Diagonalization of the thermal mass matrix

Summarized:

To improve the regularity of the solution in the problems of transient thermal, one of the solutions consists with “lumper” (i.e.: to condense on the diagonal) the thermal mass matrix (matrix of capacity).

This possibility is accessible by modelizations PLAN_DIAG, AXIS_DIAG and 3D_DIAG for the THERMAL phenomenon. It is activated at the time of the call to the commands of thermal computation THER_LINEAIRE and THER_NON_LINE.

When these modelizations are used, only the linear finite elements (2D and 3D) have a lumped mass matrix. Indeed, the direct diagonalization does not give of results satisfactory for the quadratic finite elements. Consequently, for the quadratic finite elements 2D, one carries out a cutting in linear elements, which are lumped. On the other hand, for the quadratic finite elements 3D, one does not make a diagonalization of the mass matrix.

The theoretical results are illustrated by the thermomechanical computation of a cylinder subjected to a thermal shock.

Contents

1 Introduction

One is interested in transitory thermal computations where intervene of abrupt variations of the loadings - for example, thermal shocks. In certain cases, it is noted that the temperature oscillates spatially and temporally. Moreover, if one observes a profile of temperature at a given time of the transient, the temperature can in some nodes exceeding the limits min. and max. imposed by the initial conditions and the boundary conditions. This result physically unacceptable viol what is called the “principle of the maximum”.

The diagonalization of the mass matrix can solve these problems of going beyond the maximum. This is detailed in the note [bib1]. One is satisfied here to recall the principal results of them.

One points out the principle of the maximum in the continuous case, then one expresses sufficient conditions which make it possible to check it for the discrete equations. It is shown in particular that the diagonalization of the thermal mass matrix is one of these sufficient conditions and one presents various methods for diagonalizing $\mathbf{M}$.

Another sufficient condition depends on the thermal stiffness matrix (conduction). One more particularly studies from this point of view the finite elements of thermal used in *Code_Aster*.

It results from it that in the case of the linear elements, all the sufficient conditions to check the principle of the maximum are gathered. In particular, the diagonalization of the mass makes it possible indeed to obtain a regular solution. On the other hand, for the quadratic elements, one cannot prevent the oscillations.

One thus describes the solution suggested in *Code_Aster* : the modelizations developed in 2D (AXIS_DIAG, PLAN_DIAG) function with linear elements (if the mesh is of order 2, one cuts out it in linear elements for thermal computation). In 3D, only the linear elements are treated.

A numerical study of a thermal shock on a cylinder makes it possible to illustrate these results.

2 Principle of the Stated

2.1 maximum of the principle for the continuous case

One gives here one of the statements possible of the principle of the maximum for the operator of heat (in the absence of terms of source, and in isotropic homogeneous linear thermal) [bib2].

That is to say Ω open limited of $\mathbb{R}^n$ border Γ , whose dependancy is noted $\bar{\Omega}$.

Maybe $u(x,t)$ such as:

$$\frac{\partial u}{\partial t}-\Delta u\leq 0 \text{ sur } \Omega\times]0,T[,(T>0)$$

of class C^2 compared to x and u of class C^1 compared to t on $\Omega'\times]0,T[$

Then $\underset{\bar{\Omega}\times[0,T]}{Max}\ u=\underset{P}{Max}\ u$, where $P=(\bar{\Omega}\times\{0\})\cup(\Gamma\times[0,T])$ is the border of the cylinder $\Omega\times]0,T[$.

This result thus ensures that the maximum of u is inevitably reached either during the initial conditions or on an edge of the field during the transient.

2.2 Respect of the principle of the maximum at the discrete level

One considers the equation of heat (thermal conduction):

$$\operatorname{div}(\lambda.\nabla T)+s(x,t)=\rho C_p\frac{\partial T}{\partial t}$$ **+ limiting Conditions + initial condition** $T(t_0,x)=T^0(x)$

with T temperature
s heat per unit of volume (internal sources)
t variable of variable
x time of space
λ thermal coefficient of conductivity
ρC_p voluminal heat with constant pressure

Types of limiting conditions (into linear):

- Imposed temperature: condition limits of Dirichlet

$$T(x,t)=T_{imp}(x,t)\,\text{sur}\,\Gamma_{imp}$$

- imposed normal Flux: condition of Neumann defining flux entering the field

$$-\mathbf{q}(x,t).\mathbf{n}=f(x,t)\,\text{sur}\,\Gamma_{flux}$$

- Exchanges: limiting condition of Fourier modelling the convective exchanges on edges of the field

$$-\mathbf{q}(x,t).\mathbf{n}=h(x,t)\left(T_{ext}(x,t)-T(x,t)\right)\text{sur}\,\Gamma_{échange}$$

the variational formulation of the problem is the following one: [bib3]

$$\int_\Omega \rho C_p\frac{\partial T}{\partial t}.v\,d\Omega+\int_\Omega \lambda\nabla T.\nabla v\,d\Omega+\int_{\Gamma_{échange}} hT.v\,d\Gamma=\int_\Omega s.v\,d\Omega+\int_{\Gamma_{flux}} f.v\,d\Gamma+\int_{\Gamma_{échange}} hT_{ext}.v\,d\Gamma$$

$\forall v$ checking $v=T_{imp}(x,t)\,\text{sur}\,\Gamma_{imp}$

After discretization in space of this equation, the system is obtained:

$$\mathbf{M}\left\{\frac{\partial \mathbf{T}}{\partial t}(t)\right\}+\mathbf{K}\mathbf{T}(t)=\mathbf{F}(t).$$

with $\mathbf{T}(t)$: vector of the nodal temperatures

$\mathbf{M}$: thermal mass matrix $\quad \mathbf{M}=\sum_e\int_{\Omega_e}\rho C\,\mathbf{N}\mathbf{N}^T dV$

$\mathbf{K}$: thermal stiffness matrix $\quad \mathbf{K}=\sum_e\left(\int_{\Omega_e}\lambda\nabla\mathbf{N}.\nabla\mathbf{N}^T dV+\int_{\Gamma_{échange_e}} h\mathbf{N}.\mathbf{N}^T d\Gamma\right)$

$\mathbf{F}$: vector of the second member $\quad \mathbf{F}=\sum_e\left(\int_{\Omega_e} s\mathbf{N}\,d\Omega+\int_{\Gamma_{flux_e}} f\,\mathbf{N}\,d\Gamma+\int_{\Gamma_{échange_e}} hT_{ext}\,\mathbf{N}\,d\Gamma\right)$

$\mathbf{N}$: (shape functions)

For the discretization in time, one applies one θ - method $(\theta\in[0,1])$, which leads to:

$$(\mathbf{M}+\theta\Delta t\,\mathbf{K})\,\mathbf{T}^{n+1}=(\mathbf{M}+(\theta-1)\,\Delta t\,\mathbf{K})\,\mathbf{T}^{\mathbf{n}}+\theta\langle\mathbf{F}^{n+1}\rangle+(1-\theta)\langle\mathbf{F}^{n}\rangle$$

where $\mathbf{T}^{n}$, $\mathbf{T}^{n+1}$ are the vectors of the nodal temperatures at times t_n, t_{n+1} .

2.3 Conditions sufficient for the respect of the principle of the maximum at the discrete level

One of the characteristics of nonrespect of the principle of the maximum is the appearance of oscillations (temporal or spatial): if one observes the variation in the temperature in a node in the course of time, one notes that the solution and the oscillates and exceeds the values minimal and maximum determined by the limiting initial conditions conditions. Or, at a given time, one observes spatial oscillations.

One thus seeks sufficient conditions on Δt , $\mathbf{K}$ and $\mathbf{M}$ so that the solution does not oscillate in the course of time ([bib1], [bib4], [bib5]). Indeed, one cannot obtain requirements and sufficient. One thus seeks conditions of nonoscillation of the solution in the course of time. If those are checked, it will be checked that the spatial oscillations also disappeared, and then the respect of the principle of the maximum is assured.

Assumptions:

To be able to express these sufficient conditions of nonoscillation, two assumptions should be added:

- *one places oneself at the elementary level. The respect of the properties at the elementary level is enough so that the conditions of nonoscillation are checked for the assembled matrixes.*
- *it is considered that* ***the stiffness matrix*** $\mathbf{K}$ ***is not formed that voluminal term*** $\mathbf{K}_{\mathbf{V}}=\int_{\Omega_e}\lambda\nabla\mathbf{N}.\nabla\mathbf{N}^{T}dV$

This assumption is not valid for all the limiting conditions (see paragraph 8).

The sufficient conditions of nonoscillation amount expressing conditions on time step and the diagonal and extra-diagonal terms of $\mathbf{M}$ and $\mathbf{K}$ so that certain properties of these matrixes are checked (based on the monotony of the matrixes) [bib1]:

$$M_{ij}+\theta.\Delta tK_{ij}\leq 0 \qquad i\neq j \qquad \text{éq 2.3-1}$$

$$M_{ij}+(\theta-1)\,\Delta tK_{ij}\geq 0 \qquad i\neq j \qquad \text{éq 2.3-2}$$

$$M_{ii}+(\theta-1)\,\Delta tK_{ii}\geq 0 \qquad \forall i \qquad \text{éq 2.3-3}$$

In the general case, the extra-diagonal terms can be of unspecified sign. A fast study makes it possible to determine the conditions on Δ T according to their signs so that the preceding equations are checked:

	$K_{ij} \geq 0$	$K_{ij} \leq 0$
$M_{ij} \geq 0$	$M_{ij} + \theta . \Delta t K_{ij} \leq 0 \quad i \neq j$ [éq 2.3-1] unconditionally distorts except $M_{ij} = K_{ij} = 0$	$\max_{i \neq j} \left(\frac{M_{ij}}{-\theta K_{ij}} \right) \leq \Delta t \leq \min_{i} \left(\frac{M_{ii}}{(1-\theta) K_{ii}} \right)$
$M_{ij} \leq 0$	$M_{ij} + (\theta - 1) \Delta t K_{ij} \geq 0 \quad i \neq j$ [éq 2.3-2] unconditionally distorts except $M_{ij} = K_{ij} = 0$	$\max_{i \neq j} \left(\frac{M_{ij}}{(1-\theta) K_{ij}} \right) \leq \Delta t \leq \min_{i} \left(\frac{M_{ii}}{(1-\theta) K_{ii}} \right)$
	Whatever is Δt and forms it $\mathbf{M}$, there is risk **of** oscillations.	Interval to be respected on Δt. The diagonalization of $\mathbf{M}$ makes it possible to remove the lower limit.

The sufficient conditions to avoid the oscillations are then:

$$K_{ij} \leq 0 \quad i \neq j$$
$$\Delta t_{\min} \leq \Delta t \leq \Delta t_{\max}$$

with:

- $\Delta t_{\max} = \min_{i} \left(\frac{M_{ii}}{(1-\theta) K_{ii}} \right)$ and
- $\Delta t_{\min} = \max_{i \neq j} \left(\frac{M_{ij}}{(1-\theta) K_{ij}}, \frac{M_{ij}}{-\theta K_{ij}} \right)$,

Consequently, it is necessary initially that the elementary matrixes check $K_{ij} \leq 0$ (it is the case of the finite elements **linear** studied further).

With regard to the interval on time step:

If the oscillations are due to time step too large $\left(\Delta t > \Delta t_{\max} \right)$, one can advise:

- **that is to say to choose a diagram of integration in time of the implicit type** $\left(\theta = 1 \right)$, to eliminate the higher limit from the interval.
- **that is to say to decrease** Δt. (In practice, it is difficult to know an order of magnitude of $\Delta t_{\max}$).

Rather often, the problem of the oscillations arises for the time step small ones $\left(\Delta t<\Delta t_{\min}\right)$; indeed, to take into account the variations of the solution (for example, during a thermal shock), one is brought to choose a fine discretization in time. In this case, to avoid the oscillations, one can suggest:

- **that is to say to increase the value of time step**. In practice, this is not always possible because Dt can be imposed by the nature of the problem (fast variation of the loading). Moreover, it is difficult to have an order of magnitude of $\Delta t_{\min}$.
- **that is to say to decrease the size of meshes** and thus to increase the number of elements. Indeed, the value of $\Delta t_{\min}$ depends directly on the spatial discretization:

  The forms of the elementary matrixes are indeed:

$$M_{ij}=\int_{\Omega_e}\rho CN_i N_j dV$$
$$K_{ij}=\int_{\Omega_e}\lambda\nabla N_i\nabla N_j dV$$

  For the elements 2D, the terms of $\mathbf{M}$ are thus of the form $\rho C\times\text{Surface}$ whereas those of $\mathbf{K}$ are only function of λ . This solution remains the best if one is not limited by the cost computation, because the thermal solution and especially mechanics will be all the more precise.

- **Maybe of diagonalizing the matrix** $\mathbf{M}$, which removes the lower limit of the interval. It is the solution suggested here.

In the continuation of the study, one is interested only in the problem of the oscillations which appear for time step the too small ones: $\Delta t<\Delta t_{\min}$. One more precisely presents the method of diagonalization of the matrix $\mathbf{M}$ chosen, and the different element types to which it applies.

3 Method of diagonalization selected and element types

3.1 limiting Elements and conditions such as the extra-diagonal terms of $\mathbf{K}$ are negative

One saw that the diagonalization of $\mathbf{M}$ is effective only when the extra-diagonal terms of the stiffness matrix $\mathbf{K}$ are negative. In the contrary case, one of the sufficient conditions of non-oscillation is unconditionally false, whatever the form of $\mathbf{M}$.

For each one of the finite elements used in thermal in *Code_Aster,* one checks if the elementary matrix of stiffness of the element has negative extra-diagonal terms, while resting mainly on [bib11], who gives the analytical statements of the elementary matrixes for the conventional finite elements. One summarizes here the observations made in [bib11].

3.1.1 Linear elements

3.1.1.1 Elements TRIA3, TETRA4, PENTA6

the elementary matrix $\mathbf{K}$ is function of the cotangents of the angles. If one of the angles is blunt ($\geq 90°$), certain extra-diagonal terms of $\mathbf{K}$ are positive. If all the angles are acute, the property is checked.

One has the same type of result in 3D for the tetrahedron with 4 nodes and the pentahedron with 6 nodes.

3.1.1.2 Elements QUAD4 and HEXA8

Certain terms extra-diagonal of $\mathbf{K}$ can be positive if the element is lengthened too much in a direction. If not, the property is checked.
One has the same type of result in 3D for element HEXA8.

3.1.1.3 Element 3D pyramid with 5 nodes

For this element, the shape functions are not any more of the polynomials but rational fractions in x, y, z. For this kind of element, one does not have the statement, even approximate of $\mathbf{K}$.

3.1.2 Quadratic elements

3.1.2.1 Element TRIA6

In $\mathbf{K}$, certain extra-diagonal terms are necessarily positive.

3.1.2.2 Of the same element

QUAD9, on the analytical statement of the terms of $\mathbf{K}$, one notes that some of the diagonal terms extra - are necessarily positive.

3.1.2.3 Element QUAD8

For this element, one does not have the complete statement of $\mathbf{K}$ for the real element. But for the element of reference, one notes that certain extra-diagonal terms are positive.

3.1.3 Limiting conditions

the matrix $\mathbf{K}$ is not always reduced at the end $\mathbf{K_V} = \int_{\Omega_e} \lambda \nabla \mathbf{N} . \nabla \mathbf{N}^T dV$. According to the limiting conditions used, a surface term can be added (case of the exchange or nonlinear flux). This term can be positive.

For a condition of exchange for example, the term $\mathbf{K_{échange}} = \int_{\Gamma_{échange_e}} h \mathbf{N} . \mathbf{N}^T d\Gamma$ is added to the matrix $\mathbf{K}$. It is always positive and thus does not check properties 2.3 stated above ($K_{ij} \geq 0$). The risk of oscillation is thus always present even by diagonalisant the mass matrix. In this case, to refine the mesh where the limiting condition is applied, makes it possible to reduce the positive contribution of the limiting condition and often to remove the oscillations.

3.1.4 Conclusion on the elements: properties of the matrixes $\mathbf{K}$

For the linear elements, if the real element is not too irregular, the extra-diagonal terms of $\mathbf{K}$ are quite negative. For the quadratic elements (in 2D) and certain limiting conditions, certain extra-diagonal terms of $\mathbf{K}$ are positive. Even while diagonalisant $\mathbf{M}$, one cannot ensure that the solution will not oscillate.

In *Code_Aster*, to eliminate the problems from oscillation and going beyond the maximum, one diagonalized only mass matrixes for the thermal computations carried out on linear elements. For the quadratic elements, one saw that one could not diagonalizing directly the mass matrix. One thus cuts out these elements in linear elements which are themselves lumped. This is applied to the quadratic elements 2D in *Code_Aster*, but not to the quadratic elements 3D, not for reasons of method but because automatic cutting is difficult to implement in 3D.

3.2 Method of diagonalization: Integration with the nodes of the elements

If the elementary mass matrix is calculated by numerical integration, its terms are written in the form [bib8]

$$M_{ij}=\int_{\Omega_c} \rho CN_i N_j \,\mathrm{dV} \simeq \sum_{q=1}^{N} W_q (\rho CN_i N_j)_q$$

where $N_i \rho CN_j$ is evaluated at $\mathrm{q}^{\text{ième}}$ the point of integration
and W_q is the weight of integration associated with this point.

Classically, the points of integration are Gauss points; the position of N the points and their weight are defined so that the diagram integrates the polynomials of degree exactly $2\mathrm{N}+1$. If one chooses **the points of integration to the nodes of the element**, one obtains: $M_{ij}=0$ for $i \neq j$. This integration method is also called method of Newton-Dimensions.

Notice 1: Axisymmetric problems:

If the points of integration are with the nodes, one will have, for any type of element, of the null masses on the axis of symmetry.

Indeed, $M_{ij}=\int \rho C\, N_i N_j\, 2\pi\, r \,\mathrm{dr}\, dz$

$$M_{ij} \simeq \delta_{\mathrm{ij}}\, 2\pi\, \rho C W_i \,\mathrm{Jac}\, r(x_i)$$

If the point of integration i is a node of the axis, $r(x_i)=0$ and the corresponding mass is null.

*For the axisymmetric elements, **the integration method with the nodes is thus not adapted close to the axis** . In this case, it is necessary to integrate into Gauss points the elements which touch the axis of revolution, by means of the usual modelization (AXIS).*

Notice 2: other possible methods of diagonalization:

Other methods are studied in [bib1], to test diagonalizing in particular the quadratic elements. In practice, it are not retained at the present time in the Code_Aster.

- *Setting at the level of the diagonal terms ([bib9], [bib10]): Hinton suggests the setting at the level of the diagonal terms of the consistent $\mathbf{M}$ matrix, so that the total mass of the element is preserved. One notes that the lumped masses are always positive, even for the elements quadrangles to 8 and 9 nodes.*
- *Summation by line ([bib10]): One adds the values from M_{ij} line and one concentrates result on the diagonal. Unfortunately, this process can lead to negative masses, in particular for the quadrangle with 8 nodes.*

Notice 3:

For the quadratic elements, one notes in [bib1] that, even while diagonalisant with the method of setting at the level of the diagonal terms, one obtains oscillations. One cannot thus use these elements in the frame of the diagonalization (i.e. for a relatively coarse mesh with respect to the speed of the thermal transient).

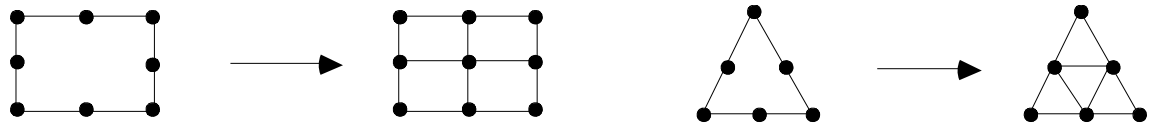
One can of course use the quadratic elements in thermal, on condition that adapting the smoothness of the mesh to the stiffness of the thermal shock.

4 Implemented in *Code_Aster*

In order to eliminate the oscillations from the temperature in space and time, modelizations AXIS_DIAG, PLAN_DIAG and 3D_DIAG carry out the diagonalization of the mass matrixes during linear thermal computation (THER_LINEAIRE) and nonlinear (THER_NON_LINE). To guarantee the effectiveness of it, it was seen that it should be carried out on linear elements.

If the mesh is linear, one carries out simply a diagonalization of the mass matrixes by integration to the nodes.

In the case of a quadratic mesh, in 2D, one carries out a thermal computation ISO-P2: computation on a QUAD9 is brought back to a computation on 4 QUAD4; in the same way, one passes from a TRIA6 to 4 TRIA3.



This makes it possible not to lose the smoothness of the discretization of the mesh, as well for the solution of the thermal problem as for that of the mechanical problem. Indeed, one shows in [bib1] that this solution is preferable with that which consists in carrying out thermal computation on the meshes linear ones which leans on meshes the nodes tops of quadratic (what is normal since the discretization is finer).

The modelizations available are thus:

4.1 Modelizations 2D

Modelization	PLAN_DIAG	AXIS_DIAG
2D	Nets	Element
Element	TRIA3	THPLTL3
THAXTL3	QUAD4	THPLQL4
THAXQL4	SEG2	THPLSL2
THAXSL2	TRIA6	THPLTL6
THAXTL6	QUAD9	THPLQL9
THAXQL9	SEG3	THPLSL3

THAXSL3 Comments on elementary computations:

For the linear elements: the terms of mass (matrix to the first member and vector with the second member) are lumped by integration with the nodes. The new elements have elementary computation options identical to the classical elements. The only modified elementary options are thus MASS_THER and CHAR_THER_EVOL.

For the quadratic elements: computation is ISO-P2. The computation on an element QUAD9 (resp. TRIA6) is brought back has a computation on 4 linear elements QUAD4 (resp. 4 TRIA3) whose terms of mass are lumped by the preceding method. The matrixes and vectors of each of the 4 linear elements are assembled on the level of the routine of elementary computation. By homogeneity, on the edge elements, one calculates the elementary terms on 2 SEG2, then one assembles.

Elements THPLTL6, THAXTL6, THPLQL9, THAXQL9 have the shape functions of the linear elements in which they are cut out.

Caution:

> *There is no element associated with mesh QUAD8. Consequently, if the mesh is composed of meshes quadratic, it is initially necessary to change the QUAD8 into QUAD9 has assistance CRÉA-MAILLAGE of the command :*

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Into axisymmetric: so elements of the mesh touch the axis, one should not integrate into the nodes which are on the axis. It is thus necessary to isolate this layer from elements and to affect modelization AXIS to him.

4.2 Modelization 3D

Modelization	**3D_DIAG**

3D	**Nets**
Element	HEXA8
THER_HEXA8_D	PENTA6
THER_PENTA6_D	TETRA4
THER_TETRA4_D	QUAD4
THER_FACE4_D	TRIA3

THER_FACE3_D Comments on elementary computations:

For the linear elements: as in 2D, the terms of mass (matrix to the 1st member and vector with the 2nd member) are lumped by integration with the nodes (3rd family of Gauss points).

For the quadratic elements, it would be necessary to cut out those in linear elements. This cutting is delicate to implement, because it results in creating a new element (PENTA18) with nodes in the middle of each quadrilateral face (and it would also be necessary to create a new element PYRAM14).

One diagonalized thus currently only elements 3D linear.

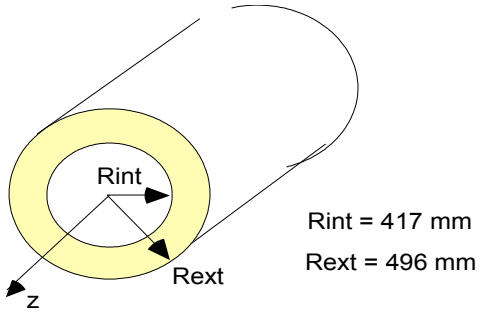
With regard to the pyramids with 5 nodes, integration with the nodes was tested but does not function well. Cf [§ 3.1.1.3] (it is not known if all the extra-diagonal terms are negative). The modelization "3D_DIAG" thus does not exist for the pyramids with 5 nodes. In any case these elements are minority in a mesh 3D: it are generated only by the voluminal free mesh generator of GIBI, which creates some with the need of them to supplement the hexahedral mesh.

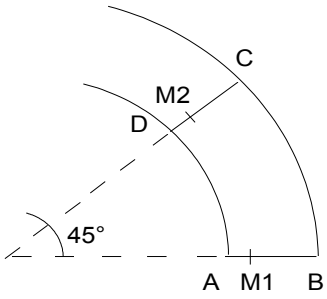
5 Thermal computation of a cylinder subjected to a cold shock

One illustrates on a numerical example what was shown previously; namely that the diagonalization is effective to check the principle of the maximum.
One takes as a starting point the industrial example of the cooling of a moulded elbow: one applies a cold thermal shock ($289\,°C$ with $20\,°C$) to a fissured elbow. During the transient of cooling, the temperature calculated in some nodes reached $310\,°$ without diagonalization of the mass matrixes. For the example treated here, one restricts with a hollow roll of the same dimension than the elbow to which one applies a cold thermal shock.

5.1 Data

a presumedly infinite hollow roll is studied. As there is no dependence compared to z (infinite cylinder), one limits the study to a plane computation. By reason of symmetry, one nets only one portion of structure.

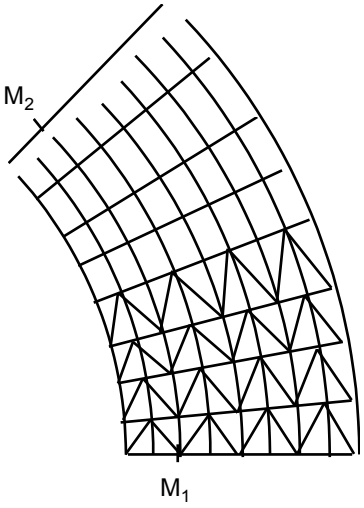




Coordinates of the points:

	$x\ (mm)$	$y\ (mm)$	$z\ (mm)$
M1	436.75	0.	0.
M2	$436.75\cos 45\,°$	$436.75\sin 45\,°$	0.

Computations are carried out on a linear mesh (meshes TRIA3-QUAD4):



Characteristics of mesh:

Many nodes: 90
Number and type of meshes: 64 TRIA3,32 QUAD4

Characteristics of the material:

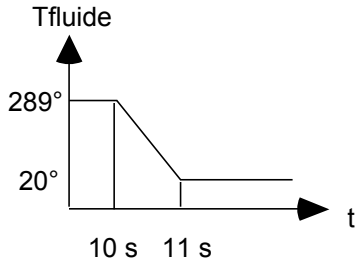
$\lambda = 19{,}97 \text{ W/m °C}$
$\rho Cp = 4.89488\,10^6 \text{ J/m}^3 \text{ °C}$

Limiting conditions and loading:

To ensure invariance by rotation, one forces null heat flux conditions on the sides AB and CD. The external wall is supposed perfectly insulated. On the internal skin AD, the heat transfer between the cylinder and the fluid are modelled by a high convective coefficient of heat exchange:

$$h = 40\,000 \text{ W/m² °C}.$$

The cold thermal shock applied to the moulded elbows is represented by a linear variation of the temperature of the fluid circulating in the pipe: $289° \rightarrow 20°$ in 12s. In order to accentuate the problem of going beyond the maximum and thus to better highlight the influence of the diagonalization, a more brutal shock is adopted: $289° \rightarrow 20°$ in 1s.



The following discretization in time is adopted:

of $t = 0\,s$ with $t = 10\,s$ 1 time step
,
of $t = 10\,s$ with $t = 11\,s$ 2 time step
,
of $t = 11\,s$ with $t = 25\,s$ 7 time step
,
of $t = 25\,s$ with $t = 60\,s$ 10 time step
,

Numerically, the value retained for the parameter of the discretization in time is $\theta = 0{,}57$.

5.2 Results

the following figures show the profiles of temperature in the thickness of the cylinder at time $t = 15\text{s}$ (urgent where the goings beyond the maximum are largest) without diagonalization of the mass matrixes.

One gives also the temporal evolution $\mathrm{T(t)}$ to the nodes $\mathrm{M_1}$ and $\mathrm{M_2}$ located at a quarter of the thickness of the internal skin.

Without diagonalization, it is noted that the temperature oscillates in time and in space exceed the maximum value of $289\,°$ at the beginning of the transient.

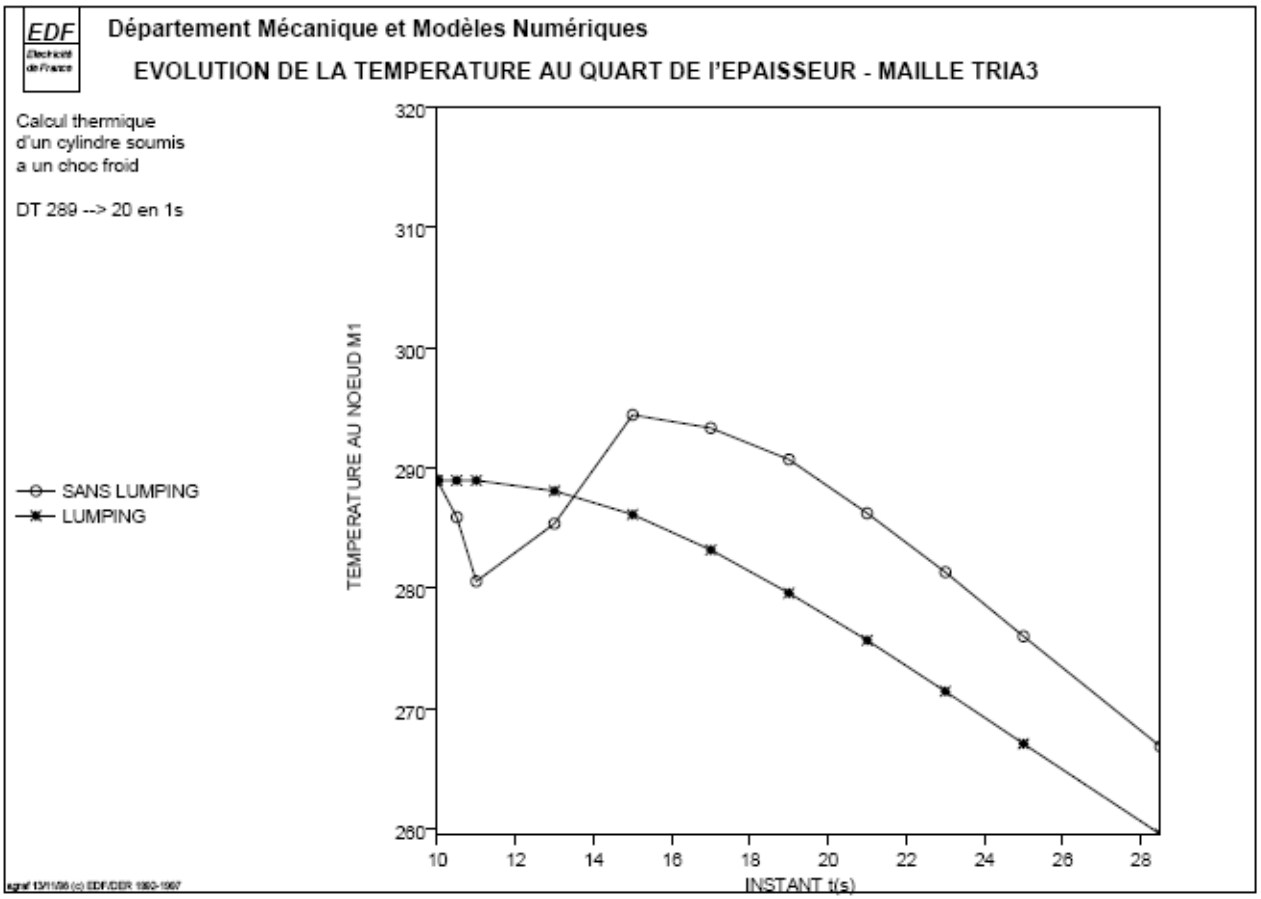
With diagonalization on the linear elements, one observes a regular solution without going beyond the maximum.

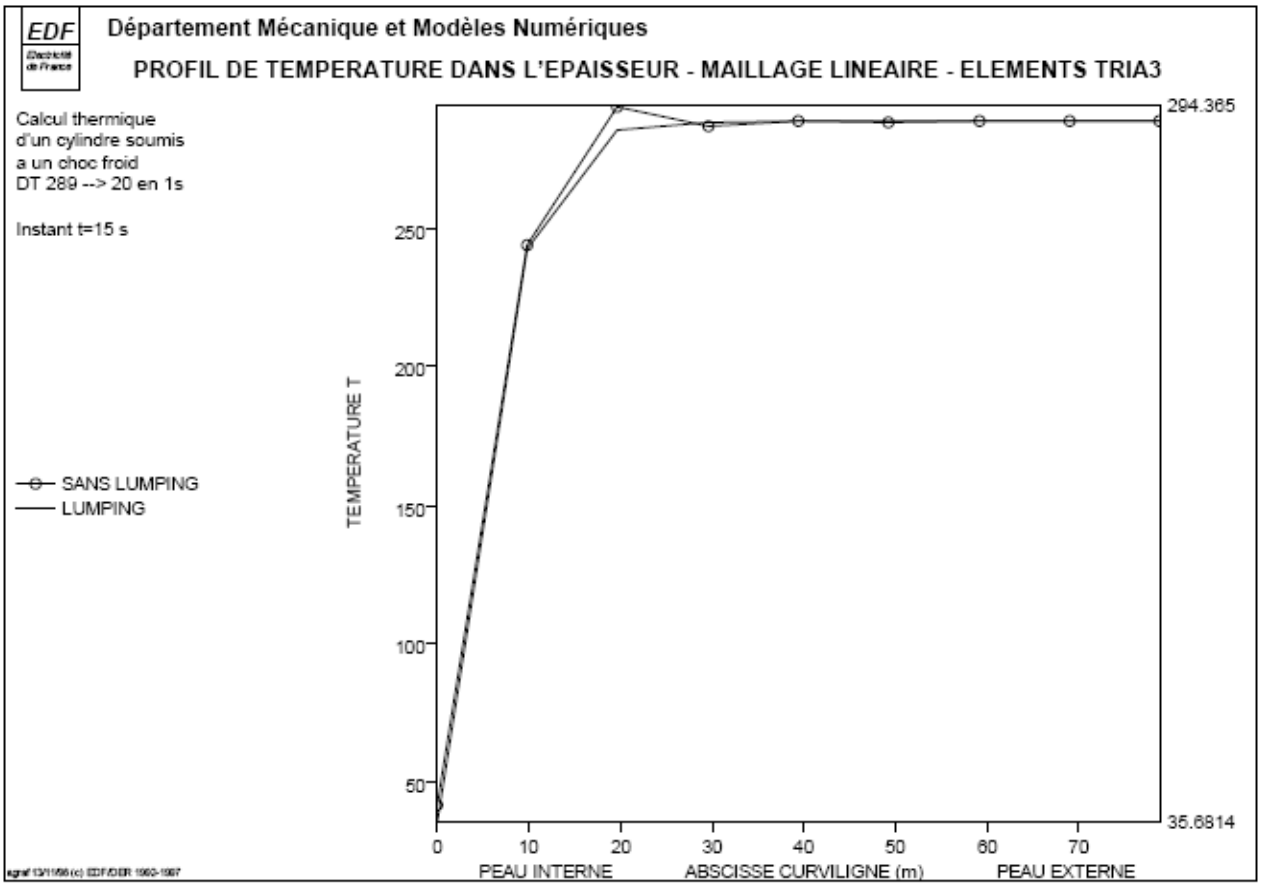
A similar study was conducted on linear elements 3D (tetrahedron with 4 nodes, pentahedron with 6 nodes, hexahedron with 8 nodes). The results lead to the same conclusions: with diagonalization, the oscillations of the temperature disappear for computation on the linear elements 3D.

Notice complementary concerning thermomechanical computation:

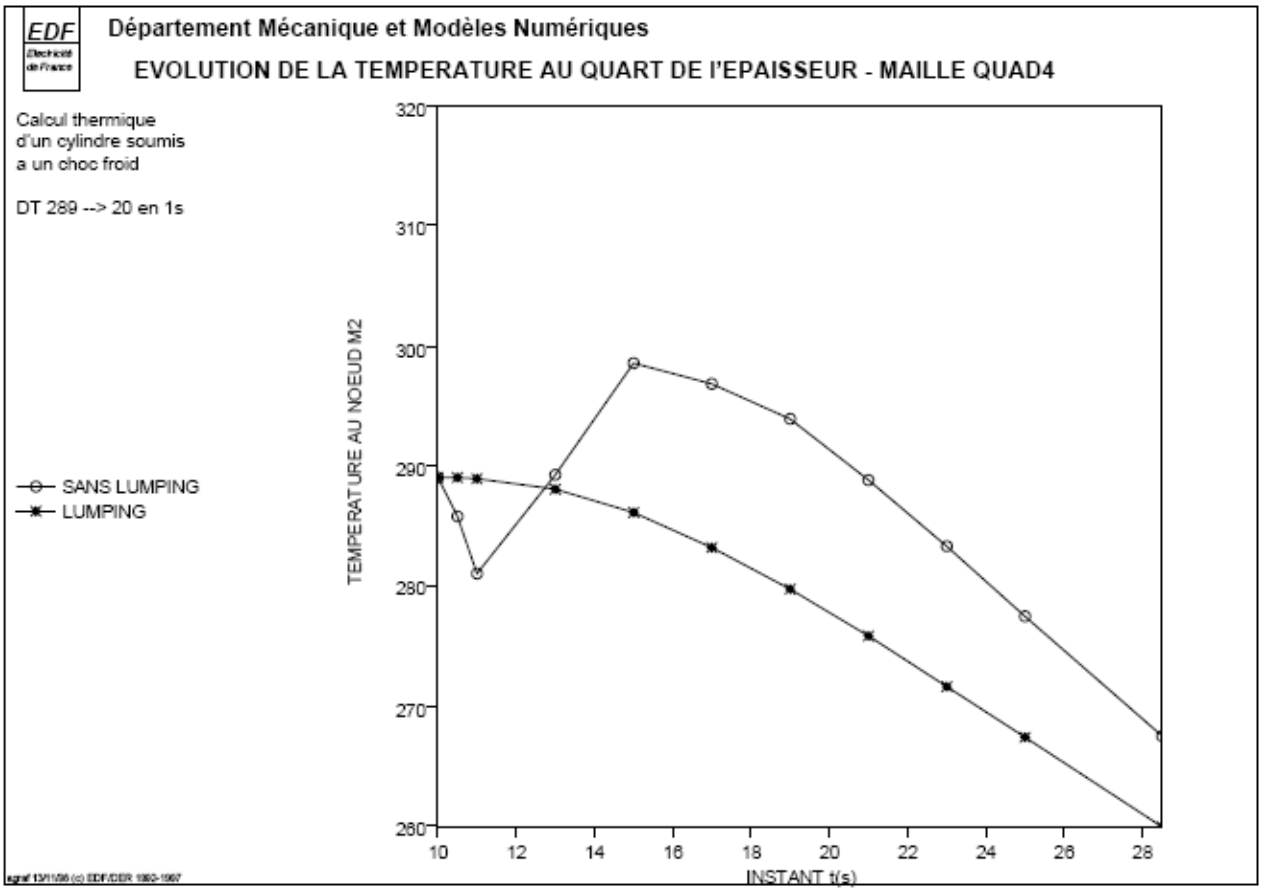
Another study was carried out in [bib1] estimating the consequences of the thermal diagonalization on the mechanical results. It is noted that computation ISO-P2 (quadratic elements divided into linear elements, whose mass matrixes are lumped) provides satisfactory results. One eliminates the spatial oscillations from the temperature. But in the studied case, with a relatively coarse mesh, the mechanical solution remains not very precise. Although the thermal solution is correct, to improve the solution in stresses, the mesh should in any case be refined.

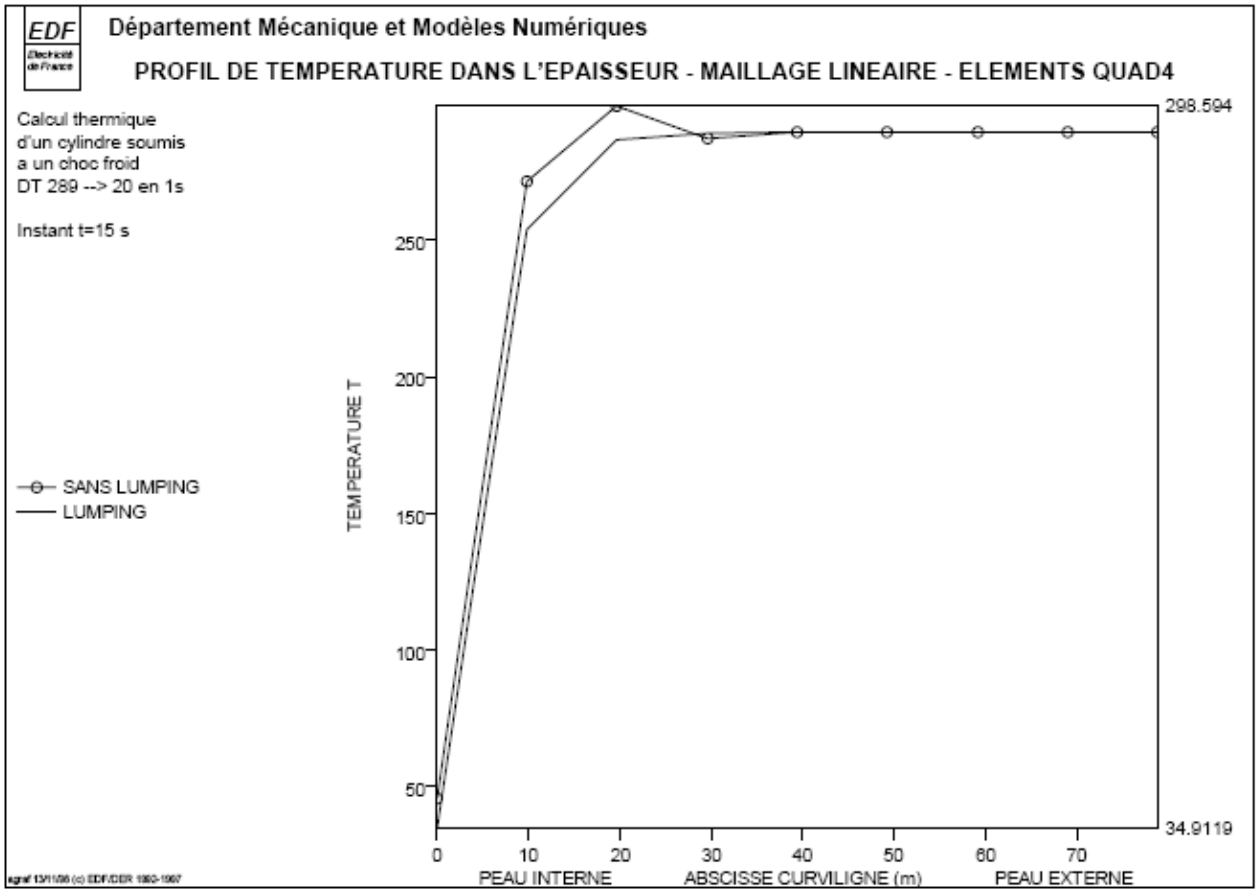
For meshes the TRIA3, the diagonalization leads to a regular solution without going beyond the maximum:





For meshes the QUAD4, the diagonalization leads to a regular solution without going beyond the maximum:





6 Conclusion

modelizations AXIS_DIAG, PLAN_DIAG and 3D_DIAG are proposed in order to solve the problems of going beyond the maximum with oscillation of the solution in space and times which during appear certain transitory thermal computations with abrupt variation of the loading.

At the discrete level, the analysis leads to a sufficient condition of NON-oscillation on the step of the discretization in time which must belong with an interval:

$$\Delta t_{\min} \leq \Delta t \leq \Delta t_{\max}$$

where the values of $\Delta t_{\min}$ and $\Delta t_{\max}$ depend on the coefficients of mass matrixes and stiffness thermals as well as parameter θ of the discretization in time.

In practice, if the oscillations come from time step too large $\left(\Delta t > \Delta t_{\max}\right)$, one suggests the choice of an implicit scheme in time $\left(\theta = 1\right)$. If time step are too small, the diagonalization of the mass matrix can make it possible to remove the oscillations.

For the linear elements, one shows that the diagonalization makes it possible indeed to avoid the oscillations of the solution. For the quadratic elements, a direct diagonalization is not enough to avoid the oscillations. For this kind of element, one cuts out them in linear elements, and one carries out a diagonalization of the linear elements resulting by integration with the nodes (this is carried out only in 2D for the moment).

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8 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
5	J.M.PROIX-R&D/AMA initial Text	