Finite elements of joint mechanical and coupled finite elements of joint hydraulic

Abstract:

This documentation relates to description of the finite elements of the linear and quadratic mechanical joint. The modelizations in pure mechanics (xxx_JOINT) support the two types of meshes, while the hydraulic modelizations coupled (xxx_JOINT_HYME) are implemented only for meshes quadratic. One carries out the elimination of fictitious degrees of freedom of pressure for the quadratic modelizations in pure mechanics. These modelizations make it possible to simulate the evolution of a crack along a predetermined path. The second type of element also takes into account the interaction of the mechanics with a fluid flow inside crack.

One presents successively the following points:

1) geometry of the elements
2) local coordinate system to the joint and transition matrix of the total reference to the local coordinate system
3) jump of displacement in the joint
4) gradient of pressure of fluid
5) nodal vector of the internal forces as well as the tangent matrix elementary

1 Geometry of the elements

1.1 Geometry of the linear joint 2D

the element of joint in 2D is a quadrangle with four nodes (QUAD4) with two small sides and two large sides [12] and [34] (see figure 1) which represent the two segments Γ^+ and Γ^- of an interface (or lips of a crack) between two pennies two-dimensional fields. To distinguish the sides Γ^+ and Γ^-, the local classification of the nodes must be made obligatorily as on figure Ci below:

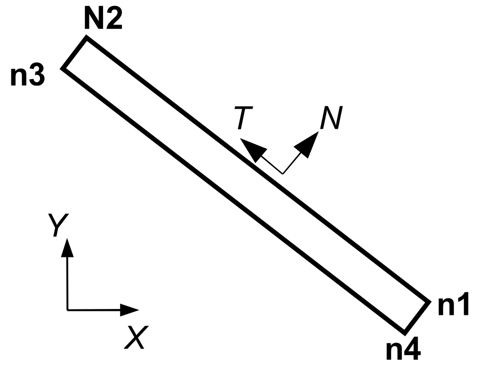

Figure 1: Element of joint 2D with good local classification.

By convention the face Γ^- is given by nodes 3 and 4 and the face Γ^+ by nodes 1 and 2. The norm n is directed face Γ^- towards the face Γ^+.

1.2 Geometry of the quadratic joint 2D

These elements are quadrangle with 8 nodes (QUAD8), nodes 1,2,3,4,5,7 carry mechanical degrees of freedom of displacement. Nodes 6 and 8 carry degrees of freedom of pressure so representing the fluid flow for the hydraulic coupled modelization. They are eliminated for the pure mechanical modelizations.

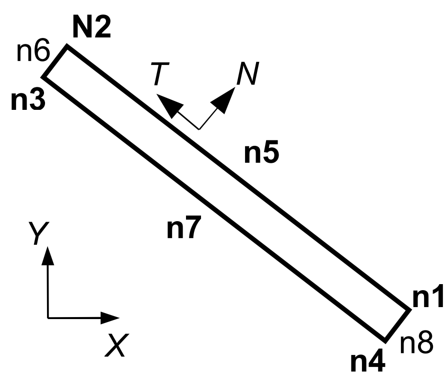

Figure 2: Element of joint HYME 2D with good local classification.

By convention the face Γ^- is given by nodes 3,4,7 and the face Γ^+ by nodes 1,2,5. The norm n is directed face Γ^- towards the face Γ^+.

1.3 Geometry of the linear joint 3D

the elements of joint in 3D make it possible to represent a surface S between two pennies voluminal fields Ω^+ and Ω^-. They are compatible with the mesh of under fields. If volume is with a grid with `HEXA8`, the joints to be used are also `HEXA8` (hexahedrons with eight nodes). If volume is with a grid with `PENTA6` or `TETRA4`, the joints to be used are `PENTA6` (pentahedral with six nodes).
To distinguish the upper surfaces S^+ (related to Ω^+) and lower S^- (related to Ω^-), it is necessary to impose a local classification of the nodes quite specific (see figure 3).

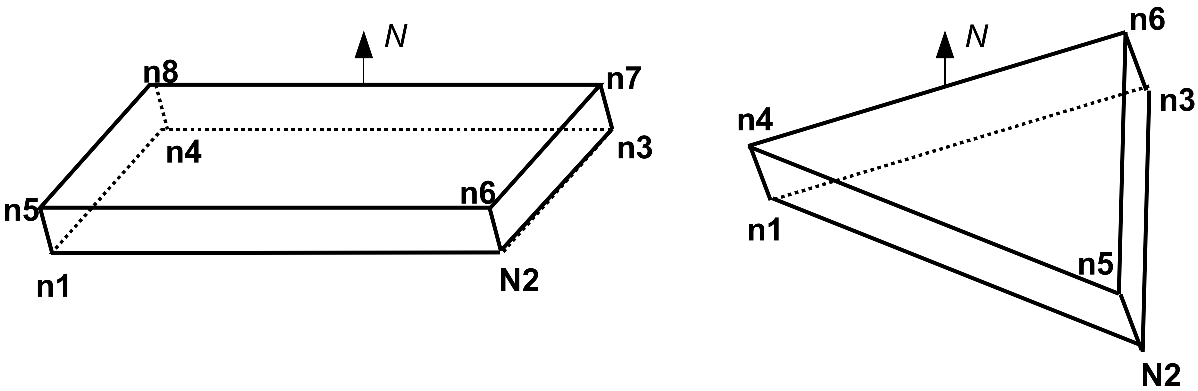

Figure 3 : Diagram of the elements of joint 3D `HEXA8` and `PENTA6` with good local classification.

By convention the face Γ^- is given by nodes 1,2,3,4 for `l'HEXA8` (or 1,2,3 for `the PENTA6`) and the face Γ^+ by nodes 5,6, 7, 8 for `l'HEXA8` (or 4,5,6 for `the PENTA6`). The norm n is directed face Γ^- towards the face Γ^+.

1.4 Geometry of the quadratic joint 3D

These elements of joint usable for modelizations are coupled `HYME` or for the modelizations in pure mechanics by the elimination of DDL of pressure. In 3D they make it possible to represent a surface S between two pennies voluminal fields *in pure mechanics* Ω^+ and Ω^-. They are compatible with the mesh of under fields. If volume is with a grid with `HEXA20`, the joints to be used are also `HEXA20` (hexahedrons with twenty nodes). If volume is with a grid with `PENTA15` or `TETRA4`, the joints to be used are `PENTA15` (pentahedral with 15 nodes).

To distinguish the upper surfaces S^+ (related to Ω^+) and lower S^- (related to Ω^-) it is necessary to impose a local classification of the nodes quite specific (see figure 4).

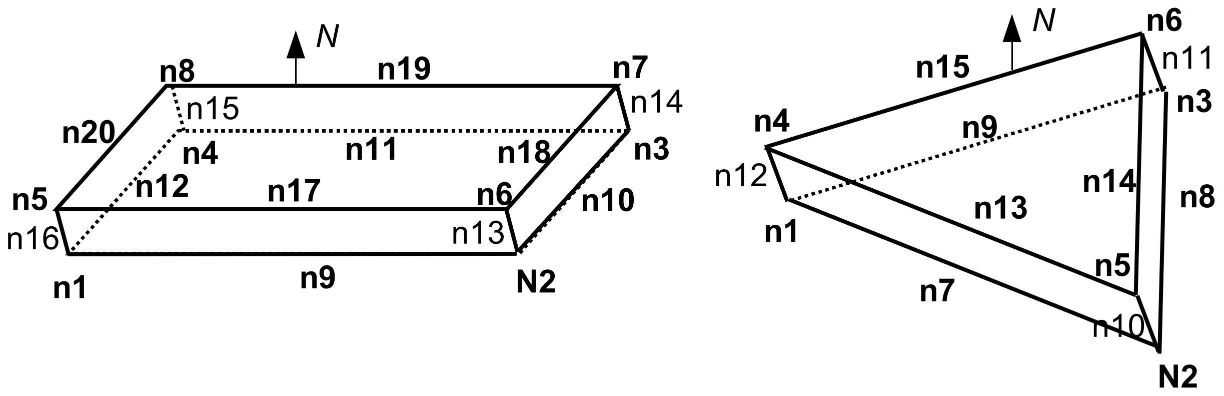

Figure 4 : Diagram of the coupled elements of joint HYME in 3D `HEXA20` and `PENTA15` with good local classification.

For `l'HEXA20`, medium nodes 13,14,15,16 carry degrees of freedom of pressure for the fluid flow, and all the others carry mechanical degrees of freedom of displacement.
For `the PENTA15`, medium nodes 10,11,12 carry degrees of freedom of pressure for the fluid flow, and all the others carry mechanical degrees of freedom of displacement.

By convention the face Γ^- is given by nodes 1,2,3,4,9,10,11,12 for `l'HEXA20` (or 1,2,3,7,8,9 for `the PENTA15`) and the face Γ^+ by nodes 5,6, 7, 8,17,18,19,20 for `l'HEXA20` (or 4,5,6,13,14,15 for `the PENTA15`). The norm n is directed face Γ^- towards the face Γ^+.

1.5 Automatic construction of meshes of joint

command `MODI_MAILLAGE` used with key word `ORIE_FISSURE` makes it possible to impose the good local classification of meshes of joint in 2D or 3D (see Doc. [U4.23.04]).

In addition techniques are available automatically to create elements of joint in a mesh which is deprived by it (see [U2.05.07]).

2 Local coordinate system and transition matrix

It is necessary to build a local coordinate system with the element to define the jump of displacement δ (given of entry of the constitutive laws: to see [R7.02.11] and [R7.01.25]). In addition, one defines the matrix transition $\boldsymbol{R}$ of the total reference in the local coordinate system. This part is valid for the modelizations of joint in pure mechanics and for the coupled modelizations hydraulic.

2.1 Case 2D

Is $(\boldsymbol{X},\boldsymbol{Y})$ the total reference. The direction given by the large sides [12] and [34] of the element of joint 2D makes it possible to define a local coordinate system $(\boldsymbol{n},\boldsymbol{t})$ in the element of joint (see figure 1):

$$\boldsymbol{t}=\frac{\vec{12}}{\|\vec{12}\|}\ ,\ \boldsymbol{n}=\boldsymbol{t}\wedge(\boldsymbol{X}\wedge\boldsymbol{Y})$$

The transition matrix of the total reference to the local coordinate system is expressed:

$$\boldsymbol{R}=\begin{bmatrix} n_x & n_y \\ t_x & t_y \end{bmatrix}$$

2.2 3D case

$(\boldsymbol{X},\boldsymbol{Y},\boldsymbol{Z})$ the total reference is noted. For the construction of the local coordinate system to the element of joint, one uses the base covariante surface element corresponding If one notes $\boldsymbol{s}(\xi^1,\xi^2)$ the parameterized position of a point of the surface element:

$$\boldsymbol{s}(\xi^1,\xi^2)=\sum_{n=1}^{Nb} N_n(\xi^1,\xi^2)\boldsymbol{s}^{\boldsymbol{n}}$$

where N_n and s^n indicate respectively the shape function and the geometrical position of the node n , and Nb the number of nodes of the surface element. One defines the local base in the following way $(\boldsymbol{a}_1,\boldsymbol{a}_2)$ covariante:

$$\boldsymbol{a}_1=\frac{\partial \boldsymbol{s}}{\partial \xi_1}=\sum_{n=1}^{Nb}\frac{\partial N_n}{\partial \xi^1}\boldsymbol{s}^{\boldsymbol{n}} \quad \boldsymbol{a}_2=\frac{\partial \boldsymbol{s}}{\partial \xi_2}=\sum_{n=1}^{Nb}\frac{\partial N_n}{\partial \xi^2}\boldsymbol{s}^{\boldsymbol{n}}$$

These two vectors are by way of vectors tangent with the element in a given point. The local direct orthonormal base $(\boldsymbol{n},\boldsymbol{t},\boldsymbol{\tau})$ is then built in the following way:

$$\boldsymbol{t}=\frac{\boldsymbol{a}_1}{\|\boldsymbol{a}_1\|} \quad \boldsymbol{n}=\frac{\boldsymbol{t}\wedge\boldsymbol{a}_2}{\|\boldsymbol{a}_2\|} \quad \boldsymbol{\tau}=\boldsymbol{n}\wedge\boldsymbol{t}$$

The transition matrix of the total reference to the local coordinate system is given by:

$$\boldsymbol{R}=\begin{bmatrix} n_x & n_y & n_z \\ t_x & t_y & t_z \\ \tau_x & \tau_y & \tau_z \end{bmatrix}$$

3 Jump of displacement

the joints have authority to represent two sides in glance, they utilize only the interpolation functions and the points of integration of the surface *elements* (in 3D) or *linear* (in 2D) corresponding:

In 2D: for the joint `QUAD4` (or joint `HYME QUAD8`), the linear element is `the SEG2`
In 3D: for the joint `PENTA6` (or joint `HYME PENTA15`) the surface element is `the TRIA3`
for the joint `HEXA8` (or joint `HYME HEXA20`) the surface element is `the QUAD4`.

One calls N_n the shape function of the node n of the element surfacique1Par[1]. U^{+n} and U^{-n} nodal displacements of the segments indicate respectively Γ^+ and Γ^- in 2D or of the sides S^+ and S^- in 3D.
In the local coordinate system, the jump of displacement $\boldsymbol{\delta}$ is discretized starting from the shape functions N_n. At the point of gauss g , it is expressed like the difference of displacements of the sides (or segments) + and -:

$$\boldsymbol{\delta}_{\boldsymbol{g}}=\sum_{n=1}^{Nb}\boldsymbol{R}(\boldsymbol{U}^{+\boldsymbol{n}}-\boldsymbol{U}^{-\boldsymbol{n}})N_n^g$$

1 the continuation, one uses the generic term: “surface” for 2D as for 3D

where Nb is the number of nodes of the surface element and where $\boldsymbol{R}$ transition matrix in 2D, in 3D, which makes it possible to express nodal displacements in the local coordinate system. One can synthesize the preceding statement in a matrix $\boldsymbol{M}_g^U$ which acts on the vector of nodal displacements of the element: $\boldsymbol{U}$, to build the jump of displacement in the local coordinate system:

$$\boldsymbol{\delta}_g = \boldsymbol{M}_g^U \boldsymbol{U}$$

The matrix $\boldsymbol{M}_g^U$ is of dimension $ndim \times Nddl_U$, with $Nddl_U$ mechanical number of *degrees of freedom* :

$Nddl_U = 8$ for the joint `2D`,
$Nddl_U = 24$ for joint `3D HEXA`
$Nddl_U = 18$ for joint `3D PENTA`
$Nddl_U = 12$ for joint `HYME 2D`
$Nddl_U = 48$ for joint `HYME 3D HEXA`
$Nddl_U = 36$ for joint `HYME 3D PENTA`

4 Gradient of the fluid pressure

the elements of joint `HYME` have, besides the mechanical degrees of freedom $\boldsymbol{U}$, of degrees of freedom of the nodal fluid pressure (one by node) noted $\boldsymbol{P}$.
For `the QUAD8`, the nodes medium nodes 6 and 8 carry these degrees of freedom of pressure. The linear element of reference used for the approximation of the pressures is `the SEG2`.
For `l'HEXA20`, medium nodes 13,14,15,16 carry these degrees of freedom of pressure. The surface element of reference used for the approximation of the pressures is `the QUAD4`.
For `the PENTA15`, medium nodes 10,11,12 carry these degrees of freedom of pressure. The surface element of reference used for the approximation of the pressures is `the TRIA3`.

The flow model of the fluid (cubic model, to see [R7.01.25]) utilized the gradient of pressure in the direction of the flow estimated with the point of gauss *G* in way classical:

$$\nabla p_g = \sum_{n=1}^{Nb} \boldsymbol{P}_{\boldsymbol{n}} \nabla N_n^g$$

where Nb is the number of nodes of pressure and N_n^g the value of the shape function of the node n at the point of gauss g. To simplify the writing one notes:

$$\nabla p_g = \boldsymbol{M}_g^P \boldsymbol{P}$$

The matrix $\boldsymbol{M}_g^U$ is of dimension $(ndim-1) \times Nddl_P$: with $Nddl_P$ fluid number of *degrees of freedom* :

$Nddl_P = 2$ for joint `HYME 2D`
$Nddl_P = 4$ for joint `HYME 3D HEXA`
$Nddl_P = 3$ for joint `HYME 3D PENTA`

5 Internal forces and tangent matrix

5.1 mechanical Case pure

the formulation of the mechanical problem (see [R7.02.11] and [R7.01.25]) utilized the work of the forces along the discontinuity, which is not other than the energy of surface related to the cracking of structure:

$$W_s(\boldsymbol{\delta})=\sum_g \omega_g \psi(\boldsymbol{\delta_g})$$

with ψ density of energy of surface and ω_g weight of the point of gauss g . That makes it possible to define the vector of the internal forces:

$$\boldsymbol{F}_{\text{int}}^{\boldsymbol{U}}=\frac{\partial W_s(\boldsymbol{\delta})}{\partial \boldsymbol{U}}=\sum_g \omega_g \frac{\partial \psi}{\partial \boldsymbol{\delta_g}} \frac{\partial \boldsymbol{\delta_g}}{\partial \boldsymbol{U}}$$

In the preceding statement, the first term is given by the cohesive constitutive law (see [R7.02.11]). That corresponds to the vector forced $\vec{\boldsymbol{\sigma}}_{\boldsymbol{g}}$ (or forces cohesive) at the point of gauss g :

$$\frac{\partial \psi}{\partial \boldsymbol{\delta_g}}=\vec{\boldsymbol{\sigma}}_{\boldsymbol{g}}$$

The second term is resulting from the definition of the jump displacement in part 5 :

$$\frac{\partial \boldsymbol{\delta_g}}{\partial \boldsymbol{U}}=\boldsymbol{M}_{\boldsymbol{g}}^{\boldsymbol{U}}$$

The nodal vector of the internal forces is thus expressed in the following way:

$$\boldsymbol{F}_{\text{int}}^{\boldsymbol{U}}=\sum_g \omega_g \boldsymbol{M}_{\boldsymbol{g}}^{\boldsymbol{U}t} \vec{\boldsymbol{\sigma}}_{\boldsymbol{g}}$$

In the frame of an algorithm of Newton, to solve the nonlinear problem of equilibrium, it is useful to have the elementary tangent matrix, i.e. the derivative of the internal forces compared to nodal displacements. In the case of the element of joint, it is expressed simply:

$$\boldsymbol{K}^{\boldsymbol{UU}}=\frac{\partial \boldsymbol{F}_{\text{int}}^{\boldsymbol{U}}}{\partial \boldsymbol{U}}=\sum_g \omega_g \boldsymbol{M}_{\boldsymbol{g}}^{\boldsymbol{U}t} \frac{\partial \vec{\boldsymbol{\sigma}}_{\boldsymbol{g}}}{\partial \boldsymbol{\delta_g}} \boldsymbol{M}_{\boldsymbol{g}}^{\boldsymbol{U}}$$

The latter leans on the tangent operator: $\dfrac{\partial \vec{\boldsymbol{\sigma}}_{\boldsymbol{g}}}{\partial \boldsymbol{\delta_g}}$ specific with the adopted cohesive constitutive law (see [R7.02.11]).

5.2 Hydraulic case coupled

joints HYME, besides the nodal forces related to the mechanics $\boldsymbol{F}_{\text{int}}^{\boldsymbol{U}}$ on which one reporte2Pour[2] pressure of fluid at the point of gauss on the normal component $\vec{\boldsymbol{p}}_{\boldsymbol{g}}=(p_g, 0, 0)$ (expressed in the local coordinate system with crack):

$$\boldsymbol{F}_{\text{int}}^{\boldsymbol{U}}=\sum_g \omega_g \boldsymbol{M}_{\boldsymbol{g}}^{\boldsymbol{U}t}(\vec{\boldsymbol{\sigma}}_{\boldsymbol{g}}-\vec{\boldsymbol{p}}_{\boldsymbol{g}})$$

have nodal forces for the fluid flow on the nodes which carry d.o.f. of pressure.

2 to take into account coupling HM

The formulation of the hydraulic problem (see [R7.01.25]) utilized the work of the forces of the fluid along the way of flow (inside crack):

$$W_F(\nabla p)=\sum_g \omega_g H(\nabla p)$$

with H density of energy of surface and ω_g weight of the point of gauss g. That makes it possible to define the vector of the internal forces:

$$\boldsymbol{F}_{\text{int}}^{\boldsymbol{P}}=\frac{\partial W_F(\nabla p)}{\partial \boldsymbol{P}}=\sum_g \omega_g \frac{\partial H}{\partial \nabla p_g}\frac{\partial \nabla p_g}{\partial \boldsymbol{P}}$$

In the preceding statement, the first term is given by the cubic constitutive law of the fluid (see [R7.01.25]). That corresponds to hydraulic flux $\vec{\boldsymbol{w}}_{\boldsymbol{g}}$ at the point of gauss g :

$$\frac{\partial H}{\partial \nabla p_g}=\vec{\boldsymbol{w}}_{\boldsymbol{g}}$$

According to the definition of the gradient of pressure in 6. The second term is given by:

$$\frac{\partial \nabla p_g}{\partial \boldsymbol{P}}=\boldsymbol{M}_{\boldsymbol{g}}^{\boldsymbol{P}}$$

The nodal vector of the internal forces is thus expressed in the following way:

$$\boldsymbol{F}_{\text{int}}^{\boldsymbol{P}}=\sum_g \omega_g \boldsymbol{M}_{\boldsymbol{g}}^{\boldsymbol{Pt}}\vec{\boldsymbol{w}}_{\boldsymbol{g}}$$

In the frame of an algorithm of Newton, to solve the nonlinear problem of equilibrium, it is useful to have the tangent matrix, i.e. the derivative of the internal forces compared to the degrees of freedom. The derivative of the internal force on the nodal degrees of freedom of displacement compared to displacement gives the term identical to that of the matrix obtained in pure mechanics in 6 :

$$\boldsymbol{K}^{\boldsymbol{UU}}=\frac{\partial \boldsymbol{F}_{\text{int}}^{\boldsymbol{U}}}{\partial \boldsymbol{U}}=\sum_g \omega_g \boldsymbol{M}_{\boldsymbol{g}}^{\boldsymbol{Ut}}\frac{\partial \vec{\boldsymbol{\sigma}}_{\boldsymbol{g}}}{\partial \boldsymbol{\delta}_{\boldsymbol{g}}}\boldsymbol{M}_{\boldsymbol{g}}^{\boldsymbol{U}}$$

In the case of the hydraulic coupling, explicitly $\boldsymbol{F}_{\text{int}}^{\boldsymbol{U}}$ depend them on the pressure (see statement above) from where:

$$\boldsymbol{K}^{\boldsymbol{UP}}=\frac{\partial \boldsymbol{F}_{\text{int}}^{\boldsymbol{U}}}{\partial \boldsymbol{P}}=\sum_g \omega_g \boldsymbol{M}_{\boldsymbol{g}}^{\boldsymbol{Ut}}\boldsymbol{X}_{\boldsymbol{g}}$$

with $\boldsymbol{X}_{\boldsymbol{g}}=\frac{\partial \vec{\boldsymbol{p}}_{\boldsymbol{g}}}{\partial \boldsymbol{P}}=(-N_g^i,0,0)$ $i=1$ à Nb, Nb many nodes of pressure (or many degrees of freedom of pressure per element) and N_n^g the value of the shape function of the node n at the point of gauss g.

The derivative of the internal forces on the nodal degrees of freedom of pressure compared to displacement gives:

$$\boldsymbol{K}^{\boldsymbol{PU}}=\frac{\partial \boldsymbol{F}_{\text{int}}^{\boldsymbol{P}}}{\partial \boldsymbol{U}}=\sum_g \omega_g \boldsymbol{M}_{\boldsymbol{g}}^{\boldsymbol{Pt}}\frac{\partial \vec{\boldsymbol{w}}_{\boldsymbol{g}}}{\partial \boldsymbol{\delta}_{\boldsymbol{g}}}\boldsymbol{M}_{\boldsymbol{g}}^{\boldsymbol{U}}$$

Lastly, the derivative of the internal forces on the degrees of freedom of pressure compared to the nodal pressures gives:

$$\boldsymbol{K}^{PP}=\frac{\partial \boldsymbol{F}_{\text{int}}^{P}}{\partial \boldsymbol{P}}=\sum_{g}\omega_g \boldsymbol{M}_g^{Pt}\frac{\partial \vec{\boldsymbol{w}}_g}{\partial \nabla p_g}\boldsymbol{M}_g^{P}$$

The elementary tangent matrix (asymmetric) is expressed in the following way:

$$\boldsymbol{K}=\begin{bmatrix}\boldsymbol{K}^{UU} & \boldsymbol{K}^{UP}\\ \boldsymbol{K}^{PU} & \boldsymbol{K}^{PP}\end{bmatrix}$$

The latter leans on the components of the tangent operator: $\frac{\partial \vec{\boldsymbol{\sigma}}_g}{\partial \boldsymbol{\delta}_g}$, $\frac{\partial \vec{\boldsymbol{w}}_g}{\partial \boldsymbol{\delta}_g}$ and $\frac{\partial \vec{\boldsymbol{w}}_g}{\partial \nabla p_g}$ specific with the adopted cohesive constitutive law (see [R7.01.25]).

6 Features and validation

See documentations [R7.02.11] and [R7.01.25].

7 Description of the versions of the document

Index document	Version Aster	Author (S) Organization (S)	Description of the modifications
B	7.2	J.Laverne, EDF-R&D/AMA	
C	8.4	J.Laverne, EDF-R&D/AMA	
D	9.1	J.Laverne, EDF-R&D/AMA	3D card-indexes 9807 integration of the elements of joint
E	10.4	J.Laverne, EDF-R&D/AMA	F Card-indexes 14831 addition of coupled modelizations
*_HYME	11.4	K.Kazymyrenko, J.Laverne EDF-R&D/AMA	Card-indexes 18711 activation of the quadratic modelizations in pure mechanics