HPLP101 - Plate fissured in thermoelasticity (plane constraints)

Summary:

This test is resulting from the validation independent of Code_Aster in breaking process (reference resulting from Murakami: Mura11-17). It makes it possible to validate the operators of breaking process for a two-dimensional problem (assumption of the plane constraints) in isotropic linear thermoelasticity.

This test understands a modeling in plane constraints in which are calculated:

- the rate of refund of energy $G$ (classical calculation by the method theta),
- coefficients of intensity of constraints $K_I$ and $K_{II}$.

These two calculations are carried out on 6 different crowns of integration.

The interest of the test is to compare the values of $G$ and $K_{II}$ compared to the reference solution and to test the invariance of calculations compared to the various crowns of integration.
1 Problem of reference

1.1 Geometry

![Diagram of a plate with a crack]

- Width of the plate: \( W = 0.6 \text{ m} \)
- Length of the plate: \( L = 0.3 \text{ m} \)
- Length of the crack: \( 2a = 0.3 \text{ m} \)

1.2 Properties of material

Notation for thermoelastic properties:

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} & 0 \\
S_{12} & S_{22} & 0 \\
0 & 0 & S_{66}
\end{bmatrix}
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} +
\begin{bmatrix}
\alpha_{11} \\
\alpha_{22} \\
0
\end{bmatrix}\cdot(T-T_{ref})
\]

- \( S_{11} = 1/E_x \)
- \( S_{22} = 1/E_y \)
- \( S_{12} = -\nu_y/E_x = -\nu_y/E_y \)
- \( S_{66} = 1/G_{xy} \)
- \( \alpha_{11} = \alpha_x \)
- \( \alpha_{22} = \alpha_y \)

One limits oneself to isotropic material, as well from the thermal point of view as mechanical:

- \( E_x = E_y = 2.10^5 \text{ MPa} \)
- \( \nu_x = \nu_y = 0.3 \)
- \( \alpha_x = \alpha_y = 1.210^{-5} \text{ C}^{-1} \)
- \( \lambda_x = \lambda_y = 54. W/m \text{ °C} \)
1.3 Boundary conditions and loading

Two models are considered:

- the half-model \( x = 0 \)
- the complete model

**Boundary conditions mechanical:**

- half-model
  \[
  UX = 0 \text{ along the axis of symmetry } X = 0 \\
  UY = 0 \text{ at the point } (W/2.)
  \]
- complete model
  \[
  UX = 0 \text{ at the point } (0, L/2.) \\
  UY = 0 \text{ at the points } (-L/2.) \text{ and } (L/2.)
  \]

**Boundary conditions thermal:**

- half-model
  \[
  T = 100^\circ C \text{ on the higher edge } Y = L/2. \\
  T = -100^\circ C \text{ on the lower edge } Y = -L/2. \\
  \text{null flow on the axis of symmetry, the free edge } X = W/2. \text{ and on the edge of the crack}
  \]
- complete model
  \[
  T = 100^\circ C \text{ on the higher edge } Y = L/2. \\
  T = -100^\circ C \text{ on the lower edge } Y = -L/2. \\
  \text{null flow on the free edges } X = \pm W/2. \text{ and on the edge of the crack} \]
2 Reference solution

2.1 Method of calculating used for the reference solution

Complex potential [bib1].

2.2 Results of reference

\[ \eta = \frac{2a}{W} \]
\[ \beta = \frac{L}{W} \]
\[ K_II = \frac{\alpha_{11} T_0}{S_{11}} \sqrt{\frac{W}{2}} \cdot F_{II} \]

where the geometrical factor of correction \( F_{II} \) is given according to \( \eta \) for each material, in the typical case \( \beta = 0.5 \) on the curves below.

The isotropic material being represented by the curve \( I \)

2.3 Uncertainty on the solution

Nondefinite precision.

2.4 Bibliographical references

3 Modeling A

3.1 Characteristics of modeling

For this modeling, the 3 topological parameters of the block crack are:

- \( NS \) : many sectors on \( 90^\circ \)
- \( NC \) : many crowns
- \( rt \) : the ray of the largest crown (with half a: length of the crack)

\[
\begin{aligned}
NS &= 8 \\
NC &= 4 \\
rt &= 0.001 \times a
\end{aligned}
\]

Values of the higher and lower rays, to specify in the order \text{CALC}_G\ are:

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<tbody>
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<tr>
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<td>1.500E-4</td>
<td>1.875E-4</td>
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</table>

3.2 Characteristics of the grid

Half-grid; grid radiating at the right end of the crack.

3831 nodes, 
1516 elements, 
884 TRI6,  
632 QUA8.

3.3 Sizes tested and results of modeling A

<table>
<thead>
<tr>
<th>Identification</th>
<th>Reference</th>
<th>Aster</th>
<th>% difference</th>
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<td>2.2814E+7</td>
<td>2.09</td>
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3.4 Remarks
In the reference, the author supposes that $K_I = 0$, but it does not check it a posteriori. With the sights of the deformations resulting from Code_Aster, the coefficient $K_I$ is different from zero, but there remains very weak compared to $K_{II}$ (the crack slips more than it does not open).

With regard to the rate of refund of energy $G$, if we suppose that $K_I = 0$, we draw the value of reference starting from the formula from IRWIN in plane constraints:

$$G_{\text{ref}} = \left(\frac{1}{E}\right) \times K_{II}^2$$

4 Summary of the results

Differences between the reference solution and the results of Code_Aster do not exceed 2% on the coefficients of intensity of constraints and 4% for the rate of refund of energy. One checks the invariance of the results compared to the various crowns of integration.