HSNV121 - Traction in great deformations plastics of a bar under loading thermics

Summary:

This quasi-static thermomechanical test consists in heating a bar of rectangular section uniformly (3D) or cylindrical (2D axisymmetric) then to subject it to a traction. One thus validates the kinematics of the great deformations in plasticity (order STAT_NON_LINE, keyword DEFORMATION: 'SIMO_MIEHE' or 'PETIT_REAC') for a relation of behavior in great deformations with linear isotropic work hardening (order STAT_NON_LINE, keyword RELATION: 'VMIS_ISOT_LINE' and 'VMIS_ISOT_TRAC') with thermomechanical loading. With modelings hull or plate, the great deformations in plasticity are accessible thanks to the keyword DEFORMATION: 'PETIT_REAC' provided that rotations remain weak.

The bar is modelled by a voluminal element (HEXA20, modeling A) or quadrangular (QUAD4, for an axisymmetric modeling, modeling B) or by elements of plate or hull (DKT for modeling C and COQUE_3D for modeling D).

The solution is analytical.

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1 Problem of reference

1.1 Geometry

![Diagram of geometry]

1.2 Properties of material

The material obeys a law of behavior in great deformations figure with linear isotropic work hardening, whose characteristics depend on the temperature. The traction diagram is given in the plan deformation logarithmic curve - rational constraint.

\[
\sigma = \frac{F}{S} = \frac{F}{S_o} \cdot \frac{l}{l_o}
\]

\[
\nu = 0.3
\]

\[
\alpha = 10^{-4} K^{-1}
\]

\[
\sigma_y = 1000 \text{ MPa}
\]

à \( T = 20^\circ \text{C} \)

\[
E = 250000 \text{ MPa}
\]

\[
E_T = 2500 \text{ MPa}
\]

à \( T = 120^\circ \text{C} \)

\[
E = 200000 \text{ MPa}
\]

\[
E_T = 2000 \text{ MPa}
\]

\( l_o \) and \( l \) are, respectively, the initial length and the current length of the useful part of the test-tube.

\( S_o \) and \( S \) are, respectively, initial and current surface. Between the temperatures 20 °C and 120 °C, the characteristics are interpolated linearly.
1.3 Boundary conditions and loadings

The bar, initial length $l_o$, blocked in the direction $Ox$ on the face [1.2] is subjected to a uniform temperature $T$ and with a mechanical displacement of traction $u^\text{meca}$ on the face [3, 4]. The sequences of loading are the following ones:

Temperature of reference: $T_{\text{ref}} = 20^\circ\text{C}$.

Note:

*Mechanical displacement is measured starting from the configuration deformed by the thermal loading ($t = 1\text{s}$). To have total displacement, it is thus necessary to add the thermal displacement obtained at time $t = 1\text{s}$.*
2 Reference solution

2.1 Result of the reference solution

For a tensile test according to the direction $x$, the tensor of Kirchhoff $\tau$ is form:

$$
\tau = \begin{bmatrix}
\tau_{xx} & 0 & 0 \\
0 & \tau_{yy} & 0 \\
0 & 0 & \tau_{zz}
\end{bmatrix}
$$

Tensor gradients of the transformation $F$ and $\bar{F}$ and the isochoric tensor of plastic deformations $G^p$ are form:

$$
F = \begin{bmatrix}
F_{xx} & 0 & 0 \\
0 & F_{yy} & 0 \\
0 & 0 & F_{zz}
\end{bmatrix}
$$

$$
\bar{F} = J^{-1/3}F = \begin{bmatrix}
\bar{F}_{xx} & 0 & 0 \\
0 & \bar{F}_{yy} & 0 \\
0 & 0 & \bar{F}_{zz}
\end{bmatrix}
$$

$$
G^p = \begin{bmatrix}
G^p_{xx} & 0 & 0 \\
0 & G^p_{yy} & 0 \\
0 & 0 & G^p_{zz}
\end{bmatrix}
$$

By the law of behavior, one obtains the following relation:

$$
\tau = \frac{3K}{2} (J^2 - 1) - \frac{9K\alpha(T - T_{ref})}{2} (J + \frac{1}{J})
$$

that is to say:

$$
J^3 - 3\alpha(T - T_{ref})J^2 - J(1 + \frac{2\tau}{3K}) - 3\alpha(T - T_{ref}) = 0
$$

The constraint of Cauchy is written:

$$
J\sigma = \tau
$$

In plastic load for an isotropic work hardening $R$ linear, such as:

$$
R(p) = \frac{EE_T}{E - E_T} p
$$

one a:

$$
p = \frac{E - E_T}{EE_T} (\tau - \sigma_T)
$$
The integration of the law of flow of the plastic deformation $G^p$ give (knowing that $G^p(p=0)=1$):

$$G^p = e^{-2p}$$

The component $\bar{F}$ gradient of the transformation is given by the resolution of:

$$F^3 - \frac{\tau}{\mu G^p} F - \frac{1}{(G^p)^{3/2}} = 0$$

The field of displacement $u$ (in the initial configuration) is form $u = u_x X + u_y Y + u_z Z$. The components are given by:

$$
\begin{align*}
  u_x &= \frac{\tilde{u}}{l_o} X \\
  u_y &= \frac{\tilde{v}}{l_o} Y \\
  u_z &= \frac{\tilde{w}}{l_o} Z
\end{align*}
$$

\[\tilde{u} = (F - 1) \cdot l_o\]
\[\tilde{v} = \sqrt{\frac{J}{F - 1}} \cdot l_o\]

\[\tilde{w} = 303 \text{ mm} \quad \tilde{v} = -110 \text{ mm}\]

With these sizes, it is possible to determine the elastic energy of the bar. Attention, the presence of thermics generates a high jacobien, requiring a specific correction as described in R5.03.21. With final, one obtains at the material point: $\Psi_{elas} = 5.6 \text{ MPa}$

### 2.2 Results of reference

One will adopt like results of reference displacements, the constraint of Cauchy $\sigma$ and cumulated plastic deformation $p$.

**At time** $t = 2$ s ($\Delta T = 100^\circ C$, traction $u$)

One seeks total displacement (thermal + mechanical) such as the constraint $\tau$ that is to say equalizes with:

$$\tau = 1500 \text{ MPa} \quad \text{(with } T = 120^\circ C\} )$$

- $3K = 500 000 \text{ MPa}$
- $J = 1.03$
- $\sigma = 1453 \text{ MPa}$
- $p = 0.2475$
- $G^p = 0.609$
- $F = 1.289$
- $F = 1.303$
- $\tilde{u} = 303 \text{ mm}$
- $\tilde{v} = -110 \text{ mm}$

With these sizes, it is possible to determine the elastic energy of the bar. Attention, the presence of thermics generates a high jacobien, requiring a specific correction as described in R5.03.21. With final, one obtains at the material point: $\Psi_{elas} = 5.6 \text{ MPa}$

### 2.3 Uncertainty on the solution

The solution is analytical. With the rounding errors near, one can consider it exact.

### 2.4 Bibliographical references

One will be able to refer to:

1) V. CANO, E. LORENTZ: Introduction into Code_Aster of a model of behavior in great deformations elastoplastic with isotropic work hardening - Note interns EDF DER HI - 74/98/006/0
3 Modeling A

3.1 Characteristics of modeling

Voluminal modeling:

1 mesh HEXA20
1 mesh QUAD8

Boundary conditions:

\[ \begin{align*}
N2: & \quad U_x = U_y = U_z = 0 \\
N1: & \quad U_x = U_z = 0 \\
N6: & \quad U_x = U_y = 0 \\
\end{align*} \]

Load: Traction on the face \([348711161915]\) + assignment of the same temperature on all the nodes.

The full number of increments is of 21 (1 increment enters \(t=0s\) and 1s, 20 increments enters \(t=1s\) and 2s).

Convergence is carried out if the residue RESI_GLOB_RELA is lower or equal to \(10^{-6}\).

3.2 Characteristics of the grid

Many nodes: 20
Many meshs: 2
1 HEXA20
1 QUAD8

3.3 Sizes tested and results

<table>
<thead>
<tr>
<th>Identification</th>
<th>Reference</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t=2) Displacement (DX (N8))</td>
<td>303</td>
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<tr>
<td>(t=2) Displacement (DY (N8))</td>
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<tr>
<td>(t=2) Displacement (DZ (N8))</td>
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<td>1,00%</td>
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<tr>
<td>(t=2) Constraints SIGXX (PG1)</td>
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With INDIPLAS the indicator of plasticity.
4 Modeling B

4.1 Characteristics of modeling

Modeling 2D axisymmetric:

\[ 1 \text{ mesh QUAD4} \]
\[ 1 \text{ mesh SEG2} \]

Boundary conditions:

\[ \begin{align*}
N1: & \quad U_y = 0 \\
N2: & \quad U_y = 0 
\end{align*} \]

Loading:

Traction on the face \[ \{3, 4\} \] (mesh SEG2) + assignment of the same temperature on all the nodes.

The full number of increments is of 21 (1 increment enters \( t = 0s \) and \( 1s \), 20 increments enters \( t = 1s \) and \( 2s \)).

Convergence is carried out if the residue \( \text{RESI}_\text{GLOB}_\text{RELA} \) is lower or equal to \( 10^{-6} \).

4.2 Characteristics of the grid

Many nodes: 4
Many meshes: 2

\[ \begin{align*}
1 \text{ QUAD4} \\
1 \text{ SEG2} 
\end{align*} \]

4.3 Sizes tested and results

<table>
<thead>
<tr>
<th>Identification</th>
<th>Reference</th>
<th>Tolerance</th>
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<td>( t=2 ) Constraints ( SIGYY \ (PG1) )</td>
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<td>( t=2 ) Variable ( P \ VARI \ (PG1) )</td>
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<tr>
<td>( t=2 ) ( \text{ENER}_\text{ELAS}, \ \text{TOTAL} )</td>
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</tr>
</tbody>
</table>
5 Modeling C

5.1 Characteristics of modeling

Modeling plates DKT of thickness 1000 mm: 1 mesh QUAD4, 2 meshes TRIA3
1 mesh SEG2

Boundary conditions:

\[ \begin{align*}
N2: & 
U_x = 0 & U_y = 0 & U_z = 0 & \theta_x = 0 & \theta_y = 0 & \theta_z = 0 \\
N1: & 
U_x = 0 & U_z = 0
\end{align*} \]

Loading:

Traction on the face [3 4] (mesh SEG2) + assignment of the same temperature on all the nodes
The full number of increments is of 21 (1 increment enters \( t = 0 \) s and 1 s, 20 increments enters \( t = 1 \) s and 2 s)
Convergence is carried out if the residue \( RESI\_GLOB\_RELA \) is lower or equal to \( 10^{-6} \).

5.2 Characteristics of the grid

Many nodes: 8
Many meshes: 4
1 QUAD4
2 TRIA3
1 SEG2

5.3 Sizes tested and results

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<tr>
<th>Identification</th>
<th>Reference</th>
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<tbody>
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<td>( t = 2 ) Variable ( p ) ( VARI ) (PG1)</td>
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<td>1.50%</td>
</tr>
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</table>

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6 Modeling D

6.1 Characteristics of modeling

Modeling COQUE_3D of thickness 1000 mm:

- 1 mesh QUAD9
- 2 meshes TRIA7
- 1 mesh SEG3

Boundary conditions:

\[ \begin{align*}
N2: & \quad U_x = 0 \quad U_y = 0 \quad U_z = 0 \quad \theta_x = 0 \quad \theta_y = 0 \quad \theta_z = 0 \\
N5: & \quad U_x = 0 \quad U_z = 0 \\
N1: & \quad U_x = 0 \quad U_z = 0
\end{align*} \]

Loading:

- Traction on the face \([3 4]\) (mesh SEG3) + assignment of the same temperature on all the nodes

The full number of increments is of 21 (1 increment enters \( t=0 \) and \( 1 \) s, 20 increments enters \( t=1 \) s and \( 2 \) s)

Convergence is carried out if the residue \( \text{RESI\_GLOB\_RELA} \) is lower or equal to \( 10^{-6} \).

6.2 Characteristics of the grid

- Many nodes: 17
- Many meshes: 4

1 QUAD9
2 TRIA7
1 SEG3

6.3 Sizes tested and results

<table>
<thead>
<tr>
<th>Identification</th>
<th>Reference</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t=2 ) Displacement ( DX ) (N3)</td>
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<tr>
<td>( t=2 ) Displacement ( DY ) (N3)</td>
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<td>( t=2 ) Constraint ( SIXX ) (PG1)</td>
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</table>

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7 Summary of the results

Results found with Code_Aster and DEFORMATION: ‘SIMO_MIEHE’ are very satisfactory with percentages of error lower than 0.4% on the constraint and with 1.2% on the variable of work hardening. For elements of plate and hull the use of DEFORMATION: ‘PETIT_REAC’ gives satisfactory results with percentages of error of 3% on the effort or the constraint and lower than 0.7% on the variable of work hardening.