Elements of Fourier for axisymmetric structures

Abstract

the elements of Fourier are intended to calculate the structure response for axisymmetric geometry solicited by nonaxisymmetric loadings broken up into Fourier series.

One exposes in this document a general theory of Analysis of Fourier with coupling of the symmetric and skew-symmetric modes in the anisotropic case. The case of the isotropic, or orthotropic materials of axis Oz , where the modes are uncoupled, is studied except for.

The elements of Fourier are usable in *Code_Aster* from modelization AXIS_FOURIER. Meshes the supports of these elements are triangles and quadrangles of degree 1 and 2.

Contents

1 Introduction

the analysis of Fourier are intended to calculate the structure response for axisymmetric geometries subjected to nonaxisymmetric loadings. In this case, it is necessary to develop the loadings in Fourier series. Generally convergence is reached for 4 or 5 harmonics, but the speed of this convergence depends on the nature of the loading: the more regular the loading is and the more quickly the corresponding series converges. The most unfavourable case is that of a concentrated force for which the practice shows that it is necessary to go to beyond (at least 7 harmonics).

In *the Code_Aster*, the decomposition of the loading in Fourier series is supposed to be made as a preliminary by the user. *The Code_Aster* makes it possible to calculate the responses with this loading, harmonic by harmonic (modelization `AXIS_FOURIER`), and overall after recombination of the harmonics between them (operator `COMB_FOURIER`).

One will expose in a first chapter the general frame of the anisotropy, while insisting on the decoupling of the modes in the orthotropic case. The second chapter clarifies the computation of the stiffness matrix in the isotropic case.

For the use of the elements of Fourier in *Code_Aster*, one returns to the note of use of the modelization Fourier [U2.01.07].

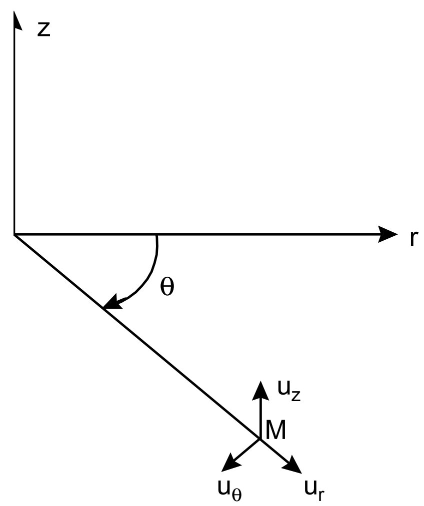
2 Analyzes of anisotropic Fourier

2.1 general Theory

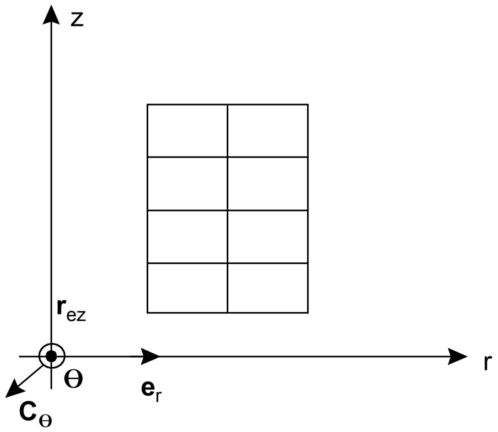
All the fields considered (forces, displacements, strains, stresses) are expressed in cylindrical coordinates with following convention on the order of the components:

1 radial component according to r
2 axial component according to z
3 tangential component according to θ

Example: $(u_r, u_z, u_\theta), (f_r, f_z, f_\theta)$



The mesh is localised in the plane (r, z), the symmetry of revolution being done around the axis Oz. The trihedron (r, z, θ) is directed in the direct meaning.



Displacement is broken up $\mathbf{u}$ (or the loading $\mathbf{f}$) according to $\mathbf{u}=\mathbf{u}^s+\mathbf{u}^a$ where $\mathbf{u}^s$ (resp. $\mathbf{u}^a$) indicates the symmetric part (resp. skew-symmetric) of the development in Fourier series from $\mathbf{u}$ ratio to the variable θ.

One obtains:

$$\left.\begin{array}{l} u_r^s=\sum_{l=0}^{\infty} u_l^s(r,z)\cos l\theta \\ u_z^s=\sum_{l=0}^{\infty} v_l^s(r,z)\cos l\theta \\ u_\theta^s=\sum_{l=0}^{\infty} w_l^s(r,z)(-\sin l\theta) \end{array}\right\}\text{partie symétrique } u^s$$

$$\left.\begin{array}{l} u_r^a=\sum_{l=0}^{\infty} u_l^a(r,z)\sin l\theta \\ u_z^a=\sum_{l=0}^{\infty} v_l^a(r,z)\sin l\theta \\ u_\theta^a=\sum_{l=0}^{\infty} w_l^a(r,z)\cos l\theta \end{array}\right\}\text{partie antisymétrique } u^a$$

A to note the choice of the sign $-$ for u_θ^s, which makes it possible to simplify later computations. If one notes $\mathbf{U}_l^s=(u_l^s, v_l^s, w_l^s)$ $(\text{resp.}\, \mathbf{U}_l^a)$ $l-\text{ième}$ the symmetric component (resp. skew-symmetric) from the development in Fourier series of $\mathbf{u}$, one obtains:

$$\mathbf{u}=\sum_{l=0}^{\infty}\left[\begin{pmatrix}\cos l\theta & & 0\\ & \cos l\theta & \\ 0 & & -\sin l\theta\end{pmatrix}\mathbf{U}_l^s+\begin{pmatrix}\sin l\theta & & 0\\ & \sin l\theta & \\ 0 & & \cos l\theta\end{pmatrix}\mathbf{U}_l^a\right] \qquad \text{éq 2.1-1}$$

If one indicates by $\boldsymbol{\varepsilon}$ the vector strain linearized, one realizes that $\boldsymbol{\varepsilon}$ can be broken up into following Fourier series:

$$\boldsymbol{\varepsilon}=\sum_{l=0}^{\infty}\begin{pmatrix}\cos l\theta\, I_4 & 0_{4,2}\\ 0_{2,4} & -\sin l\theta\, I_2\end{pmatrix}\boldsymbol{\varepsilon}_l^s+\begin{pmatrix}\sin l\theta\, I_4 & 0_{4,2}\\ 0_{2,4} & \cos l\theta\, I_2\end{pmatrix}\boldsymbol{\theta}_l^a \qquad \text{éq 2.1-2}$$

with $\boldsymbol{\varepsilon}=\{\varepsilon_r, \varepsilon_z, \varepsilon_q, \gamma_{rz}, \gamma_{rq}, \gamma_{zq}\}$

$$\boldsymbol{\varepsilon}_l^s=B_l^s\mathbf{U}_l^s \qquad \boldsymbol{\varepsilon}_l^a=B_l^a\mathbf{U}_l^a$$

(see [bib1]):

$$B_l^s=\begin{bmatrix} \frac{\partial}{\partial r} & 0 & 0 \\ 0 & \frac{\partial}{\partial z} & 0 \\ \frac{1}{r} & 0 & -\frac{l}{r} \\ \frac{\partial}{\partial z} & \frac{\partial}{\partial r} & 0 \\ \frac{l}{r} & 0 & \frac{\partial}{\partial r}-\frac{1}{r} \\ 0 & \frac{l}{r} & \frac{\partial}{\partial z} \end{bmatrix}$$

One has $B_l^a=B_l^s \ \forall l$ (this is due to the choice of the symmetric development of $\mathbf{u}$ in $(\cos, \cos, -\sin)$ instead of $(\cos, \cos, \sin)$). One will omit from now the indices a and s one will note B_l the operator allowing to calculate the strains corresponding to the harmonic l .

2.2 Coupling and decoupling of the modes symmetric and skew-symmetric

By taking again the preceding notations, one a:

$$\mathbf{u}=\sum_l \begin{pmatrix} \cos l\theta I_2 & 0_{2,1} \\ 0_{1,2} & -\sin l\theta \end{pmatrix}\mathbf{u}_l^s+\sum_l \begin{pmatrix} \sin l\theta I_2 & 0_{2,1} \\ 0_{1,2} & \cos l\theta \end{pmatrix}\mathbf{u}_l^a$$

what is written, by introducing matrixes M_l^s et M_l^a :

$$\mathbf{u}=\sum_l \left(M_l^s\mathbf{U}_l^s+M_l^a\mathbf{U}_l^a\right)$$
$$\mathbf{u}_l=M_l^s\mathbf{U}_l^s+M_l^a\mathbf{U}_l^a$$

One from of deduced that: $\boldsymbol{\varepsilon}_l=M'^s_l\boldsymbol{\varepsilon}_l^s+M'^a_l\boldsymbol{\varepsilon}_l^a$

avec $M'^{s}_{l}=\begin{pmatrix}\cos l\theta I_4 & 0_{4,2}\\ 0_{2,4} & -\sin l\theta I_2\end{pmatrix}$

$M'^{a}_{l}=\begin{pmatrix}\sin l\theta I_4 & 0_{4,2}\\ 0_{2,4} & \cos l\theta I_2\end{pmatrix}$

Computation of strain energy

$$
\begin{aligned}
W_l &= \int_0^{2\pi}\int_s {}^t\varepsilon_l D\varepsilon_l\, ds\, d\theta \quad \text{avec } ds=rdrdz\\
&= \int_0^{2\pi} d\theta\int_s {}^t\varepsilon_l^s\, {}^tM'^s_l DM'^s_l\varepsilon_l^s ds+\int_0^{2\pi} d\theta\int_s {}^t\theta_l^a\, {}^tM'^a_l DM'^a_l\varepsilon_l^a ds\\
&+\int_0^{2\pi} d\theta\int_s {}^t\varepsilon_l^a\, {}^tM'^a_l DM'^s_l\varepsilon_l^s ds+\int_0^{2\pi} d\theta\int_s {}^t\varepsilon_l^s\, {}^tM'^s_l DM'^a_l\varepsilon_l^a ds
\end{aligned}
$$

Puisque $M'^a_l DM'^s_l=\begin{pmatrix}\sin l\theta I_4 & 0\\ 0 & \cos l\theta I_2\end{pmatrix}\begin{pmatrix}D_1 & D_3\\ {}^tD_3 & D_2\end{pmatrix}\begin{pmatrix}\cos l\theta I_4 & 0\\ 0 & -\sin l\theta I_2\end{pmatrix}$

$$M'^a_l DM'^s_l=\begin{pmatrix}D_1\sin l\theta\cos l\theta & -D_3(\sin l\theta)^2\\ {}^tD_3(\cos l\theta)^2 & -D_2\sin l\theta\cos l\theta\end{pmatrix}$$

and that $\int_0^{2\pi}\sin l\theta\cos l\theta\, d\theta=0$, si $D_3=0$ there is thus no term $({}^t\varepsilon_l^a,\varepsilon_l^s)$ ou $({}^t\varepsilon_l^s,\varepsilon_l^a)$ in W. There is then no coupling (U^a,U^s) ou (U^s,U^a).

2.3 Computation of the stresses

Just as $\boldsymbol{\varepsilon}$, $\boldsymbol{\sigma}$ can be broken up into following Fourier series:

$$\boldsymbol{\sigma}=\sum_l\left(M'^s_l\boldsymbol{\sigma}_l^s+M'^a_l\boldsymbol{\sigma}_l^a\right)$$

Hooke's law $\boldsymbol{\sigma}=D\boldsymbol{\varepsilon}$, one deduces:

$$\mathbf{s}=\sum_l\begin{pmatrix}\cos l\theta D_1 & -\sin l\theta D_3\\ \cos l\theta D_3^t & -\sin l\theta D_2\end{pmatrix}\boldsymbol{\varepsilon}_l^s+\begin{pmatrix}\sin l\theta D_1 & \cos l\theta D_3\\ \sin l\theta D_3^t & \cos l\theta D_2\end{pmatrix}\boldsymbol{\varepsilon}_l^a$$

Maybe, while revealing the matrixes M'^s_l et M'^a_l :

$$
\begin{aligned}
\boldsymbol{\sigma}= &\sum_l M'^s_l\left[\begin{pmatrix}D_1 & 0_{4,2}\\ 0_{2,4} & D_2\end{pmatrix}\boldsymbol{\varepsilon}_l^s+\begin{pmatrix}0_{4,4} & D_3\\ -D_3^t & 0_{2,2}\end{pmatrix}\boldsymbol{\varepsilon}_l^a\right]\\
&+M'^a_l\left[\begin{pmatrix}0_{4,4} & -D_3\\ D_3^t & 0_{2,2}\end{pmatrix}\boldsymbol{\varepsilon}_l^s+\begin{pmatrix}D_1 & 0_{4,2}\\ 0_{2,4} & D_2\end{pmatrix}\boldsymbol{\varepsilon}_l^a\right]
\end{aligned}
$$

While posing $D^s\begin{pmatrix} D_1 & 0_{4,2} \\ 0_{2,4} & D_2 \end{pmatrix}$ and $D^a=\begin{pmatrix} 0_{4,4} & D_3 \\ -D_3^t & 0_{2,2} \end{pmatrix}$, one from of deduced the parts symmetric and skew-symmetric of the stress relating to the harmonic l :

$$\begin{cases} \boldsymbol{\sigma}_l^s = D^s \boldsymbol{\varepsilon}_l^s + D^a \boldsymbol{\varepsilon}_l^a = D^s B_l \mathbf{u}_l^s + D^a B_l \mathbf{u}_l^a \\ \boldsymbol{\sigma}_l^a = -D^a \boldsymbol{\varepsilon}_l^s + D^s \boldsymbol{\varepsilon}_l^a = -D^a B_l \mathbf{u}_l^s + D^s B_l \mathbf{u}_l^a \end{cases}$$ éq 2.3-1

Note::

In the case of the orthotropy compared to Oz, one has $D^a=0$ and [éq 2.1-1] is reduced to:

$$\begin{cases} \boldsymbol{\sigma}_l^s = D^s B_l \mathbf{u}_l^s \\ \boldsymbol{\sigma}_l^a = D^s B_l \mathbf{u}_l^a \end{cases}$$

I.e. if displacements are symmetric (or skew-symmetric), the stresses are it too.

3 Computation of the stiffness matrix

3.1 general Case

Are $\mathbf{u}$ and $\boldsymbol{\varepsilon}$ two unspecified kinematically admissible fields. By applying the principle of the virtual wors to the volume element v, one can write:

$$\int_v \left({}^t \delta \boldsymbol{\varepsilon} . \mathbf{s} \right) dv = \int_v \left({}^t \delta \mathbf{u} . f \right) dv$$

After decomposition in Fourier series and integration compared to θ, one obtains, for unspecified $\varepsilon_l^s, \varepsilon_l^a, u_l^s, u_l^a$ fields A.C. and any harmonic l :

$$\int_{s_l} \left({}^t \delta \varepsilon_l^s . \sigma_l^s + {}^t \delta \varepsilon_l^a . \sigma_l^a \right) ds_l = \int_{s_l} \left({}^t \delta u_l^s . f_l^s + {}^t \delta u_l^a . f_l^a \right) ds_l$$

Maybe, by means of [éq 2.3-1] and while posing:

$$K_l^s = \int_{s_l} {}^t B_l D^s B_l ds_l$$

$$K_l^a = \int_{s_l} {}^t B_l D^s B_l ds_l = K_l^s = K_l$$

$$K_l^{as} = \int_{s_l} {}^t B_l D^a B_l ds_l$$

The following system of equations is obtained:

$$\begin{cases} K_l u_l^s + K_l^{as} u_l^a = f_l^s \\ {}^t K_l^{as} u_l^s + K_l u_l^a = f_l^a \end{cases}$$ éq 3-1

where ${}^t K_l^{as} = -K_l^{as}$ it is seen that if $D_a \neq 0$, the decoupling of the modes in symmetric and skew-symmetric harmonics is not possible any more. On the other hand, if $D_a = 0$ (orthotropy compared to Oz) then $K_l^{as} = 0$ and [éq 3-1] is reduced to:

$$\begin{cases} K_l u_l^s = f_l^s \\ K_l u_l^a = f_l^a \end{cases}$$

While taking for vectors displacement (resp. force) corresponding to the harmonic l the vectors:

$$\mathbf{u}_l=\left\{u_r^s, u_z^s, u_\theta^s, u_r^a, u_z^a, u_\theta^a\right\}_l$$
$$\mathbf{f}_l=\left\{f_r^s, f_z^s, f_\theta^s, f_r^a, f_z^a, f_\theta^a\right\}_l$$

One has then:

$$K_l^g\,\mathbf{u}_l=\mathbf{f}_l \text{ avec } K_l^g=\begin{pmatrix} K_l & K_l^{as} \\ {}^t K_l^{as} & K_l \end{pmatrix}$$

3.2 Computation of K_l^g in the isotropic case

In this case one thus has $K_l^{as}=0$. One details in the continuation the computation of $K_l=\int_{s_l} {}^t B_l D^s B_l ds_l$

In the isotropic case, one a:

$$D=D^s=\begin{bmatrix} D1 & D2 & D2 & 0 & & \\ D2 & D1 & D2 & 0 & & 0 \\ D2 & D2 & D1 & 0 & & \\ 0 & 0 & 0 & D3 & & \\ & & 0 & & D3 & 0 \\ & & & & 0 & D3 \end{bmatrix}$$

avec $D1=\dfrac{E(1-\nu)}{(1+\nu)(1-2\nu)}$

$D2=\dfrac{E\nu}{(1+\nu)(1-2u)}$

$D3=\dfrac{E}{2(1+\nu)}$

One can write:

$$\begin{Bmatrix} \varepsilon_r \\ \varepsilon_z \\ \varepsilon_\theta \\ \gamma_{rz} \\ \gamma_{r\theta} \\ \gamma_{z\theta} \end{Bmatrix}=B_l\begin{Bmatrix} u_r^s \\ u_z^s \\ u_\theta^s \end{Bmatrix}=B_l'^{\,t}\left\{\frac{u_r}{r},\frac{u_z}{r},\frac{u_\theta}{r},\frac{\partial u_r}{\partial r},\frac{\partial u_z}{\partial r},\frac{\partial u_\theta}{\partial r},\frac{\partial u_r}{\partial z},\frac{\partial u_z}{\partial z},\frac{\partial u_\theta}{\partial z}\right\}$$

← fcts of form → ← derived from the fcts of form →

$$\text{avec } B'_l=\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & -l & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ l & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & l & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

While indicating by $\{W_J\}_{J=1 \text{ à } 4n}$ the shape functions of the element considered, one a:

$$U=\begin{bmatrix} \frac{u_r}{r} \\ \frac{u_z}{r} \\ \frac{u_\theta}{r} \\ \frac{\partial u_r}{\partial r} \\ \frac{\partial u_z}{\partial r} \\ \frac{\partial u_\theta}{\partial r} \\ \frac{\partial u_r}{\partial z} \\ \frac{\partial u_z}{\partial z} \\ \frac{\partial u_\theta}{\partial z} \end{bmatrix} = \begin{bmatrix} \cdots & \frac{W_J}{r} & \overbrace{0}^{\text{noeud } J} & 0 & \cdots \\ \cdots & 0 & \frac{W_J}{r} & 0 & \cdots \\ \cdots & 0 & 0 & \frac{W_J}{r} & \cdots \\ \cdots & \frac{\partial W_J}{\partial r} & 0 & 0 & \cdots \\ \cdots & 0 & \frac{\partial W_J}{\partial r} & 0 & \cdots \\ \cdots & 0 & 0 & \frac{\partial W_J}{\partial r} & \cdots \\ \cdots & \frac{\partial W_J}{\partial z} & 0 & 0 & \cdots \\ \cdots & 0 & \frac{\partial W_J}{\partial z} & 0 & \cdots \\ \cdots & 0 & \underbrace{0}_{\text{bloc } P_J} & \frac{\partial W_J}{\partial z} & \cdots \end{bmatrix} \begin{Bmatrix} . \\ . \\ . \\ u_r(J) \\ u_z(J) \\ u_\theta(J) \\ . \\ . \\ . \end{Bmatrix}$$

One notes $(P)=(P_1, \ldots, P_N)$ where N is the number of nodes of the element.

Then $K_l=\int_{s_l} {}^t P \, {}^t B_l^{'} D B_l^{'} P \, ds_l$

K_l symmetric and is formed by blocks $(K_l)^{I,J}$ 3×3 :

$$(K_l)^{I,J}=\int_{s_l} {}^t P_I \, {}^t B_l^{'} D B_l^{'} P_J \, ds_l$$

The computation blocks $(K_l)^{I,J}$ is clarified below:

$$tB_l^{'}DB_l^{'}=\begin{bmatrix} D1+1^2D3 & 0 & -l(D1+D3) & D2 & 0 & lD3 & 0 & D2 & 0\\ 0 & l^2D3 & 0 & 0 & 0 & 0 & 0 & 0 & lD3\\ -l(D1+D3) & 0 & l^2D1+D3 & -lD2 & 0 & -D3 & 0 & -lD2 & 0\\ D2 & 0 & -lD2 & D1 & 0 & 0 & 0 & D2 & 0\\ 0 & 0 & 0 & 0 & D3 & 0 & D3 & 0 & 0\\ lD3 & 0 & -D3 & 0 & 0 & D3 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & D3 & 0 & D3 & 0 & 0\\ D2 & 0 & -lD2 & D2 & 0 & 0 & 0 & D1 & 0\\ 0 & lD3 & 0 & 0 & 0 & 0 & 0 & 0 & D3 \end{bmatrix}$$

$$^tP_I\,^tB_l^{'}DB_l^{'}P_J=\left(K_{ij}^l\right)_{3\times 3}^{I,J}=\begin{bmatrix} K_{11}^{I,J} & K_{12}^{I,J} & K_{13}^{I,J}\\ K_{21}^{I,J} & K_{22}^{I,J} & K_{23}^{I,J}\\ K_{31}^{I,J} & K_{32}^{I,J} & K_{33}^{I,J}\end{bmatrix} \quad \text{avec}$$

$$\begin{cases}
K_{11}^{I,J}=\left(\frac{D1+l^2D3}{r^2}\right)W_IW_J+D1\frac{\partial W_I}{\partial r}\frac{\partial W_J}{\partial r}+D3\frac{\partial W_I}{\partial z}\frac{\partial W_J}{\partial z}+\frac{D2}{r}\left(W_I\frac{\partial W_J}{\partial r}+W_J\frac{\partial W_I}{\partial r}\right)\\
K_{22}^{I,J}=\left(\frac{l^2D3}{r^2}\right)W_IW_J+D3\frac{\partial W_I}{\partial r}\frac{\partial W_J}{\partial r}+D1\frac{\partial W_I}{\partial z}\frac{\partial W_J}{\partial z}\\
K_{33}^{I,J}=\left(\frac{l^2D1+D3}{r^2}\right)W_IW_J+D3\left(\frac{\partial W_I}{\partial r}\frac{\partial W_J}{\partial r}+\frac{\partial W_I}{\partial z}\frac{\partial W_J}{\partial z}\right)-\frac{D3}{r}\left(W_I\frac{\partial W_J}{\partial r}+\frac{\partial W_I}{\partial r}W_J\right)\\
K_{12}^{I,J}=D2\frac{\partial W_I}{\partial r}\frac{\partial W_J}{\partial z}+D3\frac{\partial W_I}{\partial z}\frac{\partial W_J}{\partial r}+\frac{D2}{r}W_I\frac{\partial W_J}{\partial z}\\
K_{21}^{I,J}=D3\frac{\partial W_I}{\partial r}\frac{\partial W_J}{\partial z}+D2\frac{\partial W_I}{\partial z}\frac{\partial W_J}{\partial r}+\frac{D2}{r}W_J\frac{\partial W_I}{\partial z}\\
K_{13}^{I,J}=-\frac{l}{r^2}(D1+D3)W_IW_J-\frac{l}{r}D2W_J\frac{\partial W_I}{\partial r}+\frac{l}{r}D3W_I\frac{\partial W_J}{\partial r}\\
K_{31}^{I,J}=-\frac{l}{r^2}(D1+D3)W_IW_J-\frac{l}{r}D2W_I\frac{\partial W_J}{\partial r}+\frac{l}{r}D3W_J\frac{\partial W_I}{\partial r}\\
K_{23}^{I,J}=-\frac{l}{r}D2\frac{\partial W_I}{\partial z}W_J+\frac{l}{r}D3W_I\frac{\partial W_J}{\partial z}\\
K_{32}^{I,J}=-\frac{l}{r}D2W_I\frac{\partial W_J}{\partial z}+\frac{l}{r}D3\frac{\partial W_I}{\partial z}W_J
\end{cases}$$

The blocks $K^{I,J}$ are not symmetric except for $I=J$ (on the diagonal of K). One notices in fact that the blocks $K^{I,J}$ can be written for any harmonic ($l=0$ including).

$$
\begin{cases}
K_{11}^{I,J} = K0_{11}^{I,J} + l^2 \frac{D3}{r^2} W_I W_J \\
K_{22}^{I,J} = K0_{22}^{I,J} + l^2 \frac{D3}{r^2} W_I W_J \\
K_{33}^{I,J} = K0_{33}^{I,J} + l^2 \frac{D1}{r^2} W_I W_J \\
K_{12}^{I,J} = K0_{12}^{I,J} \\
K_{21}^{I,J} = K0_{21}^{I,J} \\
K_{13}^{I,J} = -lK0_{13}^{I,J} \\
K_{31}^{I,J} = -lK0_{31}^{I,J} \\
K_{23}^{I,J} = -lK0_{23}^{I,J} \\
K_{32}^{I,J} = -lK0_{32}^{I,J}
\end{cases}
$$

where the blocks $K0^{I,J}$ are independent of the harmonic l .

4 Loadings

It is supposed that the loading was broken up according to the same base which displacements, that is to say:

$$
\mathbf{f} = \sum_{l=0}^{\infty} \left[\begin{pmatrix} \cos l\theta & & 0 \\ & \cos l\theta & \\ 0 & & -\sin l\theta \end{pmatrix} \mathbf{F}_l^s + \begin{pmatrix} \sin l\theta & & 0 \\ & \sin l\theta & \\ 0 & & \cos l\theta \end{pmatrix} \mathbf{F}_l^a \right]
$$

There is not coupling for the same harmonic between the parts symmetric and skew-symmetric of the loading because of orthogonality of the goniometrical functions $\sin l\theta$ and $\cos l\theta$, this for all the types of loading. This wants to say in particular that the equivalent nodal forces are the same ones for the harmonics symmetric and skew-symmetric if the amplitudes $F_l^s \text{et} F_l^a$ are the same ones.

For the nature of the acceptable loadings with the modelization Fourier, one returns to the note of use [U2.01.07].

5 Conclusion and Prospects

Currently, it is supposed that the decomposition of the loading was made as a preliminary by the user, i.e. $\left\{F_l^s, F_l^a\right\}_{l\geq 0}$ is known. This decomposition could be realized by an operator of *Code_Aster* which would project the loading on the modes of Fourier.

For time, only the nonanisotropic case is established, i.e. there is never coupling of the modes. The extension to the anisotropy can constitute a later development.

6 Bibliography

1 DUVAUT G.: “Mechanical of the continuums” p282

2 ASKA HS.: “Axisymmetric Structures in Fourier series”, May 1982, ISD

7 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
5	X.Desroches EDF-R&D/AMA	initial Text