SSND105 - Law of behavior visco-élasto-plastic with effect of memory

Summary:

The problem is quasi-static non-linear in mechanics of the structures. Laws tested, VMIS_CIN2_MEMO and VISC_CIN2_MEMO, are laws with non-linear kinematic work hardening, isotropic work hardening, and memory of maximum work hardening. One analyzes the answer in a material point, with a pre-work hardening, then a cyclic loading.

Modeling A makes it possible to validate the effect of memory with VMIS_CIN2_MEMO in a case where work hardening is purely isotropic, for a simple traction. The reference solution for this modeling is analytical.

Modeling B compares the results got with effect of memory, and without effect of memory between the laws VISC_CIN2_MEMO and VISCOCHAB, for a cyclic loading with pre-work hardening.

Modeling C is similar to modeling B, and makes it possible to validate the two models into axisymmetric.

Modeling D is similar to modeling C, and makes it possible to check that the models being able to take into account the effect of nonproportionality give in this case results identical to the preceding models.
1 Problem of reference

1.1 Geometry

Face YZ : (1, 4, 5, 8)
Face XZ : (1, 2, 5, 6)
Face 1YZ : (2, 3, 6, 7)
Face 1XZ : (4, 3, 8, 7)

1.2 Properties of materials

Isotropic elasticity  $E = 145\,000\, MPa$  $\nu = 0.3$

Elastoplasticity with effect of memory (modeling A): model VISC_CIN2_MEMO

Isotropic work hardening
\[ R_0 = 35\, MPa \quad B = 12 \]

Memory
\[ \text{DRIVEN} = 19 \quad Q_0 = 140\, MPa \]
\[ \text{ETA} = 0.5 \quad Q_M = 460\, MPa \]

Kinematic work hardening (modeling A)
\[ C1 = 0 \quad G1_0 = 0 \]
\[ C2 = 0 \quad G2_0 = 0 \]

Viscoplasticity with effect of memory (modelings B and C): model VISC_CIN2_MEMO

Parameters identical to the preceding values, except:

\[ \text{LEMAITRE} \]
\[ \frac{1}{70(\, MPa\, S^{1/N})^{-1}} = 0.0142857 \quad \text{NR} = 24 \]

Kinematic work hardening (modeling B)
\[ C1 = 1950\, MPa \quad G1_0 = 50 \]
\[ C2 = 65000\, MPa \quad G2_0 = 1300 \]

Model viscoplasticity VISCOCHAB (modelings B and C)

\[
\begin{array}{cccccc}
 k & 35\, MPa & B & 12 & \text{ETA} & 0.5 & C2 & 65000\, MPa \\
 A_K & 0 & M_R & 1 & C1 & 1950\, MPa & M_2 & 1 \\
 A_R & 1 & G_R & 0 & M_1 & 1 & D2 & 1 \\
 K_0 & 70\, MPa\, S^{1/N} & \text{DRIVEN} & 19 & D1 & 1 & G_{X2} & 0 \\
 NR & 24 & Q_M & 460 & G_{X1} & 0 & G2_0 & 1300\, MPa \\
 ALF & 0\, MPa & Q_0 & 40\, MPa & G1_0 & 50\, MPa & A_1 & 1 \\
\end{array}
\]

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1.3 Boundary conditions and loadings

\[ N_6 \quad dy = dz = 0 \]
\[ N_2 \quad dy = 0 \]
\[ FACE1YZ \quad dx = 0 \]

Traction (modeling A):
\[ FACEYZ \quad F_x = -0.25 \times \text{coef} \quad \text{Coef} = 120 \quad \text{for} \quad t = 8s \]

Pre-work hardening (modeling B)
\[ FACEYZ \quad S_{xx} = 250 \, \text{MPa} \times \text{coef2} \quad S_{xx} = 250 \, \text{MPa} \times \text{coef2} \]
\[ \text{coef2} = 1 \quad \text{for} \quad t = 10s \], then discharge \( (\text{coef2} = 0) \) for \( t = 11s \).

From 11s, 20 cycles in imposed deformation (+ 0.5%)
In this case, at the time of the first uniaxial load in direction X:

\[ \xi^- = 0 \]
\[ q^- = 0 \]
\[ \Delta q = \eta \varepsilon_x^p \]

In this case, \( q = \frac{1}{2} \Delta \varepsilon_{x\text{max}} \), implies that \( \eta = \frac{1}{2} \). In this case, \( \Delta \xi = \frac{1}{2} (\varepsilon_x^p) \)

Moreover, in the case of a cycle of symmetrical traction compression (in plastic deformation), one obtains, during the first symmetrical discharge (with \( \eta = \frac{1}{2} \)):

\[ \xi^- = \frac{1}{2} \varepsilon_{x\text{max}}^p \]
\[ q^- = \frac{1}{2} \varepsilon_{x\text{max}}^p \]
\[ \Delta q = \eta \left( \frac{2}{3} J_2 (\varepsilon_x^p) - q^- \right) = \eta \left[ \varepsilon_{x\text{min}}^p - \xi^- \right] = \frac{1}{2} \varepsilon_{x\text{max}}^p \]
\[ q = q^- + \Delta q = \varepsilon_{x\text{max}}^p = \frac{1}{2} \Delta \varepsilon_{xx}^p \]
\[ \Delta \tilde{\xi} = \left( 1 - \eta \right) \Delta q (\varepsilon_x^p - \xi^-) / \eta q^- + \Delta q = -\frac{1}{2} \Delta \varepsilon_{x\text{max}}^p \]

\( \tilde{\xi} = \xi^0 + \Delta \tilde{\xi} = 0 \) what corresponds well to the expected result (cf [bib2]): field \( F = 0 \) centered on the origin, and of ray the half-amplitude of plastic deformation.

In the case of an increasing traction, and if kinematic work hardening is neglected, the equations to be solved become:

\[ \sigma \leq R_0 + R(p) \]

The function thus should be calculated \( R(p) \), such as:

\[ dR = b(Q - R)dp \]
\[ Q = Q_0 + (Q_m - Q_0) \left( 1 - e^{-2\eta q} \right) \]

Moreover, it is considered that one is in load, therefore \( F(\varepsilon_x^p, \xi, q) = 0 \)

\[ dq = \eta dp \]

The differential equation thus should be integrated.
\[ dR = b(Q_{m} - Q_{0}) [1 - e^{-2\mu p}] - R dp \]

what is integrated in the following way:

\[ dR + bR = 0 \implies R = \lambda e^{-bp} \]

Method of variation of the constant:

\[ \lambda e^{-bp} = b(Q_{m} - Q_{0}) e^{-2\mu p} dp \]

\[ d\lambda = bQ_{m} e^{-bp} dp + b(Q_{0} - Q_{m}) e^{(b-2\mu)p} dp \]

while integrating:

\[ \lambda = Q_{m} e^{-bp} + b(Q_{0} - Q_{m}) \frac{e^{(b-2\mu)p}}{(b-2\mu)} + K \]

from where

\[ R(p) = Q_{m} + \frac{b(Q_{0} - Q_{m})}{(b-2\mu)} e^{2\mu p} + K e^{-bp} \]

The constant \( K \) is defined by the initial conditions: for \( p = 0, R = 0 \)

\[ 0 = Q_{m} + \frac{b(Q_{0} - Q_{m})}{(b-2\mu)} + K \]

that is to say \( K = \frac{bQ_{0} - Q_{m}}{(b-2\mu)} - \frac{bQ_{m}}{(b-2\mu)} \)

Finally:

\[ R(p) = Q_{m} + \frac{bQ_{0} - Q_{m}}{(b-2\mu)} e^{2\mu p} + \frac{2\mu Q_{m} - bQ_{0}}{(b-2\mu)} e^{-bp} \]

One thus has in load:

\[ \sigma = R_{0} + R(p) \]

2.2 Results of reference

Modeling a:

Value of \( SIXX \) at the final moment:

\[ \sigma = R_{0} + R(p) \]

with \( R(p) = Q_{m} + \frac{bQ_{0} - Q_{m}}{(b-2\mu)} e^{2\mu p} + \frac{2\mu Q_{m} - bQ_{0}}{(b-2\mu)} e^{-bp} \)

\( t = 8s \), one must find \( SIXX = 120 \text{ Mpa} \).

For that one calculation \( R(p) \) starting from the value of \( p \) at the moment \( t = 8s \).

Modeling b:

One will compare the results obtained with \( \text{VISC CIN2 MEMO} \) with those obtained with \( \text{VISCOCHAB} \), at the end of pre-work hardening and at the end of 10 cycles. The curves below highlight of the effect of memory (per comparison with \( \text{VISC CIN2 CHAB} \) who does not model it): after a pre-work hardening, the cycles with imposed deformation are stabilized with an amplitude of constraints higher than that obtained without effect of memory:
2.3 Uncertainty on the solution

- Analytical modeling a:
- Modeling b: intercomparison enters VISCOCHAB and VISC_CIN2_MEMO: precision of the digital integration, estimated at less 1%.
- Modeling C: validation of the behaviors in 2D AXIS; the results must be identical to those of modeling B.

2.4 Bibliographical references

[1] R5.03.04 “Behaviors élasto-visco-plastics of J.L.Chaboche”.

3    Modeling A

3.1 Characteristics of modeling
Modeling 3D, 1 hexa8. Simple traction.

3.2 Sizes tested and results

<table>
<thead>
<tr>
<th>Identification</th>
<th>Reference</th>
<th>tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{xx}$</td>
<td>120</td>
<td>0.20%</td>
</tr>
<tr>
<td>$\rho$</td>
<td>3.70925 $E-2$</td>
<td>0.10%</td>
</tr>
</tbody>
</table>

4    Modeling B

4.1 Characteristics of modeling
Pre-work hardening in traction then cycles with imposed deformation, comparison VISCOCHAB and VISC_CIN2_MEMO. 250 pas de time for 10 cycles.

4.2 Sizes tested and results

<table>
<thead>
<tr>
<th>Identification</th>
<th>Momen</th>
<th>VISCOCHAB</th>
<th>VISC_CIN2_MEMO</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{xx}$</td>
<td>10</td>
<td>220</td>
<td>220</td>
<td>0</td>
</tr>
<tr>
<td>$\sigma_{yy}$</td>
<td>11</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\sigma_{zz}$</td>
<td>113.5</td>
<td>3.75459E+02</td>
<td>3.72353E+02</td>
<td>-0.8</td>
</tr>
<tr>
<td>$\varepsilon_{xx}$</td>
<td>113.5</td>
<td>-1.87638E-02</td>
<td>-1.87638E-02</td>
<td>0</td>
</tr>
</tbody>
</table>

Essai cyclique DEPS=+/-0.5%

4.3 Remarks

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The difference of 0.8% on the constraints at the final moment grows blurred if the step of time is refined: with a step of time 2 times finer, the variation become 0.4%.

5  Modeling C

5.1 Characteristics of modeling

Pre-work hardening in traction then cycles with imposed deformation, comparison VISCOCHAB and VISC_CIN2_MEMO. 250 pas de time for 10 cycles. Modeling 2D AXIS.

5.2 Sizes tested and results

<table>
<thead>
<tr>
<th>Identification</th>
<th>Moment</th>
<th>VISCOCHAB</th>
<th>VISC_CIN2_MEMO</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{xx}$</td>
<td>113.5</td>
<td>3.75459E+02</td>
<td>3.72353E+02</td>
<td>−0.8</td>
</tr>
<tr>
<td>$\varepsilon_{xx}$</td>
<td>113.5</td>
<td>−1.87638E−02</td>
<td>−1.87638E−02</td>
<td></td>
</tr>
</tbody>
</table>

6  Modeling D

6.1 Characteristics of modeling

This modeling is identical to modeling C, with models of the type NRAD (not radiality). Results of the models VISC_MEMO_NRAD and VISC_CIN2_NRAD can be compared with those of modeling C, since the effect of nonradiality must be inoperative here. Tests of VMIS_MEMO_NRAD, VMIS_CIN2_NRAD (without viscosity) are of nonregression.

6.2 Sizes tested and results

<table>
<thead>
<tr>
<th>Behavior</th>
<th>VISC_MEMO_NRAD</th>
<th>VISC_CIN2_MEMO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{xx}$</td>
<td>113.5</td>
<td>369.679</td>
</tr>
<tr>
<td>$\varepsilon_{xx}$</td>
<td>113.5</td>
<td>−1.8773E−02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Behavior</th>
<th>VISC_CIN2_NRAD</th>
<th>VISC_CIN2_CHAB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{xx}$</td>
<td>113.5</td>
<td>269.6</td>
</tr>
<tr>
<td>$\sigma_{xx}$</td>
<td>10</td>
<td>220</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Behavior</th>
<th>VMIS_MEMO_NRAD</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{xx}$</td>
<td>113.5</td>
<td>372.2 (not regression)</td>
</tr>
<tr>
<td>$\sigma_{xx}$</td>
<td>10</td>
<td>220 (analytical)</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Behavior</th>
<th>VMIS_CIN2_NRAD</th>
<th>VISC_CIN2_MEMO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{xx}$</td>
<td>113.5</td>
<td>225.254 (not regression)</td>
</tr>
</tbody>
</table>
Summary of the results

Four modelings make it possible to validate, on a material point, the behaviors of the kinematic type nonlinear for purpose of memory, in plasticity and viscoplasticity.