical solution.
3 Modeling A

3.1 Characteristics of the grid

Many nodes: 1137
Many meshes and types: 64 (PENTA15), 192 (HEXA20)

3.2 Sizes tested and results

<table>
<thead>
<tr>
<th>$T[^{\circ}C]$</th>
<th>$l - L_0[mm]$</th>
<th>$P_f$</th>
<th>$P_f$</th>
<th>% diff.</th>
<th>$R \over R_0$</th>
<th>$R \over R_0$</th>
<th>% diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>– 50</td>
<td>20.35</td>
<td>0.01465</td>
<td>0.01481</td>
<td>1.1</td>
<td>1.0447</td>
<td>1.0458</td>
<td>0.1</td>
</tr>
<tr>
<td>– 50</td>
<td>20.30</td>
<td>0.01465</td>
<td>0.01481</td>
<td>1.1</td>
<td>1.0447</td>
<td>1.0458</td>
<td>0.1</td>
</tr>
<tr>
<td>– 150</td>
<td>20.30</td>
<td>0.01465</td>
<td>0.01481</td>
<td>1.1</td>
<td>1.0447</td>
<td>1.0458</td>
<td>0.1</td>
</tr>
<tr>
<td>– 150</td>
<td>32.525</td>
<td>1.0</td>
<td>1.0</td>
<td>0.0</td>
<td>1.068</td>
<td>1.0701</td>
<td>0.2</td>
</tr>
</tbody>
</table>

4 Summary of the results

Results got by Code_Aster are very close to the analytical solutions of reference.