SSNV515 – Tensile test with the law of Rankine

Summary

One is carried out simple tensile test with the law of Rankine. The calculated solutions are compared with an analytical solution. Three modelings of which suggested:

- a modeling 0D with SIMU_POINT_MAT;
- a modeling 3D;
- Uaxisymmetric modeling 2D;
1 Problem of reference

1.1 Geometry

The test of simple traction is carried out on only one isoparametric finite element of cubic form \( \text{CUB}_4 \). The length of each edge is worth 1. The various facets of this cube are named groups of meshes \( \text{HAUT}, \text{BAS}, \text{DEVANT}, \text{DERRIERE}, \text{DROIT} \) and \( \text{GAUCHE} \). The group of meshes \( \text{SYM} \) contains the groups of meshes in addition \( \text{BAS}, \text{DEVANT} \) and \( \text{GAUCHE} \); the group of meshes \( \text{COTE} \) groups of meshes \( \text{DERRIERE} \) and \( \text{DROIT} \).

1.2 Material properties

The elastic properties are:
- module D'Young : \( E = 1 \text{ MPa} \)
- Poisson's ratio : \( \nu = 0.25 \)

Limit in traction has is equal to \( \sigma_t = 1 \text{ kPa} \)

1.3 Boundary conditions and loadings

The simple tensile test consist in imposing on the test-tube one elongation vertical all while keeping the side pressure constant and equalizes with the initial isotropic constraint \( P_0 = 10 \text{ kPa} \)

In the model considered, the cubic element represents a eighth of the sample. The limiting conditions are thus the following ones:
- Conditions of symmetry:
  - \( u_z = 0 \) on the group of mesh \( \text{BAS} \)
  - \( u_x = 0 \) on the group of mesh \( \text{GAUCHE} \)
  - \( u_y = 0 \) on the group of mesh \( \text{DEVANT} \)
- Conditions of side pressure:
  - \( P_z = P_0 = 10 \text{ kPa} \) onS groupS of meshS \( \text{DROIT} \) and \( \text{ARRIERE} \)
- Conditions of loading:
  - \( u_z = +1 \) on the group of mesh \( \text{HAUT} \)

The loading is carried out in 30 pas de time enter \( t = 0 \) and \( t = 30 \) during which displacement imposed on the group of meshes \( \text{HAUT} \) vary of \( u_z = 0 \) with \( u_z = 0.3 \) (total vertical deformation of 30%).
1.4 Results

The solutions post-are treated with the point $C'$, in terms of:

- constraint vertical $O_{zz}$;
- deformation horizontale $E_{xx}$;
- normalizes deformation plastic $e^P = ||e^P||$

They are compared with an analytical solution (described in the following paragraph) in terms of maximum variation enters $t = 0$ and $t = 20$. 

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2 Analytical solution

Let us introduce initially the notations suvantes:

\[
\begin{align*}
A &= K + \frac{4}{3}G \\
B &= K - \frac{2}{3}G \\
C &= 2 \left( K + \frac{G}{3} \right) 
\end{align*}
\]

(1)

With \( K = \frac{E}{3(1-2\nu)} \) and \( G = \frac{E}{2(1+\nu)} \) modules of compressibility and shearing, respectively.

That is to say \( C \) the tensor of elasticity of Hooke, one will have with the assumption \( \epsilon_{yy} = \epsilon_{xx} \):

\[
C \cdot d \epsilon = \begin{pmatrix}
B d \epsilon_{zz} + C d \epsilon_{xx} \\
B d \epsilon_{zz} + C d \epsilon_{xx} \\
A d \epsilon_{zz} + 2B d \epsilon_{xx}
\end{pmatrix}
\]

(2)

One will note to simplify the vertical constraint at the moment \( + \sigma^+ = \sigma_{zz}^+ \), so that the criterion of Rankine is written:

\[
\sigma^+ \leq \sigma_t
\]

(3)

One has in addition:

\[
\begin{align*}
\sigma^+ &= \sigma + n \cdot C \cdot d \epsilon \\
\sigma^+ &= \sigma + n \cdot C \cdot [d \epsilon - d \lambda n] = \sigma^{\text{pred}} - d \lambda n \cdot C \cdot n
\end{align*}
\]

(4)

With \( n = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \) and Où:

\[
d \lambda = \frac{\langle \sigma^{\text{pred}} - \sigma_t \rangle}{A}
\]

(5)

According to the law of flow associated, one also has:

\[
\begin{align*}
d \epsilon_z^p &= d \lambda = d \epsilon_v^p \\
e^p &= \frac{2}{3}d \lambda
\end{align*}
\]

(6)

Like \( n = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \), one obtains:

\[
\Delta \sigma_c = d \lambda \cdot A
\]

(7)

Combination of the equations (7), (5) and (4) we gives the constraint \( \sigma_{zz}^+ \). Lbe equationS (6) and (5) we givesNT the standard of the deviatoric plastic deformation \( e^p \).

Let us try to obtain now the expression of the deformation rubber band horizontal \( \epsilon_{xx}^{\text{elas}} \).

Latéralely, there is the condition \( \sigma_{xx}^+ = P_0 \), that is to say:
\[ \sigma_{xx}^{\text{pred}} - \Delta \sigma_{xx,C} = P_0 \] (8)

With \( \Delta \sigma_{xx,C} = d \lambda \, B \)

One obtains then by using the equation (2):

\[ \sigma_{xx} + C \, d \varepsilon_{xx}^{\text{elas}} + B \, d \varepsilon_{zz} - B \, d \lambda = P_0 \] (9)

From where the increment of horizontal elastic strain:

\[ d \varepsilon_{xx}^{\text{elas}} = \frac{P_0 - \sigma_{xx} + B \, d \lambda - d \varepsilon_{zz}}{C} \] (10)
3 Modeling A

3.1 Characteristics of modeling

Modeling A is realized on a material point 0D with SIMU_POINT_MAT.

3.2 Sizes tested and results

3.2.1 Values tested

The solutions post-are treated with the point $C$, in terms of:

- constraint vertical $\sigma_{zz}$;
- deformation horizontale $\epsilon_{xx}$;
- normalizes deformation plastic déviatorique $e^p = ||e^p||$

They are compared with an analytical solution (described in the following paragraph) in terms of maximum variation enters $t=0$ and $t=20$. The results are recapitulated in the following tables:

$$Q = \sqrt{\frac{1}{2} \sum_s s^2} \ [Pa]$$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Absolute deviation</th>
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<tbody>
<tr>
<td>$\sigma_{zz}$</td>
<td>0</td>
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<td>$\epsilon_{xx}$</td>
<td>$3.10^{-5}$</td>
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</tr>
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<td>$e^p$</td>
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3.2.2 Comments

The variation with the analytical solution is very weak.
4 Modeling B

4.1 Characteristics of modeling

Modeling B is realized in 3D with STAT_NON_LINE.

4.2 Sizes tested and results

4.2.1 Values tested

The solutions post-are treated with the point \( C \), in terms of:
- constraint vertical \( \sigma_{zz} \);
- deformation horizontal \( \epsilon_{xx} \);
- normalizes deformation plastic \( e^P = ||e^P|| \)

They are compared with an analytical solution (described in the following paragraph) in terms of maximum variation enters \( t = 0 \) and \( t = 20 \). The results are recapitulated in the following tables:

\[
Q = \sqrt{\frac{1}{2} \xi : \xi} \quad [Pa]
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4.2.2 Comments

The variation with the analytical solution is very weak.
5 Modeling C

5.1 Characteristics of modeling

Modeling C is realized on a material point 2D axisymmetric with STAT_NON_LINE.

5.2 Sizes tested and results

5.2.1 Values tested

The solutions post-are treated with the point \( C \), in terms of:

- constraint vertical \( \sigma_{zz} \);
- deformation horizontale \( \epsilon_{xx} \);
- normalizes deformation plastic déviatorique \( e^p = \| e^p \| \)

They are compared with an analytical solution (described in the following paragraph) in terms of maximum variation enters \( t = 0 \) and \( t = 20 \). The results are recapitulated in the following tables:

\[
Q = \sqrt{\frac{1}{2} \sum \sigma^2} \quad [Pa]
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5.2.2 Comments

The variation with the analytical solution is very weak.
6 Modeling D

6.1 Characteristics of modeling

Modeling D is realized on a material point 3D with STAT_NON_LINE. The difference compared to modeling B is the calculation of the initial state by a thermal loading. To bring the sample to the initial isotropic constraint \( P_0 = 10 \text{kPa} \), one brings the sample of 20° to 30° celcius. Displacements of the sample are blocked, so that thermal dilation brings the sample in compression. One obtains:

\[
\sigma_0 = \frac{E}{9(1-2\nu)} \alpha \Delta T = P_0
\]

That is to say the following value of the thermal dilation coefficient:

\[
\alpha = \frac{9(1-2\nu)P_0}{E \Delta T}
\]

6.2 Sizes tested and results

6.2.1 Values tested

The solutions post-are treated with the point \( C \), in terms of:

- constraint vertical \( \sigma_{zz} \);
- deformation horizontale \( \epsilon_{xx} \);
- normalizes deformation plastic déviatorique \( e^P = ||e^P|| \)

They are compared with an analytical solution (described in the following paragraph) in terms of maximum variation enters \( t = 0 \) and \( t = 20 \). The results are recapitulated in the following tables:

\[
Q = \sqrt{\frac{1}{2} \int_{[P]} \left| \right|^2 \left[ Pa \right]}
\]

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6.2.2 Comments

The variation with the analytical solution is very weak.
7 Summary of the results

One represents in following figures evolution of the various sizes during the tensile test with the law of Rankine.

TRACTION TEST WITH RANKINE

Figure 1: Evolution of the constraints during the tensile test
TRACTION TEST WITH RANKINE

Figure 2: Evolution of the deformations during the tensile test
Figure 3: Evolution of the internal variables during the tensile test