WTNV109 - Hydrous and mechanical loading of a saturated porous environment

Summary:

One considers a three-dimensional problem of coupling thermo-hydro-mechanics of a saturated porous environment.

This test consists in studying the effect of mechanics and hydraulics on thermics. One stretches the element by imposing a displacement in the direction to him $z$, one imposes a water pressure on the whole of the field and one studies the effect of these loadings on the temperature of the model. It is a question of looking at the very weak coupling of poromecanic towards thermics. One limits oneself to the first step of time.

The studied models are 2D plans (DFQ8 and DPTR6) and 3D voluminal (HEXA20) with a linear behavior for thermal hydraulics and it.
1 Problem of reference

1.1 Presentation

One studies in this case test the behavior thermo-hydro-mechanics of a saturated porous environment consisted only one fluid: water in its liquid phase. It acts in Code_Aster of a modeling THM. The associated law of behavior of the fluid is of type LIQU_SATU.

1.2 Geometry

One considers a cube of $1m$ of with dimensions centered on the center of the axis $(-0.5\leq x\leq 0.5; -0.5\leq y\leq 0.5; -0.5\leq z\leq 0.5;)$. 

![Diagram of a cube](image)

1.3 Properties of material

<table>
<thead>
<tr>
<th>solid</th>
<th>Density (kg.m$^{-3}$)</th>
<th>2.0$\times10^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Drained Young modulus $E$ (Pa)</td>
<td>225.$\times10^6$</td>
</tr>
<tr>
<td></td>
<td>Poisson's ratio</td>
<td>0.0.</td>
</tr>
<tr>
<td></td>
<td>Thermal dilation coefficient of the solid ($K^{-1}$)</td>
<td>8.0$\times10^{-6}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Density (kg.m$^{-3}$)</th>
<th>1.0$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Heat with constant pressure ($J.K^{-1}$)</td>
<td>2.85$\times10^6$</td>
</tr>
<tr>
<td></td>
<td>Thermal dilation coefficient of the liquid ($K^{-1}$)</td>
<td>10$^{-4}$</td>
</tr>
<tr>
<td></td>
<td>Derived from the conductivity of the fluid compared to the temperature</td>
<td>0.0.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Thermics</th>
<th>Homogenized conductivity ($W.K^{-1}.m^{-1}$)</th>
<th>1.7</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Derived from the conductivity homogenized compared to the temperature</td>
<td>0.0.</td>
</tr>
</tbody>
</table>
### 1.4 Boundary conditions and loadings

- **Complete element:**
  pressure of the fluid \( PRE1 = 500.0 \, Pa \) (not of flow nor of variation of the water mass)

- **Lower face:**
  displacements \( u_x = 0.0 \, m \), \( u_y = 0.0 \, m \), \( u_z = 0.0 \, m \).

- **Higher face:**
  displacement \( u_z = 10^{-3} \, m \)

### 1.5 Initial conditions

The fields of displacement, pressure, temperature are initially all worthless, the temperature of reference is worth \( T_0 = 273 \, ^{\circ} K \).

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**Coefficients of homogenisation**

<table>
<thead>
<tr>
<th>Coefficients of homogenisation</th>
<th>Coefficient of <em>Biot</em></th>
<th>Porosity</th>
<th>Density ( (kg.m^{-3}) )</th>
<th>Heat with constant constraint ( (J.K^{-1}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homogenized coefficients</td>
<td>( 10^{-12} )</td>
<td>0.4</td>
<td>( 1.6 \times 10^3 )</td>
<td>( 2.85 \times 10^6 )</td>
</tr>
</tbody>
</table>
2 Reference solution

2.1 Method of calculating

The reference solution is unidimensional because it depends only on the vertical coordinate. One will thus reason on a linear axis according to Z.

If gravity is neglected and that it is considered that there is no external source of heat, the equation of thermics is given by the following expression:

\[ h_i^m \frac{dm_i}{dt} + \delta Q' \frac{dt}{dt} - \text{Div} \; M_i + \text{div} q = 0 \]  

(1)

Where \( h_i^m \) is the mass enthalpy of water, \( m_i \) its mass, \( M_i \) its flow, \( Q \) heat flow and \( Q' \) not convectée heat.

The conservation equation of the mass is the following one:

\[ \frac{dm_i}{dt} + \text{Div} \; M_i = 0 \]  

(2)

In the absence of hydrous flow, the equation (1) is simplified to become:

\[ \delta Q' \frac{dt}{dt} + \text{div} q = 0 \]  

(3)

In this equation, quantity \( Q' \) represent the heat received by the system in a transformation for which there are no contributions of heat per entry of fluid. One chooses a weak variation of pressure and parameters so that the temperature varies little and thus that:

\[ \frac{dQ'}{T} = \frac{dQ'}{T_0} \]

\( dQ' \) consequently this expression has:

\[ dQ' = 3 \alpha_0 K_0 T_0 d \varepsilon_v + C^c_0 dT - 3 \alpha_i^m T_0 dp_i \]

with:
- \( \alpha_0 \), the thermal dilation coefficient homogenized equivalent with that of the solid \( \alpha_s \).
- \( K_0 \), the drained coefficient of elasticity.
- \( C^c_0 \), heat has constant deformation which has as an expression \( C^c_0 = C_0^0 - 9 T_0 K_0 \alpha_0^2 \) and \( C_0^m \), heat with constant constraint.
- \( \alpha_i^m \), The relative thermal dilation coefficient of the liquid compared to the skeleton, it has as an expression: \( \alpha_i^m = (b - \phi) \alpha_s + \phi \alpha_l \) with B the coefficient of Biot, \( \phi \) porosity and \( \alpha_l \) the dilation coefficient of the liquid.
- \( d \varepsilon_v \), Voluminal variation of deformation.
- \( dp_i \), Variation of pressure of liquid.

In addition, the heat flow has the following expression: \( q = -\lambda_T \frac{T}{\partial z} \) where \( \lambda_T \) is the thermal coefficient of conductivity.

While replacing \( dQ' \) and \( Q \) by their value in the equation (3) and limiting itself to only step of time \( \Delta t \), one obtains:
By considering initially worthless temperatures and displacements, one can write:

$$\Delta T = T(t) - T(0) = \Delta T$$

and

$$\Delta \varepsilon = \varepsilon(t) - \varepsilon(0) = \varepsilon$$

and

$$\Delta p_i = p_i(t) - p_i(0) = p_i$$

Thus with the first step of time, one will have:

$$T - a \frac{\partial^2 T}{\partial z^2} = b$$

with

$$a = \frac{\lambda_i \Delta T}{C_i^0}$$

and

$$b = -\frac{3 \alpha_0 K_0 T_0 \Delta \varepsilon - 3 \alpha_i^m T_0 p_i}{C_i^0}$$

The variational formulation of this expression (in the unidimensional case) is then the following one:

$$\int_{\Omega} T \cdot \hat{T} \, dz + a \int_{\Omega} \frac{\partial T}{\partial z} \frac{\partial \hat{T}}{\partial z} \, dz - a \int_{\partial \Omega} \frac{\partial T}{\partial z} \hat{T} \, dz = \int_{\Omega} b \cdot \hat{T} \, dz \quad \text{(4)}$$

To establish the analytical solution a single element of degree 1 is considered since in modeling THM the thermohydraulic part is treated by linear elements.

That is to say a linear element:

$$\begin{align*}
N2 & : z = 0.5 \\
N1 & : z = -0.5
\end{align*}$$

One points out the boundary conditions: $$\frac{\partial T}{\partial z} = 0$$ at 2 ends (z=0.5 and Z = -0.5)

The temperature on the basis as of functions of forms is written in the following way:

$$T(z,t) = \sum_{i=1}^{2} T_i(t) \lambda_i(z)$$

with

$$\lambda_1(z) = 0.5(1 + 2z)$$

and

$$\lambda_2(z) = 0.5(1 - 2z)$$

The following matrices then are introduced:

$$[A] = [A_{ij}] ; A_{ij} = \int_{-0.5}^{0.5} \lambda_i \lambda_j \, dz$$
\[ B = B_y; B_y = \int_{-0.5}^{0.5} \lambda_z d\lambda_z \]

what leads to:

\[ A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \]

One notes then classically:

\[ T = \begin{bmatrix} T^1 \\ T^2 \end{bmatrix} \]

and

\[ \frac{\partial T}{\partial t} = \begin{bmatrix} \partial T^1 \\ \partial T^2 \\ \partial T^1 \\ \partial T^2 \end{bmatrix} \]

The equation (4) becomes then


with the boundary conditions imposed (null displacement in bottom and worthless flows of temperatures at the two ends), one a:

\[ b = \begin{bmatrix} 0 \\ b \end{bmatrix} \quad \text{et} \quad \frac{\partial T}{\partial t} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]

What with final gives us

\[ \begin{bmatrix} T^1 \\ T^2 \end{bmatrix} + a [A]^{-1}[B] \begin{bmatrix} T^1 \\ T^2 \end{bmatrix} = \begin{bmatrix} 0 \\ b \end{bmatrix} \]

Finally one obtains:

\[ \begin{bmatrix} 1+6a & -6a \\ -6a & 1+6a \end{bmatrix} T = \begin{bmatrix} T^1 \\ T^2 \end{bmatrix} \]

### 2.2 Sizes and results of reference

For a time court of 100s, one will have

\[ \begin{bmatrix} T^1 \\ T^2 \end{bmatrix} = \begin{bmatrix} 2.10^{-14} \\ 1.045 \cdot 10^{-7} \end{bmatrix} \]

One will consider \( T^1 \) no one.

### 2.3 Uncertainty on the solution

None
3 Modeling A

3.1 Characteristics of modeling

Voluminal modeling 3D\_THM

3.2 Characteristics of the grid

1 mesh HEXA20.

3.3 Sizes tested and results

Discretization in time: only one step of time of $100\,s$. The diagram in time is implicit ($\theta = 1$). The results correspond perfectly to the analytical solution.

<table>
<thead>
<tr>
<th>Node</th>
<th>Type of value</th>
<th>Moment ($s$)</th>
<th>Reference (analytical)</th>
<th>Tolerance (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N1, N3$</td>
<td>$TEMP$</td>
<td>$10^2$</td>
<td>$-1.045 \times 10^{-7}$</td>
<td>$0.1$</td>
</tr>
<tr>
<td>$N6, N8$</td>
<td>$TEMP$</td>
<td>$10^2$</td>
<td>$-1.045 \times 10^{-7}$</td>
<td>$0.1$</td>
</tr>
</tbody>
</table>
4 Modeling B

4.1 Characteristics of modeling

Plane modeling: D_PLAN_THM

4.2 Characteristics of the grid

1 mesh QUAD8.

4.3 Sizes tested and results

Discretization in time: only one step of time of 100s. The diagram in time is implicit ($9 = 1$). The results correspond perfectly to the analytical solution.

<table>
<thead>
<tr>
<th>Node</th>
<th>Type of value</th>
<th>Moment (S)</th>
<th>Reference (analytical)</th>
<th>Tolerance (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N3</td>
<td>TEMP</td>
<td>$10^2$</td>
<td>$-1.045 \times 10^{-7}$</td>
<td>0,1</td>
</tr>
<tr>
<td>N4</td>
<td>TEMP</td>
<td>$10^2$</td>
<td>$-1.045 \times 10^{-7}$</td>
<td>0,1</td>
</tr>
</tbody>
</table>
5 Modeling C

5.1 Characteristics of modeling

Plane modeling: D_PLAN_THM

5.2 Characteristics of the grid

2 meshes TRIA6.

5.3 Sizes tested and results

Discretization in time: only one step of time: $10^2 \text{s}$. The diagram in time is implicit ($\varphi = 1$). The results correspond perfectly to the analytical solution.

<table>
<thead>
<tr>
<th>Node</th>
<th>Type of value</th>
<th>Moment (S)</th>
<th>Reference (analytical)</th>
<th>Tolerance (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N3</td>
<td>TEMP</td>
<td>$10^2$</td>
<td>$-1.045 \times 10^{-7}$</td>
<td>0,1</td>
</tr>
<tr>
<td>N4</td>
<td>TEMP</td>
<td>$10^3$</td>
<td>$-1.045 \times 10^{-7}$</td>
<td>0,1</td>
</tr>
</tbody>
</table>
6 Modeling D

6.1 Characteristics of modeling

Voluminal modeling 3D_THM.

6.2 Characteristics of the grid

2 mgoes PENTA15.

6.3 Sizes tested and results

Discretization in time: only one step of time: $10^2 \text{s}$. The diagram in time is implicit ($\theta = 1$). The results correspond perfectly to the analytical solution.

<table>
<thead>
<tr>
<th>Node</th>
<th>Type of value</th>
<th>Moment (S)</th>
<th>Reference (analytical)</th>
<th>Tolerance (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N3</td>
<td>TEMP</td>
<td>$10^2$</td>
<td>$-1.045 \times 10^{-7}$</td>
<td>0,1</td>
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<tr>
<td>N4</td>
<td>TEMP</td>
<td>$10^2$</td>
<td>$-1.045 \times 10^{-7}$</td>
<td>0,1</td>
</tr>
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7 Summary of the results

The results are in coherence with the analytical solution.