Linear kinematic relations of Summarized type

RBE3:

This document describes the way in which the linear kinematic relations of type RBE3 are calculated.

Contents

1 Introduction

1.1 general Principle

a linear kinematic relation of type RBE3 imply a node says main to several node known as slaves. The relation causes to distribute the mass and the loads seen by the master node on the nodes slaves. This distribution is made so that the resultant of the load vector forces on the implied nodes is null.

1.2 Notations

X_M : coordinates of the master node

X_i : coordinates of i - ème slave node ($1\leq i\leq n$)

$\xi_i=X_i-X_M$: relative coordinates of i - ème slave node

ω_i : coefficient of i - ème slave node

$T_i=\begin{bmatrix}F_i\\M_i\end{bmatrix}$: load vector force with i - ème node, which thus contains the forces $F_i=\begin{bmatrix}F_{ix}\\F_{iy}\\F_{iz}\end{bmatrix}$ and the moments $M_i=\begin{bmatrix}M_{ix}\\M_{iy}\\M_{iz}\end{bmatrix}$ seen by this node

One will note M^T transposed of the matrix M

In addition, one considers that the nodes carry by default the components of displacements and rotation, that is to say 6 degrees of freedom per node.

2 Definitions

The computation of the relations requires a setting at the level of components of rotations, so that the relations created are not modified during a scaling of the problem. For that, the following characteristic length is defined:

$$L_c=\frac{\sum_{i=1}^{n}\|\xi_i\|}{n}$$

and the matrixes (diagonal), coefficient for the i-ème slave node:

$$W_i=\begin{bmatrix}\omega_i & 0 & 0 & 0 & 0 & 0\\ 0 & \omega_i & 0 & 0 & 0 & 0\\ 0 & 0 & \omega_i & 0 & 0 & 0\\ 0 & 0 & 0 & \omega_i L_c^2 & 0 & 0\\ 0 & 0 & 0 & 0 & \omega_i L_c^2 & 0\\ 0 & 0 & 0 & 0 & 0 & \omega_i L_c^2\end{bmatrix}$$

In addition, computations require the introduction of the matrixes which make it possible to clarify the formulas of change of point of reduction for torsors. It is pointed out that the load vector force of the master node reduced on the slave node i is written:

$$T_i=\begin{bmatrix}F_i\\M_i\end{bmatrix}=\begin{bmatrix}F_M\\\xi_i\wedge M_M\end{bmatrix}=S_i T_M$$

The matrix thus is defined S_i :

$$S_i=\begin{bmatrix}1&0&0&0&(\xi_i)_z&-(\xi_i)_y\\0&1&0&-(\xi_i)_z&0&(\xi_i)_x\\0&0&1&(\xi_i)_y&-(\xi_i)_x&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}$$

3 Statement of the kinematic relations

3.1 Obtaining the kinematic relations

the reasoning is carried out on the transmission of the forces between the master node and the nodes slaves. Initially, it is considered that all the components of the master node and all the components of the nodes slaves are implied in the relation (section 5 approaches the restriction of the relations on certain components).
That is to say T_M the load vector force of the master node which one seeks to distribute on T_i , load vector forces on the nodes slaves.
The following form of distribution is adopted for each T_i :
$T_i=W_i S_i X T_M$
where the matrix X is unknown and is used to distribute the forces on the nodes slaves. It is given by calculating the resultant of the load vector forces on the nodes slaves and while imposing that it is equal to the torsor with the master node:
$T_M=\sum_{i=1}^n S_i^T T_i=\sum_{i=1}^n S_i^T W_i S_i X T_M$

One thus obtains:
$X=(S^T W S)^{-1}$ where S is the assembly of S_i and W is the assembly Des. W_i

One notes that the matrix X must be invertible. This is ensured by a relevant choice of the degrees of freedom Master and slave.

One notes now $B_i=W_i S_i X$. Relation $T_i=B_i T_M$, one deduces by duality the kinematic relations to be imposed on the degrees of freedom of displacement:
$u_i=(B_i)^T u_M$

3.2 Dimension of the matrixes

One notes $NDDLES$ the nombre total of the degrees of freedom slaves. The matrixes which one handles have following dimensions:
–Stamp W (assembly of W_i): $NDDLES$ lines, $NDDLES$ columns
–Stamps S (assembly of S_i): $NDDLES$ lines, 6 columns
–Stamps $S^T W S$: 6 lines, 6 columns
–Stamps X : 6 lines, 6 columns
–Stamps B : $NDDLES$ lines, 6 columns

4 Restriction of the relations on certain Component

4.1 components on the nodes slaves

to restrict the components on the nodes slaves with the wanted components, only the lines of the matrixes S_i corresponding to the desired components are preserved (these matrixes become consequently of the rectangular and either square matrixes).
The matrixes W_i remain square because one removes the lines and the columns corresponding to the degrees of freedom slaves not implied in the linear relation.

4.2 Components on the master node

to restrict the components on the master node with the wanted components, only the lines of the matrixes B_i corresponding to the desired components are preserved.