HPLP300 - Plate with Young modulus function of temperature

Summary:

This thermoelastic test makes it possible to compare the solution obtained by Code_Aster with an analytical solution, when the Young modulus varies in a nonlinear way compared to the temperature.

This test is deduced from the test 3D HPLV100 describes in [V7.03.100] (parallelepiped whose Young modulus is function of the temperature).
1. Problem of reference

1.1 Geometry

1.2 Material properties

Thermal conductivity: \( \lambda = 1 \)

Young modulus: \( E = \frac{10000}{8000 - T} \), \( T = \) température

Poisson's ratio: \( v = 0.3 \)

1.3 Boundary conditions and loadings

1.3.1 Thermics

\( T(0) = 40 \)

\( \lambda \frac{\partial T}{\partial n} = -4 \) on edge \( x = h/2 \)

\( \lambda \frac{\partial T}{\partial n} = +4 \) on edge \( x = -h/2 \)

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\[ \lambda \frac{\partial T}{\partial n} = -3 \text{ on edge } y = h/2 \]

\[ \lambda \frac{\partial T}{\partial n} = +3 \text{ on edge } y = -h/2 \]

### 1.3.2 Mechanics

Not \( O \) blocked \( (u=v=0) \)

Following displacement \( x \) blocked in \( B \)

Uniform pressure \( po \) being exerted normally on contour: \( po = 1 \).

### 2 Reference solution

#### 2.1 Method of calculating used for the reference solution

The field of temperature is given by:

\[ T = -4X - 3Y + 40 \]

The field of displacements is given by:

\[ u = -Vp \left[ Bxy + \frac{C}{2} (x^2 - y^2) \right] + Dx + \frac{Ch}{4} y \]

\[ v = -Vp \left[ \frac{B}{2} (y^2 - x^2) \right] + Cxy + Dy - \frac{Ch}{4} y \]

where \( B = 0.003 \) \( C = 0.004 \) \( D = 0.76 \) \( p = \frac{1}{v} po \)

The field of deformations is given by:

\[ \varepsilon = \varepsilon_x = \varepsilon_y = -v p \left( By + Cx + D \right) \]

\( \varepsilon_y = 0 \)

The stress field is given by:

\[ \sigma = \sigma_x = \sigma_y = \left( \frac{E}{1-v} \varepsilon \right) = \left( \frac{1000}{800} \frac{vp}{1-v} \left( 0.004x + 0.003y + 0.76 \right) \right) = \frac{v}{1-v} p = po \]
2.2 Results of reference

Temperature at the points $O, A, B, C, D, B1, C1$
Displacements at the points $A, B, C, D, B1, C1$

2.3 Uncertainty on the solution

Analytical solution.

3 Modeling A

3.1 Characteristics of modeling

It is about a modeling in plane constraints.

![Diagram of a model with points O, A, B, C, D, B1, C1 and cutting: 4 x 4 elements.]

Cutting: $4 \times 4$ elements

Limiting conditions:

1) in $O$, $u = v = 0$
2) in $B$, $u = 0$
3.2 Characteristics of the grid

Many nodes: 65
Number of meshes and type: 16 QUAD8

Name of the nodes

\[ O = N38 \quad A = N1 \quad B = N23 \quad C = N16 \quad D = N3 \quad B1 = N9 \quad C1 = N30 \]

3.3 Sizes tested and results

<table>
<thead>
<tr>
<th>Localization</th>
<th>Type of value</th>
<th>Reference</th>
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<tbody>
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<tr>
<td>Not B</td>
<td>( T )</td>
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<tr>
<td>Not C</td>
<td>( T )</td>
<td>20.</td>
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<tr>
<td>Not D</td>
<td>( T )</td>
<td>5.</td>
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<tr>
<td>Not B1</td>
<td>( T )</td>
<td>55.</td>
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<tr>
<td>Not C1</td>
<td>( T )</td>
<td>60.</td>
</tr>
<tr>
<td>Not O</td>
<td>( T )</td>
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<tr>
<td>Not A</td>
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<tr>
<td></td>
<td>( v )</td>
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<tr>
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<tr>
<td>Not C1</td>
<td>( u )</td>
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</table>

3.4 Remarks

It is necessary to discretize the function finely \( E(t) \) to get satisfactory results. One took for this test 160 points of discretization, for the interval of temperatures [5. , 75.].
4 Summary of the results

Results got with Code_Aster are in concord with the analytical solution.