FDLV105 - Mass added on axisymmetric piston coupled to a column of incompressible fluid

Summary:

This test of the field of the fluids implements a modal analysis on a system coupled incompressible fluid structure of standard piston column of fluid with free surface. The piston and the fluid are modelled respectively by machine elements and thermal axisymmetric. One thus validates the calculation of mass added in axisymmetric configuration.
1 Problem of reference

1.1 Geometry

The system consists of a circular piston vibrating in contact with an annular column of fluid finished by a free surface:
- Length: 1 m
- Width: 0.03 m
- Height: 0.025 m

The axis of revolution is the axis $OY$ reference mark. $OX$ indicate the radial axis.

1.2 Properties of materials

Structure: steel - material elastic
- $E = 2.10^{13}$ Pa
- $\nu = 0.3$
- $\rho_s = 7800$ kg/m$^3$

The spring has a stiffness by radian of $K = \frac{10^5}{2\pi}$ N/m/rad

Fluid: thermal material are equivalent
- $\lambda = 1.$
- $\rho_f C_p = 1000.$

1.3 Boundary conditions and loadings

Side structure: the degree of freedom $DX$ of all the nodes of the structure is blocked: $DX: 0.0$

Fluid side: one imposes a pressure (i.e. temperature) worthless on the nodes of free surface.
2 Reference solution

2.1 Method of calculating used for the reference solution

One solves the analytically following coupled problem:

\[
\begin{align*}
    m \ddot{y} + k y &= F \\
    \frac{\partial^2 p}{\partial y^2} &= 0 \\
    \left( \frac{\partial p}{\partial y} \right) &= \rho_f \ddot{y}
\end{align*}
\]

with

- $F$ hydrodynamic compressive force on the piston
- $P$ hydrodynamic pressure in the fluid
- $m, k$: mass and stiffness of the piston by radian

The hydrodynamic field of pressure in the fluid is written:

\[ p = -\rho_f \ddot{y} (y - l) \]

from where the compressive force being exerted on the piston:

\[ F = \int_0^l p n \ r \ dr = -\rho_f \ddot{y} l \frac{e^2}{2} \]

the mass added by radian is worth:

\[ m_a = \rho_f l \frac{e^2}{2} \]

the clean mode of the coupled system is worth:

\[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m + m_a}} = 27.25 \text{ Hz} \]

2.2 Results of reference

Analytical.

2.3 References bibliographical

3 Modeling A

3.1 Characteristics of modeling

- Side structure:
  8 axisymmetric machine elements MEAXQU4,
  1 specific element of type K_T_N modelling the spring,

- fluid side:
  380 axisymmetric thermal elements THAXQU4 modelling the fluid,
  8 axisymmetric thermal elements THAXSE2 modelling the fluid interface/structure.

3.2 Characteristics of the grid

<table>
<thead>
<tr>
<th>Side structure</th>
<th>Fluid side</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 meshes QUAD4 5 maille POI1</td>
<td>8 meshes SEG2 320 meshes QUAD4</td>
</tr>
<tr>
<td>Many meshes: 337</td>
<td></td>
</tr>
</tbody>
</table>

3.3 Values tested

<table>
<thead>
<tr>
<th>Identification</th>
<th>Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>N°1 mode</td>
<td>27.25</td>
</tr>
</tbody>
</table>

4 Summary of the results

The calculation of mass added on axisymmetric elements is very well carried out. It will be noted however that the calculated quantities are masses added by radian (divided by $\frac{2\pi}{2}$).