Connection shell-beam

Summarized:

One describes here the connection shell-beam, which make it possible to connect two parts of mesh, one constituted by beam elements (or of a discrete element), and the other with a grid one in shell elements (to represent phenomena except kinematics of beam). This development thus functions under assumptions translating that it is the same kinematics of beam which is transmitted between the two meshes, on both sides of connection. It results in 6 linear relations connecting displacements of all the nodes of edge of the shell with the 6 degrees of freedom of ending node of the beam.

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1 Assumptions and Assumptions

1.1 applications and limitations

One describes here the connection shell-beam, which is used to connect two meshes, one comprising of the shell elements (or plates), the other comprising of the beam elements. This functionality makes it possible to model a slender structure in two parts: a part with a grid with classical elements of beams, representing a kinematics and a behavior of beams, and the other part with a grid in shell elements, to reveal other phenomena (ovalization, swelling, localised plasticity).

The following assumptions however are made:

•the surface of the cross-sectional area of the end of the mesh of shells is identical to the right sectional surface of the beam element which corresponds to him,
•the centers of gravity are identical,
•the sections are plane and coplanar,
•the norm with the section of shells is confused with the axis of the beam.

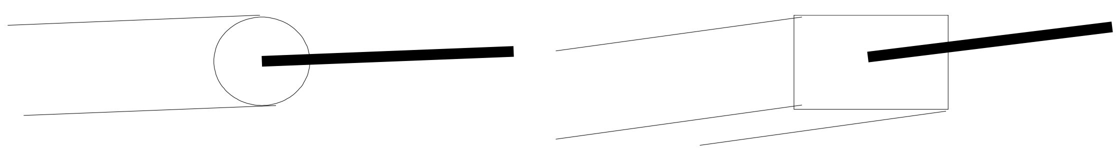
Limitations:

•one in the connection does not take account of the ovalization of the cross-sections,
•one does not take account of warping.

1.2 Applications concerned:

1.2.1 Modelization of the pipework

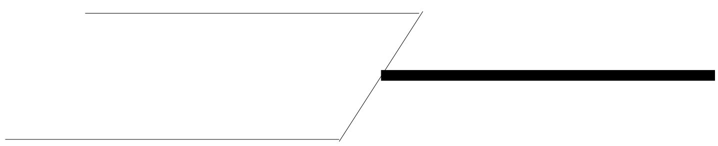
One of the major applications relates to the pipework. The bent parts or the bypasses are then with a grid in shells, which makes it possible to reveal an ovalization, a local elastoplastic behavior or a swelling in the event of internal pressure. This connection does not transmit the ovalization of the pipes since this one is not modelled in the beam elements. With this intention, it is necessary to use connection shell-pipe or to net a sufficient length of right pipework in shell elements so that ovalization on the level of the connection is negligible.



Circular pipework of section (or rectangular…) with a grid in shell then out of beam.

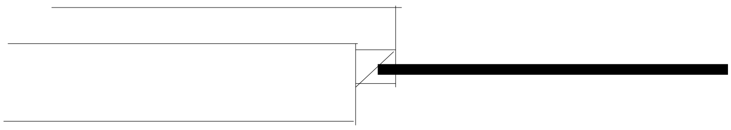
1.2.2 Connection plates beam

Connection plate-beam (mean rectangular section).



1.2.3 Beam with symmetric profile

Beam with symmetric profile with a grid partly in shells.

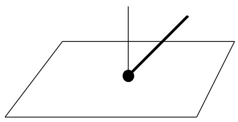


1.2.4 Application of a loading or boundary conditions of type "beam"

At the end of a slender structure with a grid in shells, it is often useful to impose either a loading of type "beam" i.e. a load vector force, or of the boundary conditions (fixed support) compatible with the kinematics of beam. One can then connect the cross-sectional area of end of the mesh shells to a discrete element to which one will apply this torsor or this fixed support.

1.2.5 Application not considered:

This functionality does not make it possible to model "" the transverse bypasses or orthogonaux' "D" a beam on a plate or a shell:



2 Application of the method of the connection 3D-beam. Equations of connection

the approach is identical to that of connection beam-3D [R3.03.03]: the connection results in 6 linear relations connecting displacements of all the nodes shell of the section of connection (6 degrees of freedom per node, compared to 3 degrees of freedom per node in 3D) to the 6 degrees of freedom of the node of beam. The section of connection of shell is made up of edge elements of shells (segments). On the section connection crosses, one breaks up the field of displacement "shell" into a part "beam" and a "complementary" part. This leads us to define the conditions of kinematical connection between beam and shell like the equality of the displacement (torsor distributer or kinematical torsor) of beam and of the beam part of the field of shell displacement

As in [R3.03.03], one introduces the space $\mathbf{T}$ of the fields associated with a kinematical torsor (defined by two vectors):

$$\mathbf{T}=\{v\in V/\exists(T,\Omega)\ \text{tel que}\ v(M)=\mathbf{T}+\boldsymbol{\Omega}\wedge\mathbf{GM}\} \quad \text{2-1}$$

Here, $\mathbf{G}$ represents the center of gravity of the section of connection (in front of being identical to that of the beam). For the fields of displacement of $\mathbf{T}$, $\mathbf{T}$ is the translation of the section (or the point $\mathbf{G}$), Ω infinitesimal rotation and the fields $\mathbf{v}$ are displacements of the space of acceptable displacements V preserving the plane section S and not deformed there (One uses still the Assumptions of NAVIER-BERNOULLI).

The vectorial subspace $\mathbf{T}$ being of finished size (equal to 6) has additional orthogonal for the scalar product defined on V :

$$\mathbf{T}^{\perp}=\left\{\mathbf{v}\in V/\int_s \mathbf{v}(M).\mathbf{w}(M)\,dS=0 \quad \forall\,\mathbf{w}\in T\right\} \qquad \text{éq 2-2}$$

Is, in a more explicit way:

$$\mathbf{T}^{\perp}=\left\{\mathbf{v}\in V/\int_s \mathbf{v}(M)\,dS=0 \text{ et } \int_s \mathbf{GM}\wedge\mathbf{v}\,dS=0\right\} \qquad \text{éq 2-3}$$

Whole field of V all in all breaks up in a single way of an element of $\mathbf{T}$ and an element of $\mathbf{T}^{\perp}$.

$$\mathbf{u}=\mathbf{u}^p+\mathbf{u}^s \quad \mathbf{u}^p\in\mathbf{T} \quad , \quad \mathbf{u}^s\in\mathbf{T}^{\perp} \qquad \text{éq 2-4}$$

One has moreover the following property:

For any couple of field shell $(\mathbf{w},\mathbf{v})$ defined on S ,

$$\begin{matrix}\mathbf{w}=\mathbf{w}^p+\mathbf{w}^s\\ \mathbf{v}=\mathbf{v}^p+\mathbf{v}^s\end{matrix} \quad \Rightarrow \quad \int_s \mathbf{v}.\mathbf{w}\,dS=\int_s \mathbf{v}^p.\mathbf{w}^p\,dS+\int_s \mathbf{v}^s.\mathbf{w}^s\,dS \qquad \text{éq 2-5}$$

Definition:

> *One calls component of displacement of beam of a field of shell* $\mathbf{u}$ *defined on the section the component* $\mathbf{u}^p$ de $\mathbf{u}$ *on the subspace* $\mathbf{T}$.

The characterization immediately is obtained:

$$\mathbf{T}_{\mathbf{u}}=\frac{1}{|S|}\int_S \mathbf{u}\,dS, \qquad \boldsymbol{\Omega}_{\mathbf{u}}=\mathbf{I}^{-1}\left(\int_s \mathbf{GM}\wedge\mathbf{u}\,dS\right) \qquad \text{éq 2-7}$$

where $|S|$ the area of the section S et $\mathbf{I}$ the geometrical tensor of inertia of surface represents S , expressed in $\mathbf{G}$.

In other words, one can as say as the computation of the beam part of a field shell $\mathbf{u}$ takes place the property of orthogonal projection by means of since $\mathbf{T}$ et $\mathbf{T}^{\perp}$ are orthogonal by definition.

If one notes $\mathbf{u}^p=\mathbf{T}_{\mathbf{u}}+\boldsymbol{\Omega}_{\mathbf{u}}\wedge\mathbf{GM}$, then:

$$\left(\mathbf{T}_{\mathbf{u}},\boldsymbol{\Omega}_{\mathbf{u}}\right)=\underset{(\mathbf{T},\Omega)}{\text{Argmin}}\int_S \left(\mathbf{u}-\mathbf{T}-\boldsymbol{\Omega}\wedge\mathbf{GM}\right)^2 \qquad \text{éq the 2-6}$$

component beam of $\mathbf{u}$ can thus be interpreted like the field of displacement of beam nearest to $\mathbf{u}$ within the meaning of the least squares.

The kinematical condition of connection sought between the field shell on S and the elements of the torsor of displacement of the beam $\mathbf{G}$ is given by it by:

$$|S|\mathbf{T}-\int_S \mathbf{u}\, dS=0 \qquad \mathbf{I}(\boldsymbol{\Omega})-\int_S \mathbf{GM}\wedge\mathbf{u}\, dS=0 \qquad \text{éq the 2-8}$$

equation [éq 2-8] shows that the situation is identical to the case 3D-beam. The linear relations will have the same form. The only difference comes from the integrals on S (which represents a curve here corresponding to the section of the shell, modelled by edge elements from shell). Moreover, the field of displacement of shell utilizes degrees of freedom of rotation.

To translate the equation [éq 2-8] into linear relations, the two integrals should be calculated:

average displacement: $\int_S \mathbf{u}\, dS$

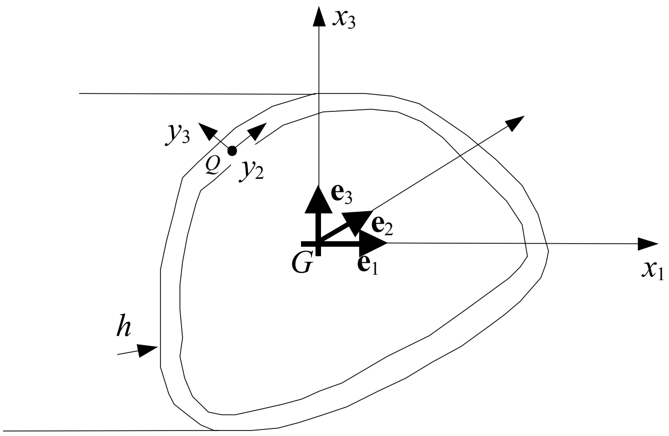
average rotation: $\int_S \mathbf{GM}\wedge\mathbf{u}\, dS$

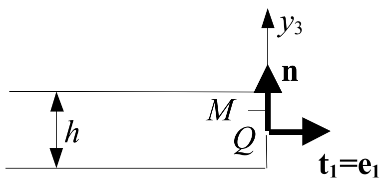
3 Integrals with calculating. Kinematics of shell.

For each node, the program calculates the coefficients of the 6 linear relations [éq 2-8] which connect:

6 degrees of freedom of the node of beam P (geometrically confused with the center of gravity $\mathbf{G}$ of the cross-sectional area of the mesh shells)
with the degrees of freedom of **all** the nodes of the list of meshes of edge of shell.

These linear relations are dualisées, like all the linear relations resulting, for example, of key word `LIAISON_DDL` of `AFFE_CHAR_MECA`. They are built as for connection 3D-beam from the assembly of elementary terms.





Bord de la section transverse (de coque) de raccordement

$S = l \times I$

l : ligne des points Q sur le feuillet moyen

$I = \left]-\frac{h}{2},\frac{h}{2}\right[$ intervalle décrivant l'épaisseur.

Kinematics of shell or linear plate in the thickness:

$$\mathbf{u}(M)=\mathbf{u}(Q)+(\boldsymbol{\theta}(Q)\wedge\mathbf{n}).\, y_3$$

- $\mathbf{u}$ is the vector displacement of mean surface in Q,
- $\mathbf{n}$ is the normal vector at the mean surface of the shell in Q,
- $\boldsymbol{\theta}$ is the vector rotation in Q norm according to the directions $\mathbf{t}_1$ et $\mathbf{t}_2$ of the tangent plane
- y_3 is the coordinate in the thickness ($y_3 \in \left]-\frac{h}{2},\frac{h}{2}\right[$).

3.1 Computation of average displacement on the section S

It acts to calculate the integral $\int_S \mathbf{u}\, dS$, where $\mathbf{u}$ is the displacement of shell (comprising 6 d.o.f. per node), S is the edge of shell of the cross-sectional area of connection.

Average displacement on the section S is written:

$$\int_s \mathbf{u}(M)\, ds = h\int_l \mathbf{u}(Q)\, ds + \int_l (\boldsymbol{\theta}(Q)\wedge\mathbf{n})\left(\int_{-h/2}^{h/2} y_3\, dy_3\right) ds$$

either $\int_s \mathbf{u}(M)\, ds = h\int_l \mathbf{u}(Q)\, ds$

One neglects in this statement the variations of metric in the thickness of the shell.

3.2 Computation of the average rotation of the section S

$$\begin{aligned}\int_s \mathbf{GM}\wedge\mathbf{u}(M)\, ds &= \int_l \int_{-h/2}^{h/2} (\mathbf{GQ}+y_3\,\mathbf{n}(Q))\wedge(\mathbf{u}(Q)+\boldsymbol{\theta}(Q)\wedge\mathbf{n}(Q)\,.\,y_3)\, ds dy_3\\ &= h\int_l \mathbf{GQ}\wedge\mathbf{u}(Q)\, ds + \int_l \mathbf{GQ}\wedge(\boldsymbol{\theta}(Q)\wedge\mathbf{n}(Q))\, ds \int_{-h/2}^{h/2} y_3\, dy_3\\ &+\int_l \mathbf{n}(Q)\wedge\mathbf{u}(Q)\left(\int_{-h/2}^{h/2} y_3 dy_3\right) ds + \int_l \mathbf{n}(Q)\wedge(\boldsymbol{\theta}(Q)\wedge\mathbf{n}(Q))\int_{-\frac{h}{2}}^{\frac{h}{2}} y_3^2\, dy_3 . ds\end{aligned}$$

is $\int_s \mathbf{GM}\wedge\mathbf{u}(M)\, ds = h\int_l \mathbf{GQ}\wedge\mathbf{u}(Q)\, ds + \frac{h^3}{12}\int_l \mathbf{n}(Q)\wedge(\boldsymbol{\theta}(Q)\wedge\mathbf{n}(Q))\, ds$.

3.3 Computation of the tensor of inertia

the tensor of inertia is defined by [R3.03.03]:

$$\mathbf{I}(\boldsymbol{\Omega}) = \int_s \mathbf{GM}\wedge(\boldsymbol{\Omega}\wedge\mathbf{GM})\, ds$$

while posing: $\mathbf{GM}=\mathbf{GQ}+\mathbf{n}(Q)\,.\,y_3$.

One obtains: $\mathbf{I}(\boldsymbol{\Omega}) = h\int_l \mathbf{GQ}\wedge(\boldsymbol{\Omega}\wedge\mathbf{GQ})\, ds + \frac{h^3}{12}\int_l \mathbf{n}(Q)\wedge(\boldsymbol{\Omega}\wedge\mathbf{n}(Q))\, ds$

3.4 Establishment of the method

The computation of the coefficients of the linear relations is done in two times:

computation of elementary quantities on the elements of the list of meshes of edges of shells (mesh of type SEG2 or SEG3):

- the 9 terms are calculated:
- $\int_{elt} ds\, ; \int_{elt} xds\, ; \int_{elt} yds\, ; \int_{elt} x^2\, ds\, ; \int_{elt} y^2\, ds\, ; \int_{elt} z^2\, ds\, ; \int_{elt} xyds\, ; \int_{elt} xzds\, ; \int_{elt} yzds$

  as of the terms resulting from $\mathbf{I}(\boldsymbol{\Omega})$: $\frac{h^3}{12}\int_l \mathbf{n}\wedge(\boldsymbol{\Omega}\wedge\mathbf{n})\, ds$

  what makes it possible to calculate: $\frac{h^3}{12}\int_l (n_y^2+n_z^2)\, ds, \frac{h^3}{12}\int_l n_x n_y\, ds$, etc…
- summation of these quantities on (S) from where the computation of:

- $A=|S|$
- position of $\mathbf{G}$
- tensor of inertia $\mathbf{I}$

knowing $\mathbf{G}$, elementary computation on the elements of the list of meshes of edges of shells of:

$$\int_{elt} N_i\,ds\,;\int_{elt} xN_i\,ds\,;\int_{elt} yN_i\,ds\,;\int_{elt} zN_i\,ds \quad \text{where} \quad \begin{array}{l}\mathbf{GM}=\{x\,,y\,,z\}\\ N_i=\text{fonctions de forme de l'élément}\end{array}$$

(It should simply be noticed that in this case, the integrals on the edge elements are to be multiplied by the thickness of the shell: $\int_{elt} N_i\,ds=h\int_{l} N_i\,dl$ where l the curvilinear abscisse of average fiber of the edge element represents of shell).

Moreover, one adds the terms additional coming from: $\frac{h^3}{12}\int_l \mathbf{n}(Q)\wedge(\boldsymbol{\Omega}\wedge\mathbf{n}(Q))\,ds$

While noting $\mathbf{n}=\begin{vmatrix}n_x\\n_y\\n_z\end{vmatrix}$ and $\boldsymbol{\theta}=\begin{vmatrix}\theta_x\\\theta_y\\\theta_z\end{vmatrix}$ in the total reference one obtains:

$$\mathbf{n}(Q)\wedge(\boldsymbol{\theta}\wedge\mathbf{n}(Q))=\begin{vmatrix}(n_y^2+n_z^2)\theta_x-n_x n_y\theta_y-n_x n_z\theta_z\\-n_x n_y\theta_x+(n_x^2+n_z^2)\theta_y-n_y n_z\theta_z=\mathbf{A}\boldsymbol{\theta}\\-n_x n_z\theta_x-n_y n_z\theta_y+(n_x^2+n_y^2)\theta_z\end{vmatrix}$$

then:

$$\frac{h^3}{12}\int_l \mathbf{n}(Q)\wedge(\boldsymbol{\Omega}\wedge\mathbf{n}(Q))\,ds=\frac{h^3}{12}\sum_{el}\left(\int_{el}\mathbf{A}(s)N_j(s)\,ds\right)\theta_j$$

"assembly" of the terms calculated above to obtain of each node of meshes of edge, coefficients of the terms of the linear relations.

4 Modelization

4.1 use

For each connection, the user must define under the key word factor LIAISON_ELEM of AFFE_CHAR_MECA:

S : the trace of the cross-section of the beam on the shell: it does it by key keys MAILLE_1 and/or GROUP_MA_1 i.e. it gives the list of meshes linear (affected of elements "edge" of modelization shell) which represent this section geometrically.

P : a node (key word NOEUD_1 or GROUP_NO_1) carrying the 6 classical degrees of freedom of beam: DX, DY, DZ, DRX, DRY, DRZ

$\mathbf{V}$: the vector defining the axis of the beam, directed shell towards the beam, and defined by its coordinates using key word AXE_POUTRE: (v1, v2, v3)

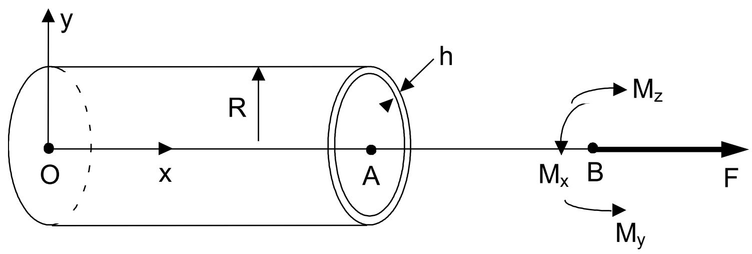
Note::

the node P can be a node of beam element or of discrete element,
the list of meshes of edge of shell, defined by MESH or GROUP_MA must represent ***exactly*** *the cross-section of the beam. It is an important stress for the mesh.*

4.2 Examples and tests

4.2.1 Test SSLX101

It is about a subjected straight beam has unit forces in B (tension, bending moments and of torsion). One takes a mean section of tube of thickness $\mathrm{h} \ll \mathrm{R}$.

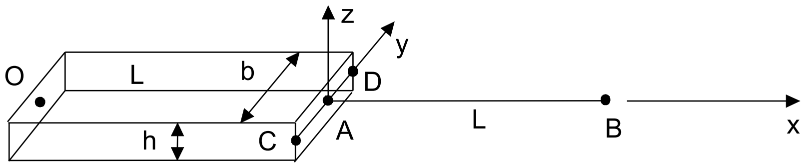


The fixed support O is carried out by it using a connection between edge of the shell and a point element located in O . This element is clamped (null translations and rotations).

This makes it possible to obtain in the shell a stress state very close to a solution "beam": there is no disturbance of the stress field. The solution differs from the analytical solution (solution RDM) of 3%, this being only due to the smoothness of the mesh in shell elements.

4.2.2 Bending of a plate

Let us consider a sufficiently long thin plate, length $2L$, of width b , of thickness h , modelled by a shell element OA and a beam element on AB :



The 1st condition of connection is written:

$$b\,h\,\mathbf{U}(A)=h\int_{CD}\mathbf{U}(y)\,dy$$

the displacement of the point A (pertaining to the beam) is the average of displacements of edge CD of the plate.

The 2nd condition of connection is written:

$$\mathbf{I}(\boldsymbol{\Omega})=h\int_{CD}\mathbf{AQ}\wedge\mathbf{U}(Q)\,ds+\frac{h^3}{12}\int_{CD}\boldsymbol{\theta}(Q)\,ds$$

In the case of a bending around y , the only non-zero term is: $\frac{h^3}{12}\int_{-\frac{b}{2}}^{b/2}\boldsymbol{\theta}(y)\,dy$

Indeed, $h\int_{CD}\mathbf{AQ}\wedge\mathbf{U}(Q)\,ds=h\left(\int_{-\frac{b}{2}}^{\frac{b}{2}}U_z\,y\,dy\right).\mathbf{x}=0$

For a bending around y , connection is thus written:

$I_y\theta_y(A)=\frac{b\,h^3}{12}\theta_y$ because of being θ_y constant on CD.

This application is put in work in test SSLX100B: mix 3D_coque_poutre.

5 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
6	J.M.PROIX-R&D/AMA	