SZLZ105 - Counting of cycles by RAINFLOW and calculation of the damage

Summary:

Transitory linear elastic problem quasi-static in mechanics of the structures.

Calculation of the final damage in an element subjected to a cyclic loading, with a linear elastic behavior.

A modeling in plane constraints and a modeling in 3D.

This test validates the method of counting of cycles (RAINFLOW) established in the operator CALC_FATIGUE as well as the method of calculating of the damage in imposed constraint (curve of Wöhler) or imposed deformation (curve of Manson-Whetstone sheath). The reference solution is an analytical solution.

It also validates the calculation of the constraints and deformations equivalent using the options SIEQ_ELGA, SIEQ_ELNO, EPEQ_ELGA, EPEQ_ELNO, EPMQ_ELGA and EPMQ_ELNO.
1 Problem of reference

1.1 Geometry

1.2 Material properties

- Linear elasticity: $E = 1 \text{ MPa}$, $\nu = 0.3$

1.3 Boundary conditions and loadings

- Blocked on face $1 - 4$ according to $X$ - node 1 blocked according to $Y$.
- In unit simple traction on the face $2 - 3$.
- Loading $F(t)$ in teeth of saw (according to the Article of Downing and Socie 1982) [bib1].

1.4 Initial conditions

Worthless constraints and deformations.
2 Reference solution

2.1 Method of calculating used for the reference solution

Analytical solution

- calculation of the constraints and deformations. For a loading in simple traction, one obtains a homogeneous state of stress uniaxial in any point:

\[
\sigma = \begin{bmatrix}
\sigma & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\quad \text{and} \quad
\varepsilon = \begin{bmatrix}
\varepsilon & 0 & 0 \\
0 & \gamma & 0 \\
0 & 0 & \gamma
\end{bmatrix}
\]

the equivalent sizes are thus

\[
\begin{align*}
\sigma_{VMIS} &= |\sigma| = \sigma_{TRESCA} \\
\sigma_{VMIS-SG} &= \sigma \\
\varepsilon_{INVA-2} &= \frac{2}{3} |\varepsilon - \gamma| \\
\varepsilon_{INVA-2SG} &= \frac{2}{3} |\varepsilon - \gamma| \times \text{sign}\left(\frac{\varepsilon + 2 \gamma}{3}\right)
\end{align*}
\]

- then manual calculation of the cycles by the method of RAINFLOW, as well as amplitudes of loading (\(\frac{\Delta \sigma}{2}\) or \(\frac{\Delta \varepsilon}{2}\)).

<table>
<thead>
<tr>
<th>cycles</th>
<th>(\Delta \sigma / 2)</th>
<th>(\Delta \varepsilon_{INVA-2}/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.00</td>
<td>0.8667</td>
</tr>
<tr>
<td>2</td>
<td>0.50</td>
<td>0.433315</td>
</tr>
<tr>
<td>3</td>
<td>1.00</td>
<td>0.8667</td>
</tr>
<tr>
<td>4</td>
<td>3.50</td>
<td>3.03335</td>
</tr>
</tbody>
</table>

- finally carryforward of these values on the curves of Wöhler or Manson-Whetstone sheath to consider the damage unit at each cycle \(i\), that is to say \(D_u_i = \frac{1}{N_i}\) (\(N_i\) being the number of cycles with rupture for a given amplitude), as well as the cumulated damage \(D = \sum_i D_u_i\) (linear rule of office plurality TO MINE).

Note:

One will use as equivalent constraint \(\sigma_{VMIS-SG}\) and like equivalent deformation

\[
\varepsilon_{INVA-2SG} = \frac{2}{3} |\varepsilon - \gamma| \times \text{sign}\left(\frac{\varepsilon + 2 \gamma}{3}\right).
\]

2.2 Results of reference

- Being given the values of the parameters of loading used, one obtains simply at the end of the loading (increment 8) \(\sigma = -3.0\), \(\varepsilon = -3.0\), \(\gamma = 0.9\), \(\varepsilon_{INVA-2} = 2.6\).

- For the calculation of the damage, one obtains:

\[
D_{\text{Wöhler}} = 4.8133 \times 10^{-3} = \sum_{i=1}^{4} D_u_i
\]

\[
D_{\text{Manson}} = 4.67 \times 10^{-3} = \sum_{i=1}^{4} D_u_i
\]

2.3 Bibliographical references

3 Modeling A

3.1 Characteristics of modeling

Modeling in plane constraints:

![Diagram of modeling in plane constraints]

3.2 Characteristics of the grid

1 mesh QUAD4.

Square width = 1
thickness = 1

3.3 Sizes tested and results

<table>
<thead>
<tr>
<th>Identification</th>
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</tr>
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<tbody>
<tr>
<td>in all nodes at the end of the loading in constraint or deformation</td>
<td></td>
</tr>
<tr>
<td>Wöhler damage</td>
<td>$4.8133 \times 10^{-3}$</td>
</tr>
<tr>
<td>Damage Manson-Whetstone sheath</td>
<td>$4.6705 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>-3.</td>
</tr>
<tr>
<td>$\sigma_{\text{VMIS}}$</td>
<td>3.</td>
</tr>
<tr>
<td>$\sigma_{\text{TRESCA}}$</td>
<td>3.</td>
</tr>
<tr>
<td>$\sigma_{\text{VMIS-SG}}$</td>
<td>-3.</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>-3.</td>
</tr>
<tr>
<td>$\varepsilon_{\text{INVA-2}}$</td>
<td>2.6</td>
</tr>
<tr>
<td>$\varepsilon_{\text{INVA-2-SG}}$</td>
<td>-2.6</td>
</tr>
</tbody>
</table>

3.4 Remarks

Fast test in time calculation.
4 Modeling B

4.1 Characteristics of modeling

Modeling in 3D:

![3D Model Diagram]

4.2 Characteristics of the grid

1 mesh **HEXA8**, cubic of width = 1

4.3 Sizes tested and results

<table>
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</tr>
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<td>$-3.$</td>
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<tr>
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</tr>
<tr>
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<td>$3.$</td>
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<td>$\sigma_{VMIS−SG}$</td>
<td>$-3.$</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>$-3.$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$0.9$</td>
</tr>
<tr>
<td>$\varepsilon_{INVA−2}$</td>
<td>$2.6$</td>
</tr>
<tr>
<td>$\varepsilon_{INVA−2,SG}$</td>
<td>$-2.6$</td>
</tr>
<tr>
<td>$(\varepsilon − \varepsilon^h)_{INVA−2}$</td>
<td>$2.6$</td>
</tr>
<tr>
<td>$(\varepsilon − \varepsilon^h)_{INVA−2,SG}$</td>
<td>$-2.6$</td>
</tr>
</tbody>
</table>

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4.4 Remarks

Same results and reference that in plane constraints.
5 Summary of the results

This test validates the method and the calculation of the damage of Wöhler and Manson-Whetstone sheath.

Results of Code_Aster are identical to those obtained analytically.