WTNV151 – Taking into account of a hydrous condition of exchange in unsaturated

Summary:

The tests presented make it possible to check that a hydrous term of exchange at the edge is correctly taken into account in modelings AXIS_*HH2*, D_PLAN_*HH2* and 3D_*HH2*. Under certain assumptions, one reduces the problem to a problem purely hydraulics in capillary pressure. This problem is written then as an equation of heat and can thus be the analogue of a problem in thermics. One thus compares the solution of the hydraulic problem with the solution thermal problem, for which the term of exchange is already well taken into account.
1 Problem of reference

It is a question here of modelling the drying of a bar with conditions of hydrous exchange at the end and which can be to compare to a thermal problem under certain conditions. Modeling will be made in 3D, 2D (plane Deformation) and axysimetry.

1.1 Geometry With

Coordinates of the points (m):

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>C</td>
<td>0.2</td>
</tr>
<tr>
<td>B</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>D</td>
<td>0</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0.01</td>
<td>0.01</td>
<td>G</td>
<td>0.2</td>
</tr>
<tr>
<td>F</td>
<td>0.2</td>
<td>0.01</td>
<td>0.01</td>
<td>H</td>
<td>0</td>
</tr>
</tbody>
</table>

1.2 Geometry B and C

Coordinates of the points (m):

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>0</td>
<td>0</td>
<td>C</td>
<td>0.2</td>
<td>0.01</td>
</tr>
<tr>
<td>B</td>
<td>0.2</td>
<td>0</td>
<td>D</td>
<td>0</td>
<td>0.01</td>
</tr>
</tbody>
</table>

1.3 Properties of material

One gives here only the properties whose solution depends, knowing that the command file contains other data of material (moduli of elasticity, heats, porosity…) who finally do not play any part in the solution of with the dealt problem.

<table>
<thead>
<tr>
<th>Material</th>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid water</td>
<td>Density (kg.m⁻³)</td>
<td>10³</td>
</tr>
<tr>
<td>Liquid water</td>
<td>Dynamic viscosity of liquid water (Pa.s)</td>
<td>μₜₜ = 8E-4</td>
</tr>
<tr>
<td>Vapor</td>
<td>Molar mass (kg.mol⁻¹)</td>
<td>0.001</td>
</tr>
<tr>
<td>Vapor</td>
<td>Viscosity of gas (kg.m⁻¹.s⁻¹)</td>
<td>5.0 × 10⁻⁷</td>
</tr>
</tbody>
</table>

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### Conditions initial

The steam pressure is negligible. It is fixed initially on all the field with \( p_{\text{vp}} = 921.6 \) Pa.

One fixes the initial gas pressure \( p_{\text{gr}}(x,0) = 10000 \) Pa and capillary pressure on all the field with \( p_{c}(x,0) = 10000 \) Pa.

What corresponds to a saturation of 0.999.

The initial temperature is of 20°C. The mechanical constraints break up into effective constraints, \( \bar{\sigma} \) and hydraulics \( \bar{\sigma}_p \):

\[
\bar{\sigma} = \bar{\sigma}' + \bar{\sigma}_p
\]

Initially the hydraulic and effective constraints are worthless.

The other generalized constraints [see U2.04.05] are initialized to zero.

### Conditions with the limits

One imposes:

- worthless displacements in all the directions;
- a temperature, \( T \) worthless;
- a hydrous condition of exchange on the flat rim \([BC]\) in 2D or face \([BCFG]\) in 3D.

\[
\lambda_H \nabla (p_c \cdot n) = h_H (p_{\text{ext}} - p_c)
\]

with \( \lambda_H = \frac{K \times kr_w}{\mu_w} \) (in \( \text{Pa} \cdot \text{s} \)), \( h_H = 1.0 \times 10^{-14} \) and

\[
p_{\text{ext}} = 1.5 \times 10^9 \text{ Pa} ;
\]

- a hydraulic condition of flow no one on all the other edges.

---

**Gas**

<table>
<thead>
<tr>
<th>Molar mass (( \text{kg} \cdot \text{mol}^{-1} ))</th>
<th>0.001</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity of gas (( \text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1} ))</td>
<td>( 5.0 \times 10^{-7} )</td>
</tr>
</tbody>
</table>

**Dissolved air**

| Constant of Henry (\( \text{Pa} \cdot \text{m}^3 \cdot \text{mol}^{-1} \)) | \( 1.30719 \times 10^5 \) |

**Coefficients homogénéisés**

<table>
<thead>
<tr>
<th>Coefficient of Biot</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient of diffusion of Fick of the liquid medium</td>
<td>0</td>
</tr>
<tr>
<td>Coefficient of diffusion of Fick of the medium gas</td>
<td>0</td>
</tr>
<tr>
<td>Intrinsic permeability ( K ) (( m^2 ))</td>
<td>( 1.0 \times 10^{-18} )</td>
</tr>
<tr>
<td>Permeability relating to water ( kr_w(S) )</td>
<td>1</td>
</tr>
<tr>
<td>Permeability relating to gas ( kr_g(S) )</td>
<td>1</td>
</tr>
<tr>
<td>Isotherm of sorption</td>
<td>( S(p_c) = 1 - p_c \times 1E-7 )</td>
</tr>
<tr>
<td>Porosity</td>
<td>0.085</td>
</tr>
</tbody>
</table>
2 Reference solution

2.1 Validation of the term of exchange hydrous starting from the term of heat exchange

ON pose a certain number of assumptions so making the analogy with the thermal case:

- the temperature is constant (not thermal calculation);
- the gas pressure is constant;
- Lsteam pressure has is negligible;
- ON neglects the diffusion of Fick;
- LE material is indeformable (not calculations mechanics);
- Gravity is worthless;
- Water is incompressible $\rho_w = \text{cste}$.

It is pointed out that the law of Darcy (see R7.01.11) for the gas phase is written:

$$M_{gz} = -\rho_{gz} \lambda_{gz} \nabla p_{gz}$$

with:

- $M_{gz}$ L E flow of gas;
- $\rho_{gz}$ density of gas;
- $\lambda_{gz}$ the hydraulic conductivity of gas;
- $p_{gz}$ gas pressure.

However:

$$p_{gz} = \text{cste} \Rightarrow \nabla p_{gz} = 0$$

Thus:

$$M_{gz} = M_{as} + M_{vp} = 0$$

As one neglects the diffusion of Fick, then:

$$M_{ad} = M_{vp} = 0$$

By (5) and (4):

$$M_{as} = 0$$

Thus one can simplify the problem which is summarized with the conservation equation of the liquid water mass:

$$\dot{m}_w + \text{Div} \left( M_w \right) = 0$$

However $m_w = \rho_w \varphi S \left( 1 + Tr \left( \varepsilon \right) \right) - \rho_w^0 \varphi^0 S^0$, with:

- $\rho_w$ density of water, $\rho_w^0$ density in an initial state;
- $\varphi = \frac{V_{vide}}{V}$ the porosity of material, $V_{vide}$ corresponding to the pores not filled, $V$ with the porosity of material in an initial state;

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By supposing that $S = \frac{V_{\text{liq}}}{V_{\text{vide}}}$ water saturation, $S^0$ initial saturation;

- $\varepsilon$ deformation of material has;

It is considered that the material is indeformable: $\varepsilon = 0$ and $\varphi = \text{cste} = \varphi^0$ and that water is incompressible, $\rho_w = \rho_w^0$. Then:

$$m_w(t) = \rho_w^0 \varphi(S(t) - S^0)$$

What implies:

$$\dot{m}_w(t) = \rho_w \varphi \dot{S}(p_c(t)) = \rho_w \varphi S'(p_c(t)) \times \dot{p}_c(t)$$

As one neglects the effects of gravity, the law of Darcy for the liquid phase is written:

$$M_{\text{liq}} = M_w = - \rho_w \lambda_w^H(S) \nabla p_w$$

However $p_c = p_{\text{gr}} - p_{\text{liq}} = p_{\text{atm}} - p_w$. Then $\nabla p_c = - \nabla p_w$ and Donc $M_w = \rho_w \lambda_w^H(S) \nabla p_c$.

While replacing $M_w$ and $\dot{m}_w$ by their last expressions in (7) and in supposing $\lambda_w^H(S) = \frac{K \times K_w(S)}{\mu}$ constant:

$$\rho_w \varphi S'(p_c(t)) \times \dot{p}_c(t) + \rho_w \lambda_w^H(S) \text{Div}(\rho_w \dot{\nabla} p_c) = 0$$

Water is supposed to be incompressible thus:

$$\rho_w \varphi S'(p_c(t)) \times \dot{p}_c(t) + \rho_w \lambda_w^H(S) \text{Div}(\nabla p_c) = 0$$

$$\rho_w \varphi S'(p_c(t)) \times \dot{p}_c(t) + \rho_w \lambda_w^H(S) \text{Div}(\nabla p_c) = 0$$

With the operator Laplacian:

$$\rho_w \varphi S'(p_c(t)) \times \dot{p}_c(t) + \rho_w \lambda_w^H(S) \Delta p_c = 0$$

Finally:

$$\partial_t p_c + \frac{\lambda_w^H(S)}{\varphi S'(p_c)} \Delta p_c = 0$$

To close the system, one adds:

- initial condition: $p_c(x, 0) := 10\,000 \, \text{Pa}$ ;
- condition mixed on [BC] (or [BCFG] according to the geometry): $\lambda_w^H \nabla (p_c \cdot n) = h^H (p_{\text{ext}} - p_c)$.

This formulation is connected with an equation of heat without source term, of the form:

$$\rho C_p \partial_t T + \text{Div}(q) = 0$$

By taking the Fourier analysis who connects the heat flow $q$ with the variation in temperature, namely:

$$q = -\lambda \nabla T$$

By supposing that $\lambda$, is a constant, characteristic of material:

$$\partial_t T - \frac{\lambda}{\rho C_p} \Delta T = 0$$

Where $\rho$ is the density of the concrete, $C_p$ specific heat of material and $\lambda$ thermal conductivity.
The thermal problem is thus already milked in **THER_NON_LINE**. One can then compare the results of the thermal problem with those of problem THM, by taking conditions of standard closing in the same way than in case THM i.e while posing:

\[
T(x,0) = \text{cste} \\
\lambda (\nabla T \cdot n) = h_T(T_{\text{ext}} - T) \text{ au bord}
\]

While making sure of more than:

- \( T(x,0) = \text{cste} : p_c(x,0) = 10000 \)
- \(-\rho C_p := \varphi S(p_c)\)
- \(\lambda := \lambda^H\)
- \(h_T := h^H\)
- \(T_{\text{ext}} := p_{\text{ext}}\)

Thereafter, one is based then on thermal modelings **AXIS**, **PLAN_DIAG**, and **3D** that one compared to modelings **AXIS_THH2MS**, **D_PLAN_THH2MS** and **3D_THH2MS**, respectively.
3 Modeling With

3.1 Characteristics of modeling With

The results presented result from the modelisation in 3D_THH2MS. They are compared with the solution obtained in linear thermics.

To supplement, calculations are also carried out in 3D_HH2MS, in 3D_THH2S ET in in 3D_HH2S. displacements and temperatures being blocked, the results must be the same ones in all the cases. The goal is here to check the good taking into account of the term of exchange.

The grid is composed of 640 elements HEXA20.

3.2 Values tested and results

One presents profiles of capillary pressure and temperature for three moments and one checks well that the results are identical.

![Comparaison HH Thermique](image)

*Figure 5.2.a: Comparison THM/thermique in 3D with a term of exchange hydrous*

One carries out tests on three values:

<table>
<thead>
<tr>
<th>Points $(x, y)$</th>
<th>Time $(s)$</th>
<th>PRE1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(0.2, 0)$</td>
<td>60 S</td>
<td>13744</td>
</tr>
<tr>
<td></td>
<td>1800 S</td>
<td>31758</td>
</tr>
<tr>
<td></td>
<td>3600 S</td>
<td>41105</td>
</tr>
</tbody>
</table>

For reasons of time-saver modelings other than 3D_THH2MS will be tested only for 60 S.
4 Modeling B

4.1 Characteristics of modeling B

It acts of the same calculation as previously but with a modeling D_PLAN. The grid is composed of 50 elements QUAD8 and 102 elements SEG3.

4.2 Values tested and results

One presents profiles of capillary pressure and temperature for three moments and one checks well that the results are identical.

Comparaison HH Thermique

![Graph showing comparison of thermodynamic properties](image)

*Figure 4.2.a: Comparison THM/thermique in plane deformations with a term of exchange hydrous*

Results in *HH2* as a reference the results of thermal calculation have. One checks the values of the flat rim of the bar to which the condition of exchange is applied.

One carries out tests on three values:

<table>
<thead>
<tr>
<th>Points (x, y)</th>
<th>Time (s)</th>
<th>PRE1</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,2; 0)</td>
<td>60 S</td>
<td>13455</td>
</tr>
<tr>
<td></td>
<td>1800 S</td>
<td>31919</td>
</tr>
<tr>
<td></td>
<td>3600 S</td>
<td>41087</td>
</tr>
</tbody>
</table>

For reasons of time-saver modelings other than D_PLAN_THH2MS will be tested only for 60 S.
5 Modeling C

5.1 Characteristics of modeling C

It acts of the same calculation as previously but with a modeling axisymetric.

5.2 Values tested and results

One presents profiles of capillary pressure and temperature for 3 moments and one checks well that the results are identical.

![Comparison HH Thermique](image)

**Figure 3.2.a: Comparison THM /thermique in axis-symmetry with a term of exchange hydrous**

Results in *HH2* as a reference the results of thermal calculation have. One checks the values of the flat rim of the bar to which the condition of exchange is applied.

One carries out tests on three values:

<table>
<thead>
<tr>
<th>Points ((x, y))</th>
<th>Time ((s))</th>
<th>PRE1</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0,2;0))</td>
<td>60 S</td>
<td>13888</td>
</tr>
<tr>
<td></td>
<td>1800 S</td>
<td>32831</td>
</tr>
<tr>
<td></td>
<td>3600 S</td>
<td>42829</td>
</tr>
</tbody>
</table>

For reasons of time-saver modelings other than **AXIS_THH2MS** will be tested only for \(60\) s.
6 Summary of the results

The solutions exits of modelings `AXIS_*HH2*`, `D_PLAN_*HH2*` and `3D_*HH2*` are similar to solutions of reference exits of modelings `AXIS`, `PLAN_DIAG`, and `3D`, respectively. The term of exchange in hydraulics is thus correctly represented.