Conditions of connection of Summarized

solid body

One presents in this documentation a way model indeformable parts of structure, in small displacements and rotations, thanks to key word LIAISON_SOLIDE of AFFE_CHAR_MECA.

Are contents

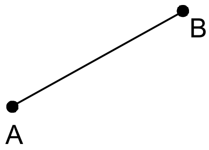
1 Introduction

key word `LIAISON_SOLIDE` of commands `AFFE_CHAR_MECA` and (`AFFE_CHAR_MECA_F`) makes it possible to model an indeformable part of a structure.

The principle selected is to write linear relations between the degrees of freedom of the "solid" part; these relations expressing the fact that the distances between the nodes are invariable.

Notice important:

The relations expressing the indeformability of a solid are in general not linear. The linearization of the problem supposes that the problem can be solved in theory of "small displacements". To be convinced some, let us take the example of a segment AB in 2D:



The indeformability of AB is written:

$$\left[\left(x_A+dx_A\right)-\left(x_B+dx_B\right)\right]^2+\left[\left(y_A+dy_A\right)-\left(y_B+dy_B\right)\right]^2=\left(x_A-x_B\right)^2+\left(y_A-y_B\right)^2$$
$$\Leftrightarrow\left(dx_B-dx_A\right)^2+2\left(x_B-x_A\right)\left(dx_B-dx_A\right)+\left(dy_B-dy_A\right)^2+2\left(y_B-y_A\right)\left(dy_B-dy_A\right)=0$$

$$\text{en notant}\left\{\begin{matrix} x_A,y_A,x_B,y_B \text{ les coordonnées de A et B} \\ dx_A,dy_A,dx_B,dy_B \text{ les déplacements de A et B}\end{matrix}\right\}$$

it is seen that the statement is quadratic in dx_A,dx_B,dy_A and dy_B . To be able to linearize it, it is necessary to eliminate the quadratic terms and for this reason, one is obliged to suppose that the elements dx_A,dx_B,dy_A and dy_B are small compared to the length of AB .

This remark wants to say that one cannot use this key word when the structure becomes deformed (or turns) too much. In such situations, "to rigidify" a solid part, one is obliged to use a "tough" material (compared to the rest of structure).

2 Principle of the use of the key word

key word `LIAISON_SOLIDE` is a key word factor répétable at will. For each occurrence key word, the user defines a "piece of model" which it wishes to rigidify.

This "piece of model" defined by key keys `GROUP_MA`, `GROUP_NO`, `MESH` and `NOEUD`, one deduces **the list from the nodes** to be rigidified.

Once this drawn up list, one writes the linear relations necessary to express that the "rigid piece" has nothing any more but the degrees of freedom of a solid (3 in 2D or 6 in 3D).

3 Which are the treated cases?

According to the degrees of freedom carried by the nodes of the list of the nodes to rigidify, one places oneself in one of the four following cases. If one does not find oneself in one of these cases, the code stops in fatal error

the cases $2DA$ and $2DB$ correspond to plane" or axisymmetric problems ".
The cases $3DA$ and $3DB$ correspond to problems 3D.

Case $2DA$ **:**

All the nodes of the list of the nodes to be rigidified carry the degrees of freedom DX, DY (and possibly DRZ) but they do not carry DRX, DRY and DZ there exists at least a node of the list of the nodes to be rigidified which carries DRZ.

Case $2DB$ **:**

All the nodes of the list of the nodes to be rigidified carry DX, DY but they do not carry DRX, DRY and DZ.

Case $3DA$ **:**

All the nodes of the list of the nodes to be rigidified carry DX DY, DZ (and possibly DRX DRY, DRZ) and it exists a node of the list of the nodes to be rigidified which carries DRX DRY DRZ.

Case $3DB$ **:**

All the nodes of the list of the nodes to be rigidified carry DX DY, DZ and it does not exist of node of the list of the nodes to be rigidified carrying at the same time DRX DRY DRZ.

4 Processing of cases 2DA and 3DA

In these 2 cases, one could find a node of the list of the nodes to be rigidified which carried **all** the degrees of freedom of solid. That is to say A this node.

in 2D: $DX \quad DY$, DRZ
in 3D: $DX \quad DY \quad DZ \quad DRX \quad DRY$, DRZ

That is to say a node M of the list of the nodes to be rigidified unspecified.

In theory of small displacements, the motion of a solid body is expressed by:

$$\boldsymbol{U}_M = \boldsymbol{U}_A + \boldsymbol{\theta} \wedge \mathbf{AM} \text{ where } \begin{Bmatrix} \boldsymbol{U}_A \text{ est de déplacement de A} \\ \boldsymbol{\theta} \text{ le vecteur rotation du solide} \end{Bmatrix}$$

4.1 Case 2DA

One writes the linear relations:

$$\forall M \neq A : \begin{Bmatrix} DX(M) - DX(A) + y\, DRZ(A) = 0 \\ DY(M) - DY(A) - x\, DRZ(A) = 0 \end{Bmatrix} \quad avec\ \boldsymbol{AM} = \begin{pmatrix} x \\ y \end{pmatrix}$$
$$+ \text{si } M \text{ porte DRZ} : DRZ(M) - DRZ(A) = 0$$

4.2 Case 3DA

$$\forall M \neq A : \begin{Bmatrix} DX(M) - DX(A) - DRY(A).z + DRZ(A).y = 0 \\ DY(M) - DY(A) - DRZ(A).x + DRX(A).z = 0 \\ DZ(M) - DZ(A) - DRX(A).y + DRY(A).x = 0 \end{Bmatrix}$$

$$+ \text{si } M \text{ porte } DRX, DRY, \text{DRZ} : \begin{Bmatrix} DRX(M) - DRX(A) = 0 \\ DRY(M) - DRY(A) = 0 \\ DRZ(M) - DRZ(A) = 0 \end{Bmatrix}$$

5 Processing of the case 2DB

5.1 general Case

$\exists A$ and B in the list of the nodes to be rigidified/ $d(A, B) \neq 0$

- determination of θ :

  That is to say $\boldsymbol{n} = \mathbf{AB} \wedge \boldsymbol{k} = \begin{pmatrix} n_x \\ n_y \end{pmatrix}$ (k unit vector according to O_z).

  $$\boldsymbol{U}_B - \boldsymbol{U}_A - \boldsymbol{\theta}\, \boldsymbol{k} \wedge \mathbf{AB} \Rightarrow \left\{ \begin{array}{l} (\boldsymbol{U}_B - \boldsymbol{U}_A) \,.\, \mathbf{AB} = 0 \\ (\boldsymbol{U}_B - \boldsymbol{U}_A) \,.\, \boldsymbol{n} - \boldsymbol{\theta} (\boldsymbol{k} \wedge \mathbf{AB}) \,.\, \boldsymbol{n} = 0 \end{array} \right\}$$

- since $\|\mathbf{AB}\| \neq 0$, one can determine $\boldsymbol{\theta}$:

  $$\Rightarrow \boldsymbol{\theta} = \frac{1}{(kL\,\mathbf{AB}) \,.\, n} \left(DX(B) \,.\, n_x - DX(A) \,.\, n_x + DY(B) \,.\, n_y - DY(B) \,.\, n_y \right)$$

  $$\text{Soit}\; \alpha = \frac{1}{(kL\,\mathbf{AB}) \,.\, n} \,;\, n' = \alpha n = \begin{pmatrix} n'_x \\ n'_y \end{pmatrix}$$

- equations to be written:
  - $(U_B - U_A) \,.\, \mathbf{AB} = 0 \,(1$ (équation pour les 2 points A et $B)$
  - $\forall M \neq (A, B) : \mathbf{AM} = \begin{pmatrix} x \\ y \end{pmatrix}$

  $$\left\{ \begin{array}{l} DX(M) - DX(A) + y\left(DX(B) \,.\, n'_x - DX(A) \,.\, n'_x + DY(B) \,.\, n'_y - DY(A)\, n'_y \right) = 0 \\ DY(M) - DY(A) - x\left(DX(B) \,.\, n'_x - DX(A) \,.\, n'_x + DY(B) \,.\, n'_y - DY(A)\, n'_y \right) = 0 \end{array} \right\}$$

5.2 Typical cases

- lists nodes to be rigidified = $\{A\} \rightarrow$ it does not have nothing there to write $\Rightarrow \langle A \rangle$ tear,
- list of the nodes to be rigidified = $\{A_i\}$ where all have A_i them the same coordinates.

  That is to say A_o the first node of the list of the nodes to be rigidified

  $\forall A_i \neq A_o$ Note: should $\left\{ \begin{array}{l} DX(A_i) - DX(A_0) = 0 \\ DY(A_i) - DY(A_0) = 0 \end{array} \right.$

  be written:

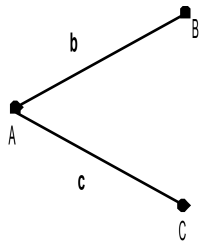
  | θ *is undetermined*

6 Processing of case 3DB

6.1 general Case

$\exists A, B, C$ in the list of the nodes to be rigidified such as ABC is a triangle of non-zero surface

6.1.1 Processing of the points A, B, C and determination of the vector rotation θ



$$\boldsymbol{b}=\mathbf{AB}\,;\boldsymbol{c}=\mathbf{AC}\,;\boldsymbol{m}=\mathbf{AM}$$

Soit $\boldsymbol{n}=\boldsymbol{b}\wedge\boldsymbol{c}$
$\boldsymbol{b}'=\boldsymbol{b}\wedge\boldsymbol{n}\,;\boldsymbol{c}'=\boldsymbol{c}\wedge\boldsymbol{n}$

$$\forall M\,,\boldsymbol{U}_M-\boldsymbol{U}_A=\boldsymbol{\theta}\wedge\boldsymbol{M} \quad \text{éq 6.1-1}$$

- for the points B and C : $\begin{cases}\boldsymbol{U}_B-\boldsymbol{U}_A=\boldsymbol{\theta}\wedge\boldsymbol{b}\\ \boldsymbol{U}_C-\boldsymbol{U}_A=\boldsymbol{\theta}\wedge\boldsymbol{c}\end{cases}$

$$\boldsymbol{U}_B-\boldsymbol{U}_A.\boldsymbol{n}=\boldsymbol{\theta}.\boldsymbol{b}' \quad \text{éq 6.1-2}$$
$$\boldsymbol{U}_B-\boldsymbol{U}_A.\boldsymbol{c}=\boldsymbol{\theta}.\boldsymbol{n} \quad \text{éq 6.1-3}$$
$$\boldsymbol{U}_B-\boldsymbol{U}_A.\boldsymbol{b}=0 \quad \text{éq 6.1-4}$$
$$\boldsymbol{U}_C-\boldsymbol{U}_A.\boldsymbol{n}=\boldsymbol{\theta}.\boldsymbol{c}' \quad \text{éq 6.1-5}$$
$$\boldsymbol{U}_C-\boldsymbol{U}_A.\boldsymbol{b}=-\boldsymbol{\theta}.\boldsymbol{n} \quad \text{éq 6.1-6}$$
$$\boldsymbol{U}_C-\boldsymbol{U}_A.\boldsymbol{c}=0 \quad \text{éq 6.1-7}$$
$$\boldsymbol{U}_B-\boldsymbol{U}_A.\boldsymbol{c}+\boldsymbol{U}_C-\boldsymbol{U}_A.\boldsymbol{b}=0 \quad \text{éq 6.1-8}$$

Of the 6 equations concerning the points B and C ,

- 3 are to be written: [éq 6.1-4], [éq 6.1-7] and [éq 6.1-8] (they do not utilize θ)
- 3 are used to determine θ :

$$\begin{Bmatrix}\boldsymbol{\theta}.\boldsymbol{b}'=\left(\boldsymbol{U}_B-\boldsymbol{U}_A\right).\boldsymbol{n}\\ \boldsymbol{\theta}.\boldsymbol{c}'=\left(\boldsymbol{U}_C-\boldsymbol{U}_A\right).\boldsymbol{n}\\ \boldsymbol{\theta}.\boldsymbol{n}'=\left(\boldsymbol{U}_B-\boldsymbol{U}_A\right).\boldsymbol{c}\end{Bmatrix} \quad \text{éq 6.1-9}$$

6.1.2 Relations concerning a point $M \neq (A, B, C)$

ls $\left\{ \begin{array}{l} U_{ABCM} \text{ le vecteur } : (U_A, V_A, W_A, \boldsymbol{U_B}, V_B, \ldots\ldots, W_C, U_M, V_M, W_M) \in R^{12} \\ \boldsymbol{\theta} \text{ le vecteur } (q_x, q_y, q_z) \in R^3 \end{array} \right\}$

the equation [éq 6.1-9] can be written: $\boldsymbol{M_1}.\boldsymbol{\theta} = \boldsymbol{M_2} \boldsymbol{U_{ABCM}}$

$$\text{avec } \boldsymbol{M_1} = \begin{bmatrix} b'_x & b'_y & b'_z \\ c'_x & c'_y & c'_z \\ n_x & n_y & n_z \end{bmatrix} M_1 \text{ est inversible car ABC est de surface non nulle}$$

$$\text{et } \boldsymbol{M_2} = \begin{bmatrix} -n_x & -n_y & -n_z & n_x & n_y & n_z & 0 & 0 & 0 & 0 & 0 & 0 \\ -n_x & -n_y & -n_z & 0 & 0 & 0 & n_x & n_y & n_z & 0 & 0 & 0 \\ -c_x & -c_y & -c_z & c_x & c_y & c_z & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{\theta} = \boldsymbol{M_1^{-1}} \boldsymbol{M_2}.\boldsymbol{U_{ABCM}} \qquad \text{éq 6.1-10}$$

the equation [éq 6.1-1] $\boldsymbol{U_M} - \boldsymbol{U_A} - \boldsymbol{\theta} \wedge \boldsymbol{M} = 0$ can be written:

$$\boldsymbol{M_4}.\boldsymbol{U_{ABCM}} + \boldsymbol{M_3}.\boldsymbol{\theta} = 0 \qquad \text{éq 6.1-11}$$

$$\text{avec } \boldsymbol{M_3} = \begin{bmatrix} 0 & -M_z & M_y \\ M_z & 0 & -M_x \\ -M_y & M_x & 0 \end{bmatrix} \text{où } \boldsymbol{M} = (M_x, M_y, M_z)$$

$$\text{et } \boldsymbol{M_4} = \begin{bmatrix} -1 & & & & & 1 & & \\ & -1 & & 0 & 0 & & 1 & \\ & & -1 & & & 0 & & 1 \end{bmatrix}$$

$$\boldsymbol{M_4}.\boldsymbol{U_{ABCM}} + \boldsymbol{M_3} \boldsymbol{M_1^{-1}} \boldsymbol{M_2}.\boldsymbol{U_{ABCM}} = 0 \Leftrightarrow \boldsymbol{M_5}.\boldsymbol{U_{ABCM}} = 0$$

$M \neq (A, B, C)$, the 3 equations should be written corresponding to the 3 lines of the matrix $\boldsymbol{M_5} = \boldsymbol{M_4} + \boldsymbol{M_3} - \boldsymbol{M_1^{-1}} \boldsymbol{M_2}$

6.1.3 Summarized of the equation to write

- computation of $\boldsymbol{b}, \boldsymbol{c}, \boldsymbol{n}, \boldsymbol{b}', \boldsymbol{c}'$
- computation of $\boldsymbol{M}_1^{-1} \boldsymbol{M}_2$
- for the points B and C: $\begin{cases} \boldsymbol{U}_B - \boldsymbol{U}_A . \boldsymbol{b} = 0 \\ \boldsymbol{U}_C - \boldsymbol{U}_A . \boldsymbol{c} = 0 \\ \boldsymbol{U}_B - \boldsymbol{U}_A . \boldsymbol{c} + \boldsymbol{U}_C - \boldsymbol{U}_A . \boldsymbol{b} = 0 \end{cases}$
- $M \neq (A, B, C)$:
  - computation of $\boldsymbol{M}_5 = \boldsymbol{M}_4 + \boldsymbol{M}_3 - \boldsymbol{M}_1^{-1} \boldsymbol{M}_2$
  - writing of the 3 equations corresponding to M

6.2 Typical cases

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- list of the nodes to be rigidified = $\{A_i\}$ where all have A_i them the same coordinates.

  That is to say A_o the first point of the list of the nodes to be rigidified

$$\forall A_i \neq A_o \begin{cases} DX(A_i) - DX(A_0) = 0 \\ DY(A_i) - DY(A_0) = 0 \\ DZ(A_i) - DZ(A_0) = 0 \end{cases}$$

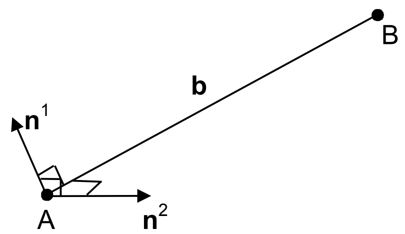
  θ is undetermined, which does not pose a problem.

- list nodes to be rigidified = $\{A_i\}$ where all them A_i are aligned (right Δ).

  The solid $\{A_i\}$ does not have any more whereas 5 possible rigid body motions.

  It misses rotation around Δ .

  That is to say:
  - two points A et $B / \|\mathbf{AB}\| \neq 0$
  - $\boldsymbol{b} = \mathbf{AB}$
  - n_1 an orthogonal non-zero vector with $\boldsymbol{b}$ (thus with Δ)
  - $\boldsymbol{n}_2 = \boldsymbol{b} \wedge \boldsymbol{n}_1$



- point: B $\boldsymbol{U_B}-\boldsymbol{U_A}=\boldsymbol{\theta}\wedge\boldsymbol{b}$

$$\left(\boldsymbol{U_B}-\boldsymbol{U_A}\right).\boldsymbol{b}=0 \qquad \text{éq 6.2-1}$$

$$\left(\boldsymbol{U_B}-\boldsymbol{U_A}\right).\boldsymbol{n_1}=\left(\boldsymbol{n_1}\wedge\boldsymbol{\theta}\right).\boldsymbol{b} \qquad \text{éq 6.2-2}$$

$$\left(\boldsymbol{U_B}-\boldsymbol{U_A}\right).\boldsymbol{n_2}=\left(\boldsymbol{n_2}\wedge\boldsymbol{\theta}\right).\boldsymbol{b} \qquad \text{éq 6.2-3}$$

- the equation [éq 6.2-1] is to be written
- the equations [éq 6.2-2] and [éq 6.2-3] are used to calculate q

the component of q on $\boldsymbol{b}$ is undetermined, one does not take account of it:

$$\boldsymbol{\theta}=\theta_1\boldsymbol{n_1}+\theta_2\boldsymbol{n_2}$$

$$\begin{cases}\left(\boldsymbol{n_1}\wedge\boldsymbol{\theta}\right)=\theta_2\boldsymbol{n_1}\wedge\boldsymbol{n_2} & \text{soit } k=\boldsymbol{n_1}\wedge\boldsymbol{n_2}.\boldsymbol{b} \qquad \boldsymbol{k}\neq 0\\ \left(\boldsymbol{n_2}\wedge\boldsymbol{\theta}\right)=\theta_1\boldsymbol{n_2}\wedge\boldsymbol{n_1}\Rightarrow\begin{cases}k\theta_1=-\left(\boldsymbol{U_B}-\boldsymbol{U_A}\right).\boldsymbol{n_2}\\ k\theta_2=-\left(\boldsymbol{U_B}-\boldsymbol{U_A}\right).\boldsymbol{n_1}\end{cases}\end{cases}$$

$$\boldsymbol{k}\theta=k\theta_1\boldsymbol{n_1}+k\theta_2\boldsymbol{n_2}$$

$$\text{Soit } \boldsymbol{n_1}=\begin{pmatrix}a\\b\\c\end{pmatrix} \;;\; \boldsymbol{n_2}=\begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix} \;;\; \mathbf{U}_{\boldsymbol{ABM}}=\left(U_A,V_A,W_A,U_B,V_B,W_B,U_M,V_M,W_M\right)\in R^9$$

$$k\boldsymbol{\theta}=\boldsymbol{M_1}\boldsymbol{U_{ABM}}\text{avec}:\boldsymbol{M_1}=\left[M_2,-M_2,0\right]$$

$$\boldsymbol{M_2}=\begin{bmatrix}0 & \beta a-b\alpha & \gamma a-c\alpha\\ b\alpha-a\beta & 0 & \gamma b-c\beta\\ c\alpha-\gamma a & c\beta-\gamma b & 0\end{bmatrix}$$

$$\forall M\left(A,B\right) \qquad \boldsymbol{U_M}-\boldsymbol{U_A}-\boldsymbol{\theta}\wedge\boldsymbol{m}=0 \qquad \boldsymbol{m}=\mathbf{AM}=\left(M_x,M_y,M_z\right)$$

$$\Rightarrow\boldsymbol{M_4}\boldsymbol{U_{ABM}}+\boldsymbol{M_3}\boldsymbol{\theta}=0$$

- $$\text{avec } \boldsymbol{M_3}=\begin{bmatrix}0 & -M_z & M_y\\ M_z & 0 & -M_x\\ -M_y & M_x & 0\end{bmatrix}\boldsymbol{M_4}=\begin{bmatrix}-1 & 0 & 0 & & & & 1 & & \\ 0 & -1 & 0 & & 0 & & & 1 & \\ 0 & 0 & -1 & & & & & & 1\end{bmatrix}$$

$$\Rightarrow\boldsymbol{M_5}\boldsymbol{U_{ABM}}=0 \qquad \text{avec } M_5=\boldsymbol{M_4}+\frac{1}{k}\boldsymbol{M_3}.\boldsymbol{M_1}$$

for each point M, the 3 equations should be written corresponding to the 3 lines of the matrix $\boldsymbol{M_5}$.

- abstract of the equations to be written:
  - computation of $\boldsymbol{b}, \boldsymbol{n}_1, \boldsymbol{n}_2, k$
  - computation of $\boldsymbol{M}_1$
  - for point: B $\boldsymbol{U}_B - \boldsymbol{U}_A . \boldsymbol{b} = 0$
  - $M \neq (A, B)$
    - computation of $\boldsymbol{M}_5 = \boldsymbol{M}_4 + \frac{1}{k} \boldsymbol{M}_3 . \boldsymbol{M}_1$
    - writing of the 3 equations corresponding to $\boldsymbol{M}_5$

7 How detecting the typical cases?

In the paragraphs [§6] and [§7], we saw that it could arrive of the typical cases when some nodes geometrically were confused or aligned on the same line.

The numerical criteria selected to detect these typical cases are:

- 2 points A and B coincide if:
  $\|\mathbf{AB}\| \leq 10^{-6} . DMIN$
- 3 points A, B, C are aligned if:
  $\left(\|\mathbf{AB} \wedge \mathbf{AC}\|\right)^{1/2} \leq 10^{-6} . DMIN$

where: $DMIN$ note the length of the smallest stops meshes mesh.

8 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
5	*J.PELLET (EDF/MTI/MMN)*	initial Text