Following Pressure for the voluminal shell elements

Abstract:

We present in this document, the model used to compute: the loading of type following pressure acting on the mean surface of the finite elements of voluminal shells corresponding to modelization COQUE_3D. Discretization of the loading led to a nodal vector of the external forces and to an asymmetric contribution in the tangent matrix of stiffness. These finite elements objects are evaluated with each iteration of the algorithm of Newton of STAT_NON_LINE.

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1 Introduction

Our analysis leave the weak formulation by the key word the equilibrium under a loading of type following pressure activated `TYPE_CHARGE: "SUIV"` in command `STAT_NON_LINE` [U4.32.01]. The difference compared to a classical geometrical linear analysis is that the pressure acts on the deformed geometry and either on the initial geometry. This new geometry is obtained from the transform of the initial mean surface subjected to large displacements and large rotations [R3.07.05]. The notations are inspired by [R3.07.05].

This transform can be parametrized exactly as initial surface by means of reduced coordinates of the associated isoparametric element: the CO-variable or counter-variable references are built in each point of deformed surface. The writing of the virtual wor of the pressure with this parameterization is made in the configuration deformed by means of the associated isoparametric elements. It results an independence from it from the field of integration with displacements which one uses to express the variation of the virtual wor of the external forces of pressure compared to known as displacements. That has an important advantage compared to the method applied for the pressure which follows the facets of the elements 3D [R3.03.04]. Indeed, this last method, based on a brought up to date Lagrangian formulation, conduit in the nonlinear terms difficult to linearize, coming from the transformation jacobienne compared to the reference configuration.

The finite elements objects obtained by linearization compared to incrémentaux displacements of the virtual wor of the external forces of pressure are to be reactualized with each iteration of the algorithm of Newton of `STAT_NON_LINE`. We underline the fact that the contribution of the following pressure to the tangent matrix of stiffness is asymmetric, and we point out that the geometrical part of the tangent matrix is already asymmetric [bib2].

2 Kinematics

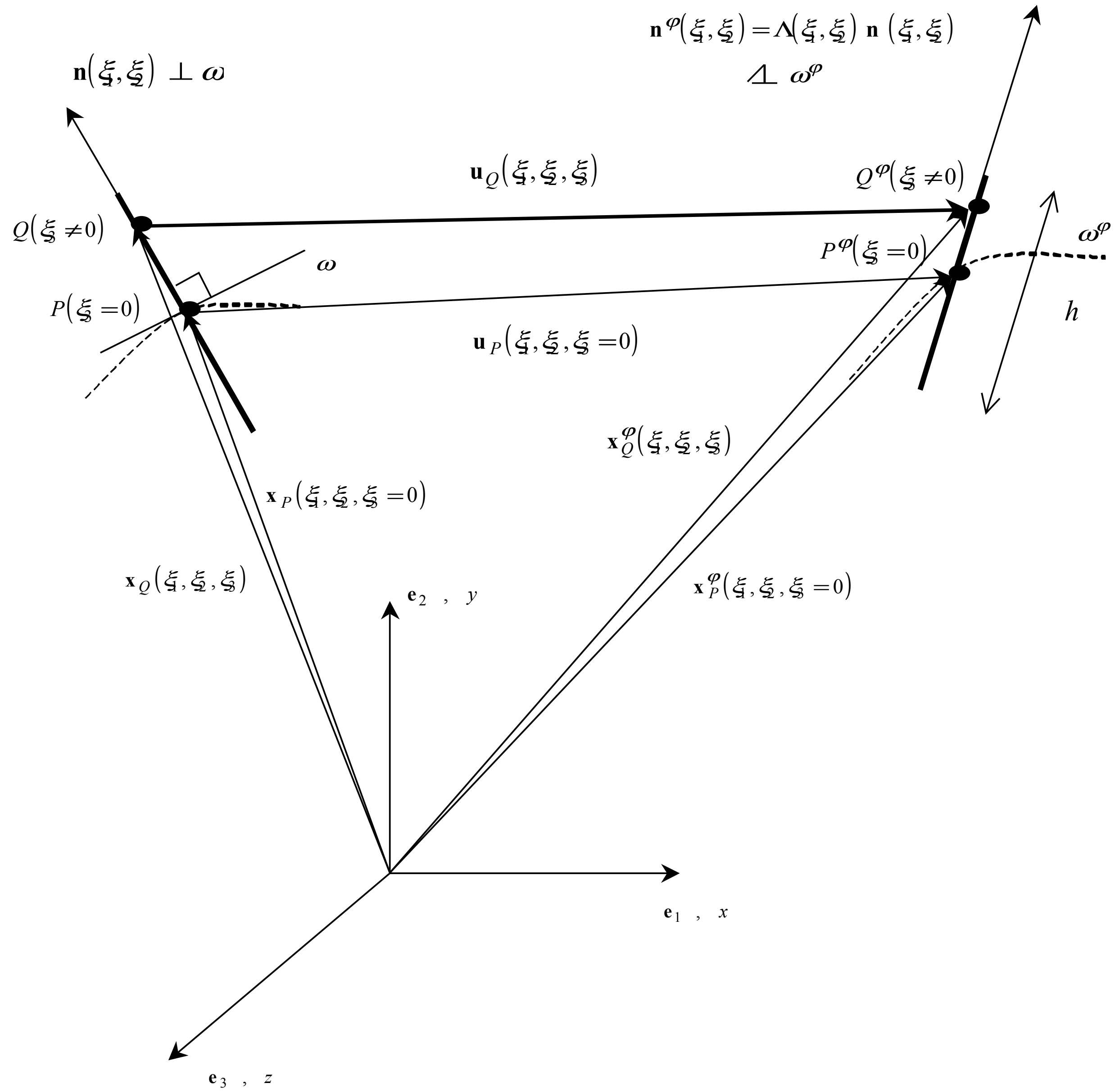
For the shell elements voluminal Ω one defines a surface of reference ω, or mean surface, left (of curvilinear coordinates $\xi_1 \xi_2$ for example) and a thickness $h(\xi_1 \xi_2)$ measured according to the norm on mean surface. The position of the points of the shell is given by the curvilinear coordinates (ξ_1, ξ_2) of mean surface ω and rise ξ_3 compared to this surface.

One points out the great transformation undergone by the shell:

> ω^{φ} (together of the points P^{φ} with $\xi_3 = 0$) is the transform of initial mean surface ω (together of the points $P\, à\, \xi_3 = 0$).

The position of the point P^{φ} on the deformed configuration can be established according to the position of the initial point p as follows:

$$x_P^{\varphi}(\xi_1, \xi_2) = x_P(\xi_1, \xi_2) + u_P(\xi_1, \xi_2) \; .$$



Appear 2-a: Voluminal shell.
Great transformations of an initially normal fiber on mean surface

2.1 Parameterization of the transform of mean surface

the transform w^j can be parametrized in a way similar to parameterization of initial surface. Thus one can define the infinitesimal element of tangent vector in w^j :

$$\mathbf{dx}_P^{\varphi}(\xi_1, \xi_2) = \frac{\partial x_P^{\varphi}}{\partial \xi_1} d\xi_1 + \frac{\partial x_P^{\varphi}}{\partial x_1} d\xi_2$$

$$\mathbf{dx}_P^{\varphi}(\xi_1, \xi_2) = d\xi_1 a_1^{\varphi}(x_1, x_2) + d\xi_2 a_2^{\varphi}(\xi_1, \xi_2)$$

where $\left[a_1^{\varphi}\left(\xi_1,\xi_2\right);a_2^{\varphi}\left(\xi_1,\xi_2\right)\right]$ represents a natural base nonorthogonal $\left(a_1^{\varphi}.a_2^{\varphi}\neq 0\right)$ and not normalized $\left(\|a_1^{\varphi}\|\neq 1;\|a_1^{\varphi}\|\neq 1\right)$ tangent on the surface ω^{φ} . The two basic vectors can be related to displacements via the following formula:

$$a_1^{\varphi}\left(\xi_1,\xi_2\right)=\frac{\partial x_P^{\varphi}}{\partial \xi_1}=\frac{\partial\left(x_p+u_p\right)}{\partial \xi_1}$$
$$a_2^{\varphi}\left(\xi_1,\xi_2\right)=\frac{\partial x_P^{\varphi}}{\partial \xi_2}=\frac{\partial\left(x_p+u_p\right)}{\partial \xi_2}$$

what makes it possible to connect them to the vectors of the natural base related to initial surface w by the relations:

$$a_1^{\varphi}\left(\xi_1,\xi_2\right)=a_1\left(\xi_1,\xi_2\right)+\frac{\partial u_p}{\partial \xi_1}$$
$$a_2^{\varphi}\left(\xi_1,\xi_2\right)=a_2\left(\xi_1,\xi_2\right)+\frac{\partial u_p}{\partial \xi_2}$$

It is important to note that these vectors are distinct from the vectors obtained by the large rotation Λ of the vectors $a_1\left(\xi_1,\xi_2\right);a_2\left(\xi_1,\xi_2\right)$:

$$a_1^{\varphi}\left(\xi_1,\xi_2\right)\neq\Lambda\left(\xi_1,\xi_2\right)a_1\left(\xi_1,\xi_2\right)$$
$$a_2^{\varphi}\left(\xi_1,\xi_2\right)\neq\Lambda\left(\xi_1,\xi_2\right)a_2\left(\xi_1,\xi_2\right)$$

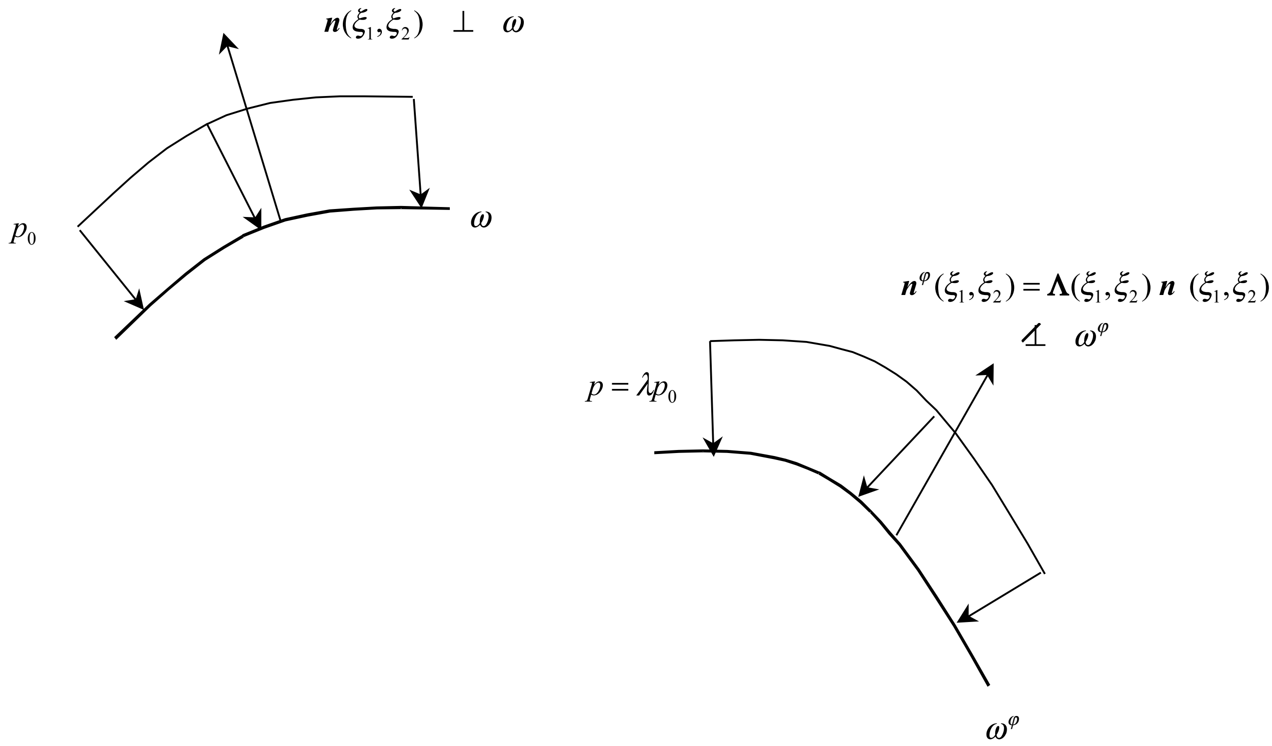
Indeed, because of strain due to the transverse shears, the turned vectors are not tangent any more with ω^{φ} . The illustration of that is given by [Figure 3.1-a].

With this parameterization, the infinitesimal vector surface element which is perpendicular to ω^{φ} can be written:

$$d\,\omega^{\varphi}\left(\xi_1,\xi_2\right)=a_1^{\varphi}\left(\xi_1,\xi_2\right)\times a_2^{\varphi}\left(\xi_1,\xi_2\right)d\,\xi_1 d\,\xi_2$$

3 Variational formulation

3.1 Virtual wor



Appears 3.1-a: Voluminal shell.
Following pressure on initial mean surface and its transform

the virtual wor of a following **pressure** p (i.e. acting on transformed mean surface and moving with) can be expressed in the form:

$$\delta\pi_{\substack{pression\\suiveuse}} = -\int_{\omega^{\varphi}} \delta u_p \,.\, p\, d\,\omega^{\varphi}$$

If one uses the isoparametric surface element corresponding to our modelization of voluminal shell, surface $d\,\omega^{\varphi}$ is expressed directly according to the isoparametric coordinates $d\,\xi_1 d\,\xi_2$ and one obtains the following simple form of the equation above:

$$\delta\,\pi_{\substack{pression\\suiveuse}} = -\int_{[-1,+1]\times[-1,+1]} \delta\,u_p \,.\, p\left(\xi_1,\xi_2\right) a_1^{\varphi}\left(\xi_1,\xi_2\right) \times a_2^{\varphi}\left(\xi_1,\xi_2\right) d\,\xi_1\, d\,\xi_2$$

3.2 Tangent operator

As the virtual wor of the following pressure depends on the current configuration, his linear variation Δ is not null and must be taken into account. The tangent operator associated with this virtual wor is written with the iteration $(i+1)$ in the form:

$$L\left[\Delta\delta\pi^{(i+1)}_{\substack{pression\\suiveuse}}\right]=\delta\pi^{(i)}_{\substack{pression\\suiveuse}}+\Delta\delta\pi^{(i)}_{\substack{pression\\suiveuse}}$$

where $\Delta\delta\pi^{(i)}_{\substack{pression\\suiveuse}}$ is the increment between two iterations of the virtual wor of the following pressure. If the pressure is given in the form:

$$p=\lambda\, p_0$$

λ being the level of load which is fixed lasting the iterations (control in load $\Delta\lambda=0$), one can write:

$$\Delta\delta\pi_{\substack{pression\\suiveuse}}=-\int_{[-1,+1]\times[-1,+1]}\delta u_p\,.\,p\left(a_1^{\varphi}\times\Delta a_2^{\varphi}-a_2^{\varphi}\times\Delta a_1^{\varphi}\right)d\xi_1\,d\xi_2$$

The incremental variations of the vectors of the tangent local base to the transform of mean surface are given by:

$$\boldsymbol{\Delta a_1^{\varphi}}=\frac{\partial}{\partial\xi_1}\boldsymbol{\Delta u_P}$$
$$\boldsymbol{\Delta a_2^{\varphi}}=\frac{\partial}{\partial\xi_2}\boldsymbol{\Delta u_P}$$

since initial mean surface “does not move” not during the iterations what involves $\boldsymbol{\Delta x_P}=0$.

These computations finally make it possible to draw up the statement of the increment of the virtual wor of following pressure in the form:

$$\Delta\delta\pi_{\substack{pression\\suiveuse}}=-\int_{[-1,+1]\times[-1,+1]}\delta u_P\cdot P\left(\left[a_1^{\varphi}\times\right]\frac{\partial}{\partial\xi_2}\Delta u_P-\left[a_2^{\varphi}\times\right]\frac{\partial}{\partial\xi_2}\Delta u_P\right)d\xi_1\,d\xi_2$$

where $\left[\boldsymbol{a_1^{\varphi}}\times\right]et\left[\boldsymbol{a_2^{\varphi}}\times\right]$ are respectively the skew-symmetric matrixes of the tangent vectors $\boldsymbol{a_1^{\varphi}}\,et\,\boldsymbol{a_2^{\varphi}}$ respectively.

Note:

In the reference [bib2], an integration by part is undertaken on the statement above. It is shown that the tangent matrix can be broken up into a symmetric part resulting from an integration on the field and an antisymmetric part resulting from integration on contour. Il is as shown as the assembly of the skew-symmetric parts of the elementary tangent matrixes leads to a matrix null when the pressure is continuous of a finite element with another, because of existence of a potential associated with work with the pressure in this case there.

4 Discretization

At the points P of mean surface, the interpolation of virtual displacement is written:

$$\boldsymbol{\delta u}\left(\xi_{1,}\xi_{2}\right)=\sum_{I=1}^{NB1} N_I^{(1)}\left(\xi_{1,}\xi_{2}\right)\begin{pmatrix}\delta u\\ \delta v\\ \delta w\end{pmatrix}_I$$

and the interpolation of incremental displacement between two iterations is written:

$$\boldsymbol{\delta u}\left(\xi_{1,}\xi_{2}\right)=\sum_{I=1}^{NB1} N_I^{(1)}\left(\xi_{1,}\xi_{2}\right)\begin{pmatrix}\Delta u\\ \Delta v\\ \Delta w\end{pmatrix}_I$$

We rewrite the two preceding equations in the matric form:

$$\delta \boldsymbol{u}\left(\xi_1,\xi_2\right)=[\boldsymbol{N}]\{\delta \boldsymbol{u}\}^e$$
$$\Delta \boldsymbol{u}\left(\xi_1,\xi_2\right)=[\boldsymbol{N}]\{\Delta \boldsymbol{u}\}^e$$

where $[N]$ is the matrix of the shape functions of translation at the mean surface, whose statement is:

$$[N]=\left[\ldots\left[N_I^{(1)}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\right]_{I=1,NB1}\ldots\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}_{NB2}\right]$$

The shape functions $N_I^{(1)}\,\text{et}\,N_I^{(2)}$ (used thereafter are given in appendix of [R3.07.04]. The nodes $I=1,NB1$ are the nodes tops and the mediums on the sides (for the quadrangle and the triangle). The node $NB2$ is with the barycenter of the element.

The vector $\{\delta \boldsymbol{u}\}^e$ is the nodal vector of virtual displacements given by:

$$\{\delta \boldsymbol{u}\}^e = \begin{pmatrix} . \\ . \\ . \\ \begin{pmatrix} \delta w \\ \delta \theta_x \\ \delta \theta_y \\ \delta \theta_z \end{pmatrix}_I \\ . \\ . \\ . \\ I = 1, NB1 \\ \begin{pmatrix} \delta \theta_x \\ \delta \theta_x \\ \delta \theta_x \end{pmatrix}_{NB2} \end{pmatrix}$$

The vector $\{\Delta \boldsymbol{u}\}^e$ is the nodal vector of displacements incremental between two iterations.

$$\{\Delta \boldsymbol{u}\}^e = \begin{pmatrix} . \\ . \\ . \\ \begin{pmatrix} \Delta u \\ \Delta v \\ \Delta w \\ \Delta \theta_x \\ \Delta \theta_y \\ \Delta \theta_z \end{pmatrix}_I \\ . \\ . \\ . \\ I = 1, NB1 \\ \begin{pmatrix} \Delta \theta_x \\ \Delta \theta_x \\ \Delta \theta_x \end{pmatrix}_{NB2} \end{pmatrix}$$

This discretization also enables us to draw up the form of derivatives of the incremental displacement of mean surface compared to the surface isoparametric coordinates in the form:

$$\frac{\partial}{\partial \xi_1} \boldsymbol{\Delta} \boldsymbol{u}(\xi_{1,} \xi_2) = \left[\frac{\partial}{\partial \xi_2} \boldsymbol{N}\right] \{\Delta \boldsymbol{u}\}^e$$
$$\frac{\partial}{\partial \xi_2} \boldsymbol{\Delta} \boldsymbol{u}(\xi_{1,} \xi_2) = \left[\frac{\partial}{\partial \xi_2} \boldsymbol{N}\right] \{\Delta \boldsymbol{u}\}^e$$

where $\left[\frac{\partial}{\partial \xi_1} \boldsymbol{N}\right] \text{et} \left[\frac{\partial}{\partial \xi_2} \boldsymbol{N}\right]$ are the matrixes derived from the shape functions of translation at the mean surface, whose statements are:

$$\left[\frac{\partial}{\partial \xi_1} \boldsymbol{N}\right] = \left[\dots \left[\frac{\partial N_I^{(1)}}{\partial \xi_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right]_{I=1,NB1} \dots \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{NB2} \right]$$
$$\left[\frac{\partial}{\partial \xi_2} \boldsymbol{N}\right] = \left[\dots \left[\frac{\partial N_I^{(1)}}{\partial \xi_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right]_{I=1,NB1} \dots \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{NB2} \right]$$

Thus one can express the virtual wor of the following pressure in the following matric form:

$$\delta \pi_{\substack{pression \\ suiveuse}} = \{\delta \boldsymbol{u}^e\} . \left\{\boldsymbol{f}^e_{\substack{pression \\ suiveuse}}\right\}$$

with $\left\{\boldsymbol{f}^e_{\substack{pression \\ suiveuse}}\right\}$ the nodal vector of the external forces which can be expressed in the following way:

$$\left\{\boldsymbol{f}^e_{\substack{pression \\ suiveuse}}\right\} = \int_{[-1,+1]\times[-1,+1]} [\boldsymbol{N}]^T \left(\boldsymbol{a}_1^{\varphi} \times \boldsymbol{a}_1^{\varphi}\right) d\,\xi_1\, d\,\xi_2$$

It is important to note that with our parameterization of the transform of mean surface, the jacobian $\det\left(\left[\boldsymbol{J}(\xi_3=0)\right]\right)$ of this surface is not implied in the computation of the finite elements objects.

It will be also noted that the pressure is discretized with an isoparametric interpolation of the values with the NB2 nodes:

$$p(\xi_1, \xi_2) = \sum_{I=1}^{NB2} N_I^{(2)}(\xi_1, \xi_2)\, p_I$$

One can also express the increment between two iterations of the virtual wor of the following pressure in the matric form:

$$\Delta\delta\pi_{\substack{pression\\suiveuse}}=-\{\delta\boldsymbol{u}^e\}\{\boldsymbol{K}^e_{\boldsymbol{T}\substack{pression\\suiveuse}}\}\{\Delta\boldsymbol{u}^e\}$$

where $\{\boldsymbol{K}^e_{\boldsymbol{T}\substack{pression\\suiveuse}}\}$ is the contribution in the tangent matrix of stiffness of the external forces which can be expressed in the form:

$$\{\boldsymbol{K}^e_{\boldsymbol{T}\substack{pression\\suiveuse}}\}=\int_{[-1,+1]\times[-1,+1]}[\boldsymbol{N}]\,p\left[\boldsymbol{a}_1^{\varphi}\times\right]\left[\frac{\partial}{\partial\xi_2}\boldsymbol{N}\right]d\xi_1\,d\xi_2-\int_{[-1,+1]\times[-1,+1]}[\boldsymbol{N}]\,p\left[\boldsymbol{a}_2^{\varphi}\times\right]\left[\frac{\partial}{\partial\xi_1}\boldsymbol{N}\right]d\xi_1\,d\xi_2$$

Note:

It is noted that the formulations finite elements exits of this approach do not utilize the degrees of freedom of rotations. The processing is thus also valid for the facets of the finite elements of three-dimensional elasticity.

5 Bibliography

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6 Description of the versions of the document

Version Aster	**Author (S) Organization (S)**	**Description of the modifications**
05/01/00	P.MASSIN EDF: R & D /MMN *AL MIKDAD (SAMTECH)*	initial Text