HPLP100 - Calculation of the rate of refund of the energy of a plate fissured in thermoelasticity

Summary

It is about a test in thermoelasticity for a two-dimensional problem. A fissured rectangular plate is considered and one places oneself on the assumption of the plane deformations.

In modeling A, the rate of refund of energy is calculated in postprocessing by two different methods:

- classical calculation by the method theta,
- calculation by the formula of IRWIN starting from the coefficients of intensity of constraints $K_I$ and $K_{II}$.

These two calculations are carried out on 4 different crowns of integration. Their interest is to compare the values of $G$ and of $G(\text{IRWIN})$ compared to the reference solution and to test the invariance of calculations compared to the various crowns of integration.
1 Problem of reference

1.1 Geometry

It is about a fissured rectangular plate (one represents only the quarter of the structure):

![Diagram of a fissured rectangular plate]

Figure 1.1-a: Fissured rectangular plate

Dimensions of this plate are the following ones:
- Half-height of the plate: \( h = 200.0 \text{ mm} \)
- Half-width of the plate: \( I = 100.0 \text{ mm} \)
- Half-length of the crack: \( a = 50.0 \text{ mm} \)

1.2 Properties of material

Thermal properties:
- \( Cp = 0 \)
- \( \lambda = 1.0 \text{ W/m } ^\circ \text{C} \)

Mechanical properties:
- \( E = 200000 \text{ MPa} \)
- \( \nu = 0.3 \)
- \( \alpha = 5.10^{-6}/^\circ \text{C} \)

We are on the assumption of the plane deformations

1.3 Boundary conditions and loadings

- Temperature imposed in \( X = 0 \): \( T = -100.0 \text{ } ^\circ \text{C} \)
- Temperature imposed in \( X = 100 \): \( T = +100.0 \text{ } ^\circ \text{C} \)
- Displacement for \( a < X < I, \ Y = 0 \): \( v = 0 \)
- Displacement for \( 0 < X < I, \ Y = H \): \( v = 0 \)
Displacement for $X = 0, Y = H : u = 0.$
2 Reference solution

2.1 Method of calculating used for the reference solution

The reference solution is resulting from WILSON and YU [bib1]:

\[ K_I = \frac{E \alpha T_0}{1-\nu} F \sqrt{a} \quad F = 0.154 \]

\[ a \text{ en mm} \]

\[ E \text{ en N/mm}^2 \]

\[ K_I = 92.0291 \]

In plane deformations, the formula of IRWIN gives:

\[ G = \frac{(1 - \nu^2)}{E} \left( K_I^2 + K_{II}^2 \right) \]

that is to say numerically: \( G = 3.8535 \times 10^{-1} \)

2.2 Results of reference

The results of reference are those resulting from the reference solution from WILSON and YU [bib1]:

\[ G = 3.8535 \times 10^{-1} \]

\[ K_I = 92.0291 \]

\[ K_{II} = 0. \]

2.3 Bibliographical references

1) The Uses of J-Integrals in thermal stress ace problems - International Newspaper of Fracture (1979) WILSON and YU.

2) Qualification complementary to codes INCA/MAYA in linear thermoelasticity. Technical note DRE/STRE/LMA 84/598
3 Modeling A

3.1 Characteristics of modeling

There are 4 crowns defined by the order \texttt{CALC\_G}:

Crown 1: \( \text{Rinf} = 10, \quad \text{Rsup} = 40 \).
Crown 2: \( \text{Rinf} = 15, \quad \text{Rsup} = 45 \).
Crown 3: \( \text{Rinf} = 5, \quad \text{Rsup} = 47 \).
Crown 4: \( \text{Rinf} = 3, \quad \text{Rsup} = 48 \).

The bottom of crack is defined by \texttt{DEFI\_FOND\_FISS}, and for each crown one carries out:

- a calculation of \( G \) classic (option \texttt{CALC\_G} of \texttt{CALC\_G}),
- a calculation of \( G \) by the formula of IRWIN starting from the coefficients of intensity of constraints \( K_I \) and \( K_{II} \) (option \texttt{CALC\_K\_G} of \texttt{CALC\_G}).

3.2 Characteristics of the grid

Many nodes: 853
Many meshes and types: 359 meshes \texttt{TRIA6} and 27 meshes \texttt{QUAD8}

3.3 Values tested and results of modeling A

The values tested are those of \( G \) obtained by the classical method and that of \( G_{IRWIN} \) obtained by the formula of IRWIN starting from the coefficients of intensity of constraints:

<table>
<thead>
<tr>
<th>Identification</th>
<th>Reference</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crown 1 ( G )</td>
<td>( 3.8535 \times 10^{-1} )</td>
<td>8,00%</td>
</tr>
<tr>
<td>Crown 1 ( G_{IRWIN} )</td>
<td>( 3.8535 \times 10^{-1} )</td>
<td>8,00%</td>
</tr>
<tr>
<td>Crown 2 ( G )</td>
<td>( 3.8535 \times 10^{-1} )</td>
<td>8,00%</td>
</tr>
<tr>
<td>Crown 2 ( G_{IRWIN} )</td>
<td>( 3.8535 \times 10^{-1} )</td>
<td>8,00%</td>
</tr>
<tr>
<td>Crown 3 ( G )</td>
<td>( 3.8535 \times 10^{-1} )</td>
<td>8,00%</td>
</tr>
<tr>
<td>Crown 3 ( G_{IRWIN} )</td>
<td>( 3.8535 \times 10^{-1} )</td>
<td>8,00%</td>
</tr>
<tr>
<td>Crown 4 ( G )</td>
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</tr>
</tbody>
</table>

3.4 Remarks

The digital values are stable compared to the various crowns of integration and almost identical for the two methods of calculating. Nevertheless the variation with the values of reference is about 6 with 7%, which seems high.
4 Summaries of the results

At the time of the first modeling, the variation with the values of reference is from 6 to 7%. The validation independent of the breaking process batch should bring brief replies on the validity of $G$ in thermoelasticity.