Connections 3D – Beam, 2D – Beam

Summarized:

This document explains the principle retained in *Code_Aster* to connect a modelization continuum 3D or 2D and a beam modelization.

In 3D, this connection results in 6 linear relations connecting displacements of the set of nodes 3D (3 degrees of freedom per node) dependant with the node of beam with the 6 degrees of freedom of this node.

In 2D, this connection results in 3 linear relations connecting displacements of the set of nodes 2D (2 degrees of freedom per node) dependant with the node of beam with the 3 degrees of freedom of this node.

Contents

1 Presentation

the command employed to treat the connections between beam elements and elements 2D or 3D is `AFFE_CHAR_MECA` [U4.25.01], key words `LIAISON_ELEM` and `OPTION = "3D_POU"` or `"2D_POU"`.

2 The connection excluded

2.1 3D-beam Purposes and solutions

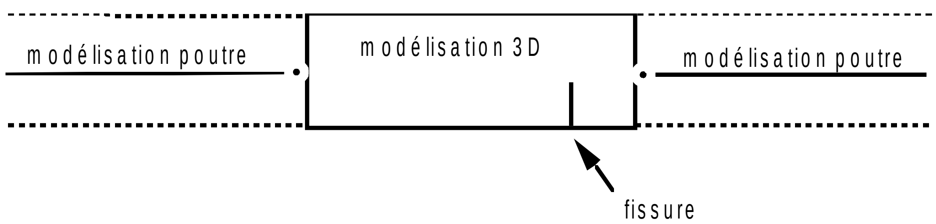
When one wishes to finely analyze part of a slender structure complexes [Figure 2.1-a], one can, to minimize the size of the mesh to be handled, to want to represent structure by a beam "far" from the part being analyzed. The goal of schematization by a beam is 3D to bring realistic boundary conditions to edges of the part modelled and with a grid in continuum. The connection 3D - Beam must thus meet the following requirements:

P1 Power to transmit the forces of beam (torsor) to the mesh 3D

P2 not to generate "parasitic" stresses (even of stress concentration), because it would then be necessary to place the connection far from the zone to be sufficiently analyzed so that these disturbances are attenuated in the zone of study.

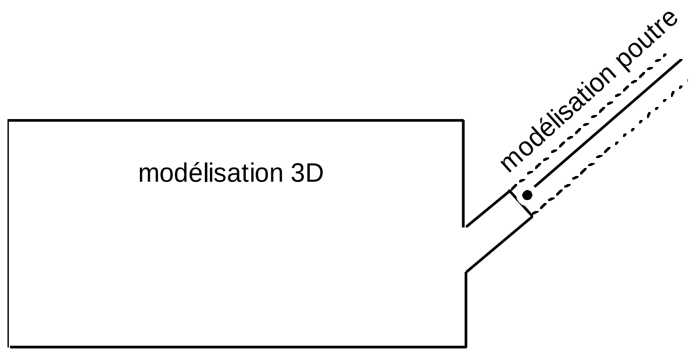
P3 not to support the kinematical conditions or the static conditions of connection one compared to the other. It must be equivalent to bring back a load vector force or of displacement to the limits of the field 3D.

P4 Admettre unspecified behaviors on both sides of connection (elasticity, plasticity…) and to also allow a dynamic analysis.



Appear 2.1-a

If these goals are achieved one will be able to also use the rules of connection 3D to deal with the problem of the fixed support of a beam in a solid mass. However the distribution of the stresses in the solid mass around the fixed support will remain rather coarse and will have to be used with precaution. It is preferable to net the connection in 3D then to prolong the starter of the mesh 3D of the section of beam by one of the beam elements with connection 3D/Poutre [Figure 2.1-b].



Appear 2.1-b

Within sight of purposes 1 to 4, one can eliminate two current techniques of connection right now:

1) the first which brings back all the connection to the processing of conditions of connections between the points in opposite to the intersection of the neutral axis of the beam and solid 3D. Except the difficulty in defining correctly the “specific” rotation of the material point belonging to solid 3D, one concentrates the forces (concentrated reaction, couple) in this point and one breaks kinematical/static symmetry by privileging a particular kinematics.
2) the second solution which completely imposes a displacement of beam (NAVIER - BERNOULLI) on the points of the solid mass 3D being at the intersection of solid 3D and of the section of the beam. In elasticity, it is known that the assumption of indeformability of the sections in their plane is only one approximation. Correct from the energy point of view for the beam, it leads to stress concentrations in the vicinity of the limits of the section of junction for solid 3D.

Note:

It goes without saying that all that is presented here is valid only on the assumption of the small disturbances (small displacements).

2.2 Directional sense

We will leave the machine elements of the connection:

the field of definite vector $\boldsymbol{\sigma}.n$ forced on the trail of the section S of the beam on the solid mass 3D, n being the norm with the plane of S,
and the field of displacement $\mathbf{u}^{\mathbf{3D}}$ defined on this same field,

for three-dimensional solid, like:

the torsor $\mathbf{T}$ of the forces of beam in the geometrical center of inertia G of S ,
and the torsor $\mathbf{D}$ of displacements of beam in this same point,

for the beam.

These mechanical magnitudes are connected by:

conditions of kinematical continuity,
equilibrium conditions of the connection.

The first conditions are the conditions of connections to impose in an approach "displacement", the seconds result from the weak formulation of the equilibrium via the virtual wor of the actions of contact between beam and solid mass (which is not other than the statement of the "principle" of the action and the reaction writes for the interface S). On surface S , one has indeed for any licit $(\mathbf{v}, T, \boldsymbol{\Omega})$ virtual displacement:

$$\int s.n.v\, dS = \mathrm{F}.\mathrm{T} + \mathrm{M}.\boldsymbol{\Omega} \qquad \text{éq 2.2-1}$$

where:

T et $\boldsymbol{\Omega}$ are respectively the translation and the infinitesimal rotation of beam: $\mathbf{D}=(\mathrm{T}, \boldsymbol{\Omega})$
F et M are respectively the resultant and the moment in the beam at the point of connection: $\mathbf{T}=(\mathrm{F}, \mathrm{M})$

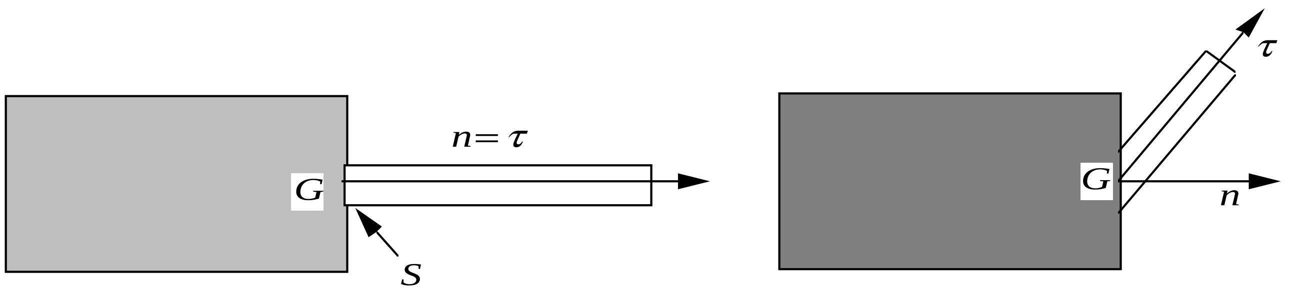
The first member of this equality will provide the scalar product thanks to which one will define the "component beam" of a field of displacement 3D definite on S . By means of this scalar product, one will ensure the symmetry of the approach between kinematical conditions and statics of connection (P3) as well as the possibility of treating unspecified behaviors on both sides of connection (P4) since no aspect of behavior appears in the equality of equilibrium used.

The approach:

One 3D will break up the field of displacement into a part "beam" and a "complementary" part. This will lead us to rather naturally define the conditions of kinematical connection between beam and solid 3D like the equality of the displacement (torsor) of beam and of the beam part of the field of displacement 3D [(§ 2.3)]. Once this made, the equality [éq 2.2-1] will enable us to interpret in static term the conditions of connection and to thus reach the conditions of static connection [(§2.4)].

2.3 Decomposition of displacement 3D on the interface

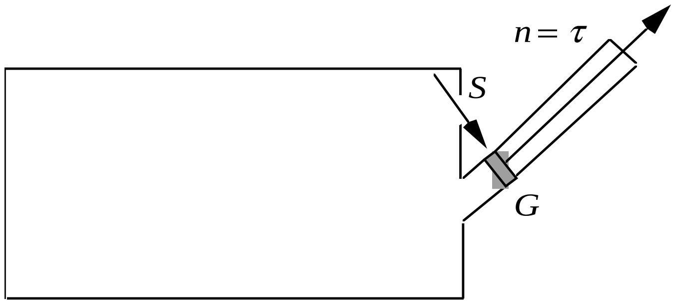
the junction between three-dimensional solid B and the beam of section S is supposed to be plane and of norm n parallel with the tangent τ with the beam at the point of contact G , geometrical center of inertia of the section S [Figure 2.3-a].



(a) Normale au solide = tangente à la poutre (b) Normale au solide ° tangente à la poutre

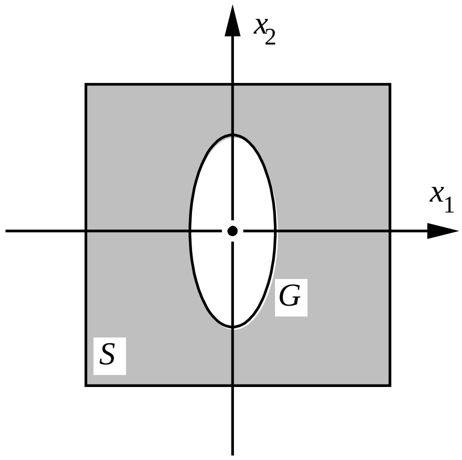
Appear 2.3-a

One thus excludes the case (b) where the beam "does not leave" by perpendicular to surface solid. It should well be understood that this restriction is necessary to the coherence of the connection such as it is considered here since the theory of the beams knows only cuts norm with average fiber: the equilibrium condition [éq 2.2-1] does not have any meaning if S is not the cross-section of the beam. If this condition is violated, one will be able to modify the mesh to carry out it as diagram Ci - indicates it below.



Appear 2.3-b

One notes:



$(G, \mathbf{e}_1\,\mathbf{e}_2)$ **a principal reference of inertia geometrical** of S for origin the center of inertia G, (x_1, x_2) coordinated having and the associated,

n or $\mathbf{e}_3$ the norm with the plane S, outgoing with the solid mass 3D,

$\boldsymbol{\varepsilon}^{\alpha\beta 3}=(\mathbf{e}_\alpha, \mathbf{e}_\beta, \mathbf{e}_3)$ the alternate shape of the mixed product of the basic vectors,

finally $\mathbf{I}$ the geometrical tensor of inertia of S (diagonal in the reference $(\mathbf{e}_1\,\mathbf{e}_2)$) and $A=|S|$ the area of the section S .

Let us recall that the tensor of inertia $\mathbf{I}$ can be defined in an equivalent way by a linear application (mixed representative):

$$\mathbf{I}(\mathbf{U})=\int_S \mathbf{GM}(x)\wedge(\mathbf{U}\wedge\mathbf{GM}(x))\,dx$$

or a symmetric bilinear application (covariant representative):

$$\mathbf{I}(\mathbf{U},\mathbf{V})=\int_S (\mathbf{U}\wedge\mathbf{GM}(x)).(\mathbf{V}\wedge\mathbf{GM}(x))\,dx$$

These two statements will be useful, in the reference $(G, \mathbf{e}_1 \mathbf{e}_2 \mathbf{e}_3)$ the matrix representative of $\mathbf{I}$ is:

$$[\mathbf{I}]=\begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_1+I_2 \end{bmatrix}$$

with I_α geometrical main moment of inertia from S ratio with the axis $(G, \mathbf{e}_\alpha)$. By convention the Greek indices take values 1 or 2.

Useful space for the fields of displacements and definite vectors forced on S is $V=L^2(S)^3$. One introduces the space $\mathbf{T}$ of the fields associated with a torsor (defined by two vectors):

$$\mathbf{T}=\{\mathbf{v}\in V / \exists(\mathrm{T}, \boldsymbol{\Omega}) \text{ tel que } \mathbf{v}(M)=\mathrm{T}+\boldsymbol{\Omega}\wedge\mathbf{GM}\} \qquad \text{éq 2.3-1}$$

For the fields of displacement of S, T is the translation of the section (or the point G), $\boldsymbol{\Omega}$ infinitesimal rotation and the fields $\mathbf{v}$ are displacements preserving the plane section S and not deformed (Assumptions of NAVIER-BERNOULLI).

For the fields of vectors forced, $|S|\mathrm{T}$ is the resultant F of the actions applied to S, and $\mathbf{I}(\boldsymbol{\Omega})$ is the resulting moment M in G. The fields $\mathbf{v}$ correspond then to distributions of stresses closely connected in the section. Indeed, one a:

$$\mathrm{F}(\sigma)\equiv\int_S s\,.\,n\,dS=\int_S \mathrm{T}\,dS=|S|\mathrm{T}$$
$$\mathrm{M}(\sigma)\equiv\int_S \mathbf{GM}(x)\wedge\boldsymbol{\sigma}\,.\,n\,dS=\int_S \mathbf{GM}(x)\wedge(\Omega\wedge\mathbf{GM})\,dS=\mathbf{I}(\boldsymbol{\Omega})$$

One used the fact here that G is geometrical center of inertia thus: $\int_S x_\alpha dS=0$. The vectorial subspace $\mathbf{T}$ being of finished size (equal to 6) has additional orthogonal for the scalar product defined on V :

$$\mathbf{T}^{\perp}=\left\{\mathbf{v}\in V / \int_S \mathbf{v}\,.\,\mathbf{w}\,dS=0\ \forall\ \mathbf{w}\in\mathbf{T}\right\} \qquad \text{éq 2.3-2}$$

Is, in a more explicit way:

$$\mathbf{T}^{\perp}=\left\{\mathbf{v}\in V / \int_S \mathbf{v}\,dS=0 \text{ et } \int_S \mathbf{GM}\wedge\mathbf{v}\,dS=0\right\} \qquad \text{éq 2.3-3}$$

Whole field of V all in all breaks up in a single way of an element of $\mathbf{T}$ and an element of $\mathbf{T}^{\perp}$.

$$\mathbf{u}=\mathbf{u}^{\mathbf{p}}+\mathbf{u}^{\mathbf{s}} \quad \mathbf{u}^{\mathbf{p}}\in\mathbf{T}, \quad \mathbf{u}^{\mathbf{s}}\in\mathbf{T}^{\perp} \qquad \text{éq 2.3-4}$$

One has moreover the following property:

For any couple of field 3D $(\mathbf{u},\mathbf{v})$ definite on S ,

$$\begin{array}{l}\mathbf{u}=\mathbf{u}^{\mathbf{p}}+\mathbf{u}^{\mathbf{s}}\\ \mathbf{v}=\mathbf{v}^{\mathbf{p}}+\mathbf{v}^{\mathbf{s}}\end{array} \Rightarrow \int_S \mathbf{v}.\mathbf{w}\,dS=\int_S \mathbf{v}^{\mathbf{p}}.\mathbf{w}^{\mathbf{p}}\,dS+\int_S \mathbf{v}^{\mathbf{s}}.\mathbf{w}^{\mathbf{s}}\,dS \quad \text{éq 2.3-5}$$

the following definition is thus natural:

Definition:

One calls component of displacement of beam of a field u defined on the section the component u^p of u on the subspace.

The computation of the beam part of a field 3D $\mathbf{u}$ the property of orthogonal projection takes place by means of since $\mathbf{T}$ and $\mathbf{T}^{\perp}$ are orthogonal by definition.

If one notes $\mathbf{u}^{\mathbf{p}}=\mathrm{T}_{\mathrm{u}}+\boldsymbol{\Omega}_{\mathbf{u}}\wedge\mathbf{GM}$, then:

$$\left(\mathrm{T}_{\mathrm{u}},\boldsymbol{\Omega}_{\mathbf{u}}\right)=\underset{\left(\mathrm{T},\boldsymbol{\Omega}\right)}{\mathrm{Argmin}}\int_S\left(\mathbf{u}-\mathrm{T}-\boldsymbol{\Omega}\wedge\mathbf{GM}\right)^2 \quad \text{éq 2.3-6}$$

One will on the way note the interpretation of the component beam of $\mathbf{u}$: it is the field of displacement of beam nearest to $\mathbf{u}$ within the meaning of the least squares. The computation minimum leads immediately to the characterization:

$$\mathrm{T}_{\mathrm{u}}=\frac{1}{|S|}\int_S \mathbf{u}\,dS,\qquad \boldsymbol{\Omega}_{\mathbf{u}}=\mathbf{I}^{-1}\left(\int_S \mathbf{GM}\wedge\mathbf{u}\,dS\right) \quad \text{éq 2.3-7}$$

the kinematical condition of connection sought is thus the following linear constraint between the field 3D on S and the elements of the torsor of displacement of the beam in G :

$$|S|\mathbf{T}-\int_S \mathbf{u}\,dS,\qquad \mathbf{I}(\boldsymbol{\Omega})-\int_S \mathbf{GM}\wedge\mathbf{u}\,dS=0 \quad \text{éq 2.3-8}$$

2.4 Statement of the static condition of connection

While returning to the weak formulation of the equilibrium of the interface [éq 2.2-1], one can deduce from them the requirements and sufficient from static connection. Indeed, there a:

$$\int_S \sigma.n.\mathbf{v}\,dS=\mathbf{R}.\mathbf{T}_{\mathbf{v}}+\mathbf{M}.\boldsymbol{\Omega}_{\mathbf{v}}\qquad \forall\,\mathbf{v}\in V \quad \text{éq 2.4-1}$$

Thanks to the statements [éq 2.3-7] and the decomposition of space V , and the property [éq 2.3 - 5], one are immediately the three equations:

$$\begin{aligned} &\mathrm{F}=\int_S \sigma . n\, dS \\ &\mathrm{M}=\int_S \mathbf{GM}(x)\wedge\sigma . n\, dS \\ &(\sigma . n)^{\sigma}=0 \quad \text{ou de manière équivalente} \quad \int_S \sigma . n . \mathbf{v}\, dS=0 \quad \forall\, \mathbf{v}\in T^{\perp} \end{aligned}$$ éq the 2.4-2

conditions of static connection are thus:

transmission of the torsor of the forces of beam, (satisfied the property P1),
nullity of the complementary part (“not beam”) of the field of vector forced 3D on the section of solid 3D (satisfied the property P2).

One will also notice static and kinematical symmetry (P3 property) since the conditions of connection are also interpreted like:

equality within the meaning of the least squares between displacement 3D and the displacement of the beam,
the equality within the meaning of the least squares between the field of vector forced and the end cells of the torsor of the forces of beam.

2.5 Establishment of the method of connection

For each connection, the user must define:

S: the trace of the section of the beam on the solid mass 3D: it does it by key keys MAILLE_1 and/or GROUP_MA_1 ; i.e. it meshes gives the list of (lma) surface (affected of elements “edge” of modelization 3D) which represents this section geometrically.
P: a node (key word NOEUD_1 or GROUP_NO_1) carrying the 6 classical degrees of freedom of beam: $DX \quad DY \quad DZ \quad DRX \quad DRY$, DRZ

Note:

the node P can be a node of beam element or of discrete element,
the list of meshes lma must represent the section of the beam exactly. It is an important stress for the mesh.

For each node, the program calculates the coefficients of the 6 linear relations [éq 2.3-8] which connect:

6 degrees of freedom of the node P ,
with the degrees of freedom of **all** the nodes of lma .

These linear relations will be dualisées, like all the linear relations resulting for example from key word LIAISON_DDL of AFFE_CHAR_MECA.

The computation coefficients of the linear relations is done in several stages:

computation of elementary quantities on the elements of lma : (OPTION: CARA_SECT_POUT3)

- surface = $\int_{elt} 1 ; \int_{elt} x ; \int_{elt} y ; \int_{elt} x2 ; \ldots$

summation of these quantities on (S) from where the computation of:

- $A=|S|$
- position of G
- tensor of inertia $\boldsymbol{\Omega}$

knowing G , elementary computation on the elements of lma : (OPTION: CARA_SECT_POUT4)

$$\int_{elt} Ni ; \int_{elt} xNi ; \int_{elt} yNi ; \int_{elt} \mathrm{zNi} \quad \text{où :} \quad \begin{array}{l} \mathbf{GM}=\{x , y , z\} \\ Ni=\text{fonctions de forme de l'élément} \end{array}$$

"assembly" of the terms calculated above to obtain of each node of lma, coefficients of the terms of the linear relations.

3 The connection 2D-beam

3.1 Purpose

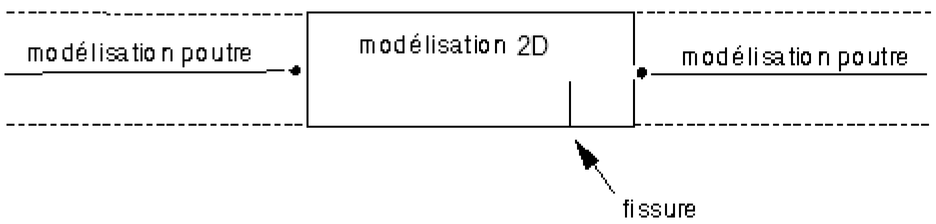
As for the connection 3D/poutre, the purpose is to be able to represent part of a slender structure complexes ([Figure 3.1-a]) by a beam of which the part to be analyzed is relatively "far". The goal of schematization by a beam is 2D to bring realistic boundary conditions to edges of the part modelled and with a grid in continuum. These boundary conditions can be brought by a beam, but also by discrete elements. The connection 2D/poutre must thus meet the following requirements:

P1 Power to transmit the forces of beam to the mesh 2D

P2 not to generate "parasitic" stresses (even of stress concentration), because it would then be necessary to place the connection far from the zone to be sufficiently analyzed so that these disturbances are attenuated in the zone of study.

P3 not to support the kinematical conditions or the static conditions of connection one compared to the other. It must be equivalent to bring back a load vector force or of displacement to the limits of the field 2D.

P4 Admettre unspecified behaviors on both sides of connection (elasticity, plasticity…) and to also allow a dynamic analysis.



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approach:

As for the connection 3D/poutre, one 2D breaks up the field of displacement into a part "beam" and a "complementary" part. This leads us to define the conditions of kinematical connection between the beam and structure 2D like the equality of displacement of beam and the beam part of the field of displacement 2D. Once this made, one then interprets in static term the conditions of connection and one to reach the conditions of static connection thus.

The reader is invited to consult paragraph 2 which describes 3D-beam) explicitly the method of the approach opposite. He will easily establish the link with the case 2D.

3.2 Establishment of the method of connection

For each connection, the user must define:

S: The edge of surface 2D: it is done by key keys MAILLE_1 and/or GROUP_MA_1 ; i.e. it meshes gives the list of (lma) linear (affected of elements “edge” of modelization 2D) which represents this section geometrically.

P: a node (key word NOEUD_1 or GROUP_NO_1) carrying the 3 classical degrees of freedom of beam: DX DY , DRZ

Note:

> *the node P can be a node of beam element or of discrete element,*
> *the list of meshes lma must represent the section of the beam.*

For each node, the program calculates the coefficients of the 3 linear relations [éq 2.3-8] which connect:

3 degrees of freedom of the node P ,
with the degrees of freedom of **all** the nodes of lma .

These linear relations will be dualisées, like all the linear relations resulting for example from key word LIAISON_DDL of AFFE_CHAR_MECA.

The computation coefficients of the linear relations is done in several stages:

computation of elementary quantities on the elements of lma : (OPTION: CARA_SECT_POUT3)

- $\int_{elt} 1 ; \int_{elt} x ; \int_{elt} y ; \int_{elt} x^2 ; \int_{elt} y^2$

summation of these quantities on edge (S) from where the computation of:

- $A=|S|$
- position of G
- tensor of inertia $\boldsymbol{\Omega}$

knowing G , elementary computation on the elements of lma : (OPTION: CARA_SECT_POUT4)

$$\int_{elt} Ni ; \int_{elt} xNi ; \int_{elt} yNi ; \quad \text{où :} \quad \begin{array}{l} \mathbf{GM}=\{x , y\} \\ Ni=\text{fonctions de forme de l'élément} \end{array}$$

“assembly” of the terms calculated above to obtain of each node of lma , coefficients of the terms of the linear relations.

4 Which uses can one make this modelization?

In addition to the two aimed uses to [§2] [Figure 2.1-a] and [Figure 2.1-b], this connection can also be used for:

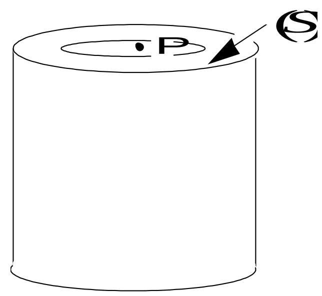
3D to apply a load vector force to a known surface of a modelization:

For that, the user defines the surface of load application (lma), it "connects it" with a node (P) of discrete element (`DIS_TR_N`) **without stiffness** then it applies the torsor wanted to this node (`FORCE_NODALE`).

In this way, the torsor is applied in "softness", without generating secondary stresses to surface.

"to retain" a structure without too much the encaster:

For example, if one has with a grid 3D in a pipe and that one wants to prevent his motions of solid body



one connects (S) to P then one blocks the 6 degrees of freedom of P .

The structure is then retained, without (S) is clamped. In particular, the section (S) can be ovalized.

5 Bibliography

1) S. ANDRIEUX: "Connections 3D/Poutre, 3D/Coques and other imaginations" (notes to appear).

6 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of modifications
3	*J. PELLET (EDF/IMA/MMN)*	initial Text