Linear modelization of the elements of continuous medium in thermal

Summarized:

One describes the statement of the elementary terms intervening in the linear modelization of the equation of heat and in postprocessings. One gives the mathematical statement of the integral to be evaluated, and for each element one provides the number of points of integration used.

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1 the options of linear modelization

In this document, one consider only the linear modelization of the physical phenomenon of change of the temperature in one continuum. All the coefficients intervening in the equation of heat will be constants or many functions being able to depend on time or space. The boundary conditions could be only linear functions of the temperature.

By default the material is supposed to be isotropic, the Fourier analysis connecting heat flux to the variation in temperature utilizes a scalar coefficient λ thermal conductivity:

$$q=-\lambda\nabla T$$

In the general case, unspecified medium, this relation is expressed with a thermal tensor of conductivity. The definite associated matrix being positive, it is always possible to be brought back to a diagonal matrix in the reference associated with the clean directions. The processing of the thermal anisotropy (see [feeding-bottle 1] is thus carried out in *Code_Aster* by providing the values of thermal conductivity for each principal direction and the clean reference. The evaluating of the elementary terms is carried out then by recovering the various coefficients and while changing reference. Two types of anisotropy are treated in *Code_Aster* , it acts:

- Cartesian anisotropy where the privileged directions remain fixed in a cartesian coordinate system, the data of the three nautical angles α , β and γ makes it possible to pass from the total reference to the principal reference of anisotropy,
- of the cylindrical anisotropy where privileged directions remain fixed in a cylindrical coordinate system, the data of the two nautical angles α and β defining the direction of the axis and of the three punctual coordinates of this axis allows to pass from the total reference to the principal reference of anisotropy.

The variational formulation of the linear equation of heat (cf [R5.02.01]) led to the evaluating of a certain number of statements in the form of integrals which constitute finally a matric system. The matrix and the second member are built from various bricks: the computation options which gather one or more integrals. The options described here are common to the group of the finite elements isoparametric. Their evaluating depends on the type of element: degree of the shape functions, number and family of points of integration used.

One will be able to refer to the documents [U3.23.01], [U3.23.02] and [U3.24.01] concerning the various modelizations (type of mesh supporting the finite elements).

2 Statement of the elementary terms for the various computation options

2.1 general Notations

We indicate by:

Ω	open of $\Re^3$ or $\Re^2$ border Γ ,
t	the variable representing time,
Δt	time step used,
r	the variable of space,
T	the temperature (unknown of the problem),
T^n	the temperature at previous time (known),
T^*	the function test,
ρ	density,
C_p	specific heat with constant pressure,
$c_p = \rho C_p$	heat capacity with constant pressure per unit of volume,
θ	the parameter of θ - method for the transitory thermal analysis.

2.2 Elementary terms making a thermal

2.2.1 contribution Stiffness

Term utilizing variations in temperatures and the coefficient of conduction λ in the case of isotropic mediums (denomination used by analogy at the end of stiffness intervening in the equation of the modelization of the mechanical phenomenon of elasticity). The coefficient λ can depend on time.

- mathematical statement: $\int_\Omega \theta \lambda(t) \nabla T . \nabla T^* d\Omega$,
  when the medium is anisotropic, the evaluating of flux $\lambda(t)\nabla T$ is carried out in the principal reference of anisotropy after a first change of reference (the thermal tensor of conductivity is diagonal there) then by a change locates opposite, one returns in the total reference,
- denomination of the option in the catalogs: `RIGI_THER`,
- many points of integration used: (first family of points of integration cf [R3.01.01]).

net support	many nodes	many points
triangle	3	1
	6	3
quadrangle	4	4
	8 or 9	9
tetrahedron	4	4
	10	15
pentahedron	6	6
	15	21
hexahedron	8	8
	20	27
	27	27

Table 2.2.1-1

2.2.2 Masses thermal

Term utilizing the coefficient of heat capacity to constant pressure $c_p = \rho C_p$ (denomination used by analogy at the end of mass intervening in the equation of the modelization of the equations of the dynamics). The coefficient C_p can depend on time.

- mathematical statement: $\int_{\Omega} \frac{1}{\Delta t} \rho C_p(t) T . T^* d\Omega$
- denomination of the option in the catalogs: MASS_THER
- many points of integration used: (second family of points of integration)

mesh support	many nodes	many points
triangle	3	3
	6	6
quadrangle	4	4
	8 or 9	9
tetrahedron	4	4
	10	15
pentahedron	6	6
	15	21
hexahedron	8	8
	20	27
	27	27

Table 2.2.2-1

2.2.3 Stiffness due to the boundary conditions of Term

exchange utilizing the coefficient of heat exchange h having for origin a boundary condition modelling the convective exchanges with edge of the field. The coefficient h can depend on time and space.

- mathematical statement: $\int_G \theta h(r,t) T . T^* dG$
- denomination of the option in the catalogs: RIGI_THER_COEF_R or RIGI_THER_COEF_F
- many points of integration used:

net support	many nodes	many points
segment	2	4
	3	4
triangle	3	3
	6	4
quadrangle	4	4
	8 or 9	9

Table 2.2.3-1

2.2.4 Stiffness due to the conditions of exchange between walls

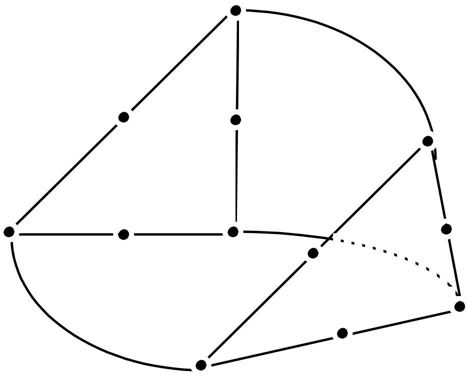
Term due to the boundary condition of the Neumann type bringing into play two pennies left the border in opposite and utilizing a single coefficient of heat exchange h. This kind of boundary condition creates new relations between the degrees of freedom of the border.

In this case, a particular finite element is used whose mesh support is obtained by associating two meshes identical edge or from face, the shape functions used and the points of integration are those of the mesh of departure.

Into three-dimensional, meshes the support of the elements of face are of type TRIA3-TRIA3, QUAD4-QUAD4, TRIA6-TRIA6, QUAD8-QUAD8 or QUAD9-QUAD9.

Into two-dimensional, they are of type SEG2-SEG2 or SEG3-SEG3.

One will be able to refer to [U4.25.02 § 3.1.3] for the description of the algorithm of search of meshes in opposite.



- mathematical statement:

  $\int_{\Gamma_1} (h(r,t+\Delta t)\theta(T_2-T_1)).T^* d\Gamma_1$ and $\int_{\Gamma_2} (h(r,t+\Delta t)\theta(T_1-T_2)).T^* d\Gamma_2$

  where Γ_1 and Γ_2 are two pennies left the border in opposite.
- denomination of the option in the catalogs: `RIGI_THER_PARO_R` or `RIGI_THER_PARO_F`
- many points of integration used: cf [Table 2.2.3-1].

2.3 Elementary terms making a contribution to the second member

2.3.1 Discretization in time

Term due:

- with the discretization of derivative in time utilizing part of the term of mass with the coefficient of heat capacity ρC_p ,
- with the θ - method utilizing part of the stiffness in the second member with the coefficient of conduction λ ,
- mathematical statement in the case of isotropic mediums: $\int_{\Omega} \frac{1}{\Delta t} \rho C_p T^n . T^* d\Omega - \int_{\Omega} (1-q)\lambda \nabla T^n . \nabla T^* d\Omega$

  when the medium is anisotropic, the evaluating of flux $\lambda(t)\nabla T$ is carried out in the principal reference of anisotropy after a first change of reference (the thermal tensor of conductivity is diagonal there) then by a change locates opposite, one returns in the total reference,
- denomination of the option in the catalogs: `CHAR_THER_EVOL`,
- many points of integration used: cf [Table 2.2.2-1].

2.3.2 Term of Term

volumic source due to the volumic source of heat.

- mathematical statement: $\int_{\Omega} (\theta s(r,t+\Delta t)+(1-q)s(r,t)).T^* d\Omega$,
- denomination of the option in the catalogs: `CHAR_THER_SOUR_R` or `CHAR_THER_SOUR_F`,
- many points of integration used: cf [Table 2.2.1-1].

2.3.3 Term of Term

convective exchange due to the boundary condition of convective exchange utilizing the coefficient of heat exchange h and the temperature of the "external" medium T_{ex} .

- mathematical statement:

$$\int_{\Gamma}(\theta h(r,t+\Delta t)T_{ex}(r,t+\Delta t)+(1-\theta)h(r,t)(T_{ex}(r,t)-T^{n})).T^{*}d\Gamma$$

- denomination of the option in the catalogs: `CHAR_THER_R` or `CHAR_THER_F`,
- many points of integration used: cf [Table 2.2.3-1].

2.3.4 Term of normal flux imposed

Term due to the flux boundary condition imposed according to the norm on the border, utilizing a function being able to depend on the variables r and t .

- mathematical statement: $\int_{\Gamma}(\theta f(r,t+\Delta t)+(1-\theta)f(r,t)).T^{*}d\Gamma$,
- denomination of the option in the catalogs: `CHAR_THER_FLUN_R` or `CHAR_THER_FLUN_F`,
- many points of integration used: cf [Table 2.2.3-1].

2.3.5 Term of exchange between walls

Term due to the boundary condition of the Neumann type bringing into play two pennies left the border in opposite and utilizing a single coefficient of heat exchange h.

- mathematical statement:

  $\int_{\Gamma_1}(h(r,t)(1-\theta)(T_2^n-T_1^n)).T^{*}d\Gamma_1$ and $\int_{\Gamma_2}(h(r,t)(1-\theta)(T_1^n-T_2^n)).T^{*}d\Gamma_2$

  where Γ_1 and Γ_2 are two pennies left the border in opposite
- denomination the option in the catalogs: `CHAR_THER_PARO_R` or `CHAR_THER_PARO_F`,
- many points of integration used: cf [Table 2.2.3-1].

3 Bibliography

1) N.RICHARD: Development of the thermal anisotropy in the software *Aster*. Note EDF/DER HM-18/94/0011 of the 7/5/1994.

4 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
3	*J.P. LEFEBVRE, X. DESROCHES (EDF/IMA/MMN)*	initial Text