FDLV100 - Piston coupled to a column of fluid incompressible

Summary:

This test of the field of the fluids (coupling fluid-structure) validates the calculation of mass added on modal basis and carries out, within the framework of a modal analysis, the calculation of the Eigen frequency of a system piston-arises coupled to a column of incompressible fluid. To model the fluid, one uses thermal elements plans; to model the piston, one uses machine elements 2D in deformation planes and a discrete element to model a spring. Lastly, the fluid interface/structure is modelled by thermal linear elements modified to introduce a boundary condition of type “acceleration” into the fluid. The cas-test comprises only one modeling, two-dimensional. The Eigen frequency of the coupled system is found to 0.01% of the analytical result.
1 Problem of reference

1.1 Geometry

Steel piston connected to the solid mass by a spring and coupled to a column of incompressible fluid:

- length: \( L = 1.0 \, \text{m} \)
- width: \( d = 0.25 \, \text{m} \)
- width \( AB \) piston: \( 0.05 \, \text{m} \)

X-coordinates of the points (in \( m \)):

\[
\begin{array}{ccc}
A & B & L \\
0.05 & 0. & 1. \\
\end{array}
\]

1.2 Material properties

Fluid:
- Water: \( \rho_0 = 1000.0 \, \text{Kg.m}^{-3} \)

Solid:
- Steel: \( \rho_S = 7800.0 \, \text{Kg.m}^{-3} ; \ E = 2.11 \times 10^5 \, \text{Pa} ; \ \nu = 0.3 \)

Spring connecting the piston to the solid mass:
- Discrete element of the type \( K_{T_D_L} \): \( K = (1.5, 1.5, 1.5) \, \text{N/m} \)

1.3 Boundary conditions and loading

One imposes a pressure (i.e. by analogy thermal a worthless temperature [R4.07.03]) in all the nodes of the end of the fluid column.

One imposes the embedding of the spring on the solid mass and one imposes a displacement of the null piston according to \( O_y \).
## Reference solution

### 2.1 Method of calculating used for the reference solution

**Analytical calculation:**

When the structure vibrates in the fluid, it modifies the field of pressure which obeys an equation of Laplace with boundary conditions of Von Neuman [R4.07.03].

In our case, taking into account symmetries of the problem, the field of pressure depends only on the variable $x$ and checks:

\[
\begin{align*}
\frac{\partial^2 p}{\partial x^2} &= 0 \\
\left( \frac{\partial p}{\partial x} \right)_{x=0} &= -\rho_f \ddot{x}_S \cdot n \\
p &= 0 \quad \text{en} \quad x = L
\end{align*}
\]

One notes thus that the field of pressure is a function closely connected of the X-coordinate $x$. The two boundary conditions on the pressure imply: $p = -\rho_f \ddot{x}_S \cdot n (x - L)$

The compressive force which is exerted on the structure writes:

\[
F = \int \rho_f L \dot{x}_S \cdot n \, n \, d \Gamma
\]

As the problem is unidimensional, this force can be expressed in an algebraic way according to the component of acceleration according to $Ox$ structure:

\[
F = -\ddot{x} \int \rho_f L \, d \Gamma = -\rho_f L \ddot{x} = -m_a \ddot{x} \quad \text{with} \quad m_a = \rho_f L d
\]

It is the linear mass added of the fluid on the structure: it is noticed that it corresponds to the mass of fluid in the column, i.e. with the mass of fluid moved by the piston.

The equation of the movement of the piston projected on $Ox$ is written (free vibration not damped taking into account the presence of the fluid):

\[
m \ddot{x} + K x = F = -m_a \ddot{x} \Leftrightarrow (m + m_a) \ddot{x} + K x = 0
\]

The Eigen frequency of this immersed system is thus written:

\[
f = \frac{1}{2 \pi} \sqrt{\frac{K}{m + m_a}}
\]

The effect of the fluid is thus to lower the Eigen frequency of the system in air.

Practically, in Aster, the matrix of added mass is given on the basis of modal structure in the vacuum: To calculate the added mass given above, one restricts oneself with the calculation of the clean mode of the system piston-arises which corresponds to a translatory movement normalized with the unit: one truncates consequently the modal base of the structure to only one mode in air (operator `CALC_MODES` option 'PLUS_PETITE'). One determines thanks to this mode the mass added on the piston.

\[
K = 10^5 \text{N/m} \quad m_a = 200 \text{kg/m} \quad m = 78 \text{kg/m}
\]

The Eigen frequency of the system piston-arises immersed is thus $f = 3.018 \text{Hz}$

### 2.2 Results of reference

**Analytical**

### 2.3 Bibliographical references

3 Modeling A

3.1 Characteristics of modeling

Thermal formulation planes for fluid (QUAD4 and SEG2)
Plane and discrete deformation formulation for solid (QUAD4 and SEG2)

Cutting =
21 meshes QUAD4 according to the axis x
4 meshes QUAD4 according to the axis y
4 meshes SEG2 on the fluid interface/piston
1 mesh SEG2 representing the spring binding the piston to the solid mass

Boundary conditions:
DDL_IMPO: (GROUP_NO: noeupist DY: 0.)
DDL_IMPO: (GROUP_NO: embed DX: 0. DY: 0. DZ: 0.)

Name of the nodes:
The group of nodes NOEUPIST is made up by the ten nodes WITH, B, C, D, E, F, G, H, I, J

The group of nodes EMBED is made up by the node K

3.2 Characteristics of the grid

Many nodes: 111 nodes
Many meshes and types: 84 QUAD4, 5 SEG2
4 Results of modeling A

4.1 Values tested

<table>
<thead>
<tr>
<th>Identification</th>
<th>Reference (Hz)</th>
<th>% tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order of the clean mode (i:1)</td>
<td>3.018</td>
<td>0.1%</td>
</tr>
</tbody>
</table>

4.2 Remarks

Calculations of modes carried out by:

```
CALC_MODES
    OPTION='PLUS_PETITE',
    CALC_FREQ=_F (NMAX_FREQ=1)
```