Quadrangular element at a point of integration, stabilized by the method "Assumed Strain"

Summarized:

Despite everything the potential of the linear isoparametric quadrilateral element for computation by finite elements, the application of a classical bilinear formulation to define its field of displacement led to poor results. If under integration of the element allows to improve its performances, it however reveals parasitic modes which make computations unstable. This document shows the main steps of a method of stabilization of computations named "assumed strain method" and explains the way in which it was established in the computer code *Code_Aster*. Many results leaning on the stabilized element are compared and commented on in order to conclude on the performances from the method.

1 Setting at fault of element 2D QUAD4:

In order to propose the need for a more powerful plane element, we carried out computations leaning on two cases tests, which make it possible to highlight blockings of the quadrilateral isoparametric element 4 nodes.

1.1 Case test n°1

the first series of computations is resulting from the modelization of a beam clamped and subjected to a shearing force fsy at its loose lead [Figure 1.1-a]. This beam has the following characteristics material: $E=100$ $\nu=0.4999$. This case test takes as a starting point that written in [bib4] (assumption of plane strains).

Boundary conditions:

Displacements on AD :

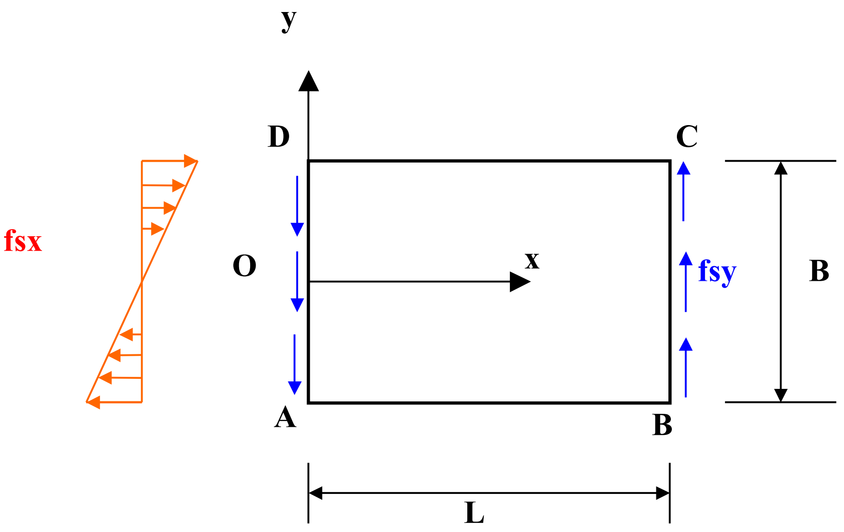
$Ux(A)=Uy(A)=0$
$Ux(D)=0$
$fsx=8\times L\times y/B^2$

Forces on BC :

$Fy=fsy=1-4\times y^2/B^2$
$Fx=0$

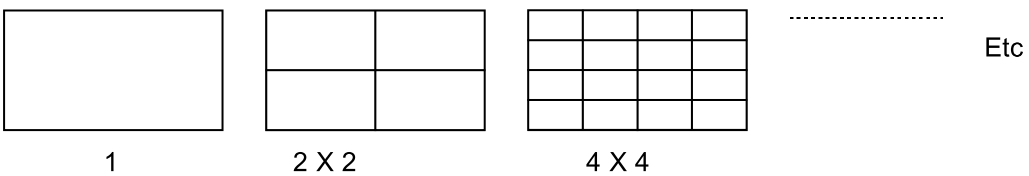
Geometry:

$L=100$
$B=50$



Appear 1.1-a

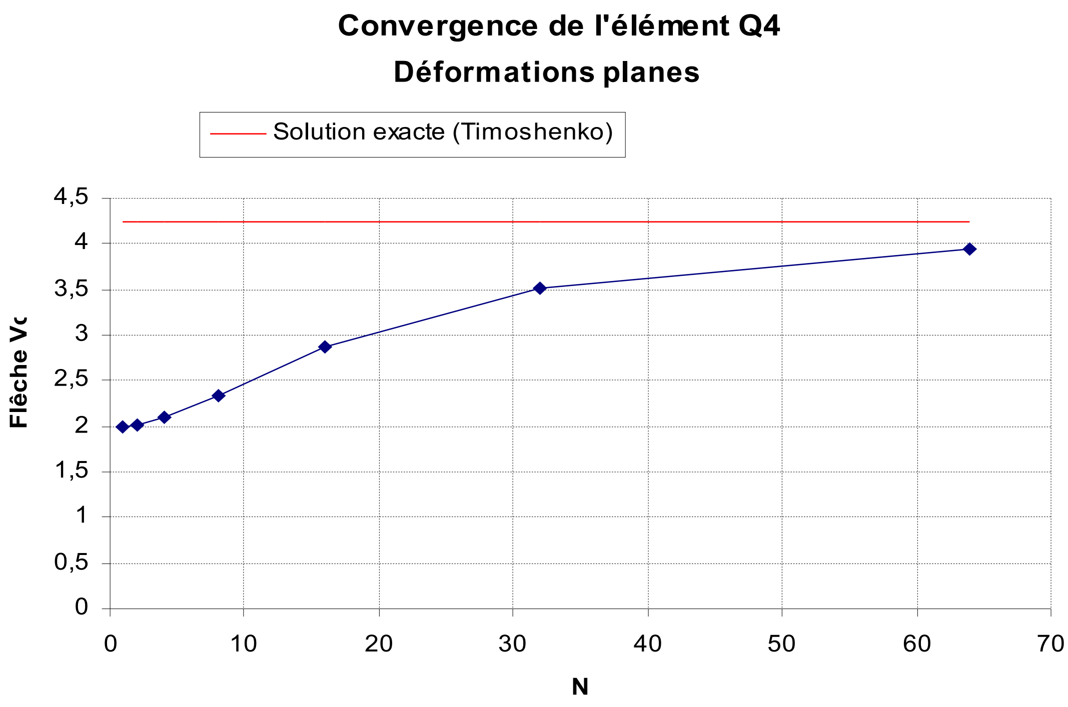
We carried out this computation seven times by multiplying the number of meshes on each edge by two each time:



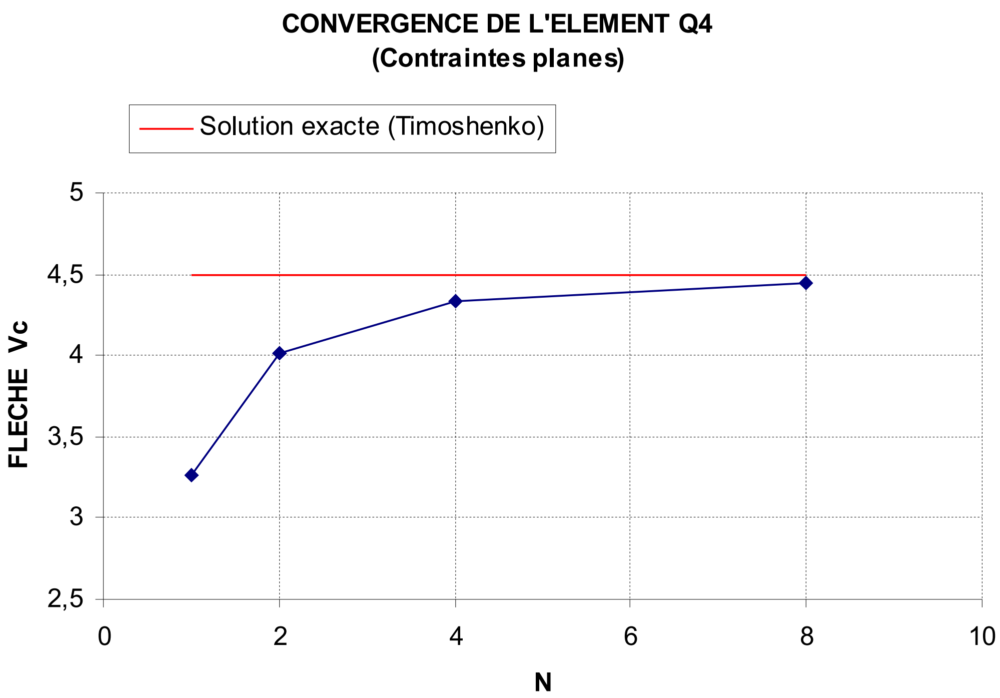
Number of meshes on an edge

At the conclusion of each computation, we brought the deflection in C with that closer to the "exact" solution of the theory of the beams of Timoshenko.

[Figure 1.1-b] shows us that the convergence of computation towards the theoretical solution largely insufficient taking into account the number of meshes is used for the modelization.
However, by carrying out computations in plane stresses, we note that the results of computation converge towards the theoretical solution satisfactorily ([Figure 1.1-c], quadratic convergence).

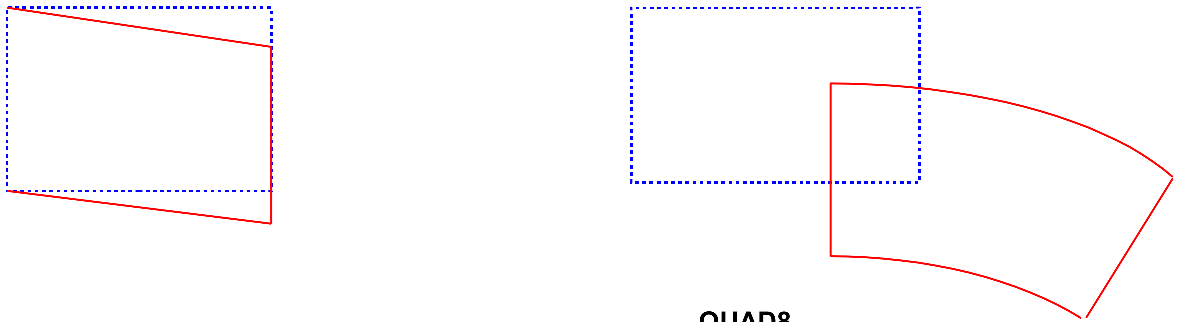


Appear 1.1-b: convergence of element QUAD4 in strain plane



Appears 1.1-c: convergence of element QUAD4 in plane stresses

This request is said to dominant bending. Through the first series of computations, we highlighted impossibility for the QUAD4 of representing the modes of strain in bending [Figure 1.1-d] in plane strain and for a coefficient ν close to $0{,}5$. This results in an excessive stiffness of the element due under the terms of shears of the operator discretized gradient.



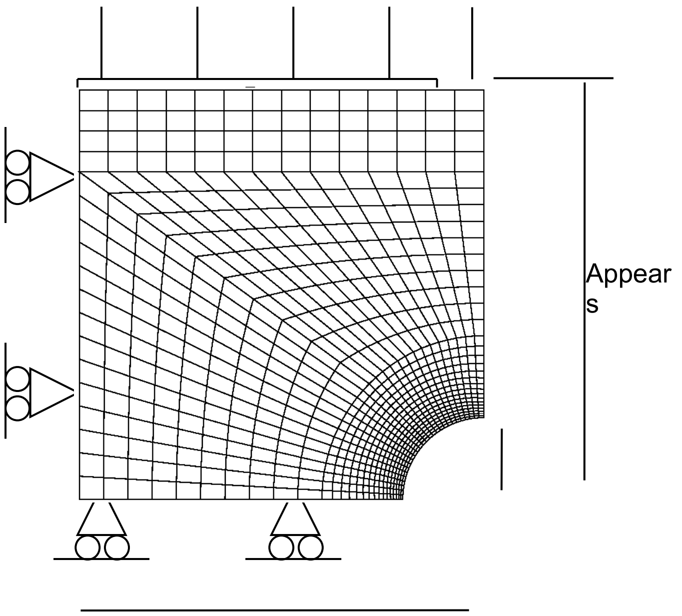
QUAD4
(important shears)

QUAD8

Appears 1.1-d

1.2 Case test n°2

The second series of computation leans on the modelization of a notched sample [Figure 1.2-a] solicited by an imposed displacement.



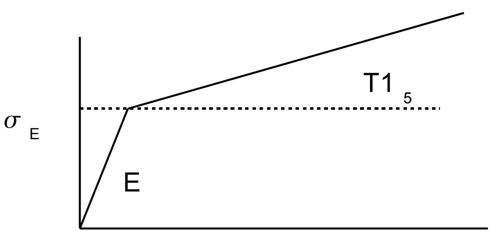
Characteristics material:

$\nu=0.4999$
$E=200\,Gpa$

$\sigma_y=0{,}1$
$E_T=10$

Plasticity criterion:

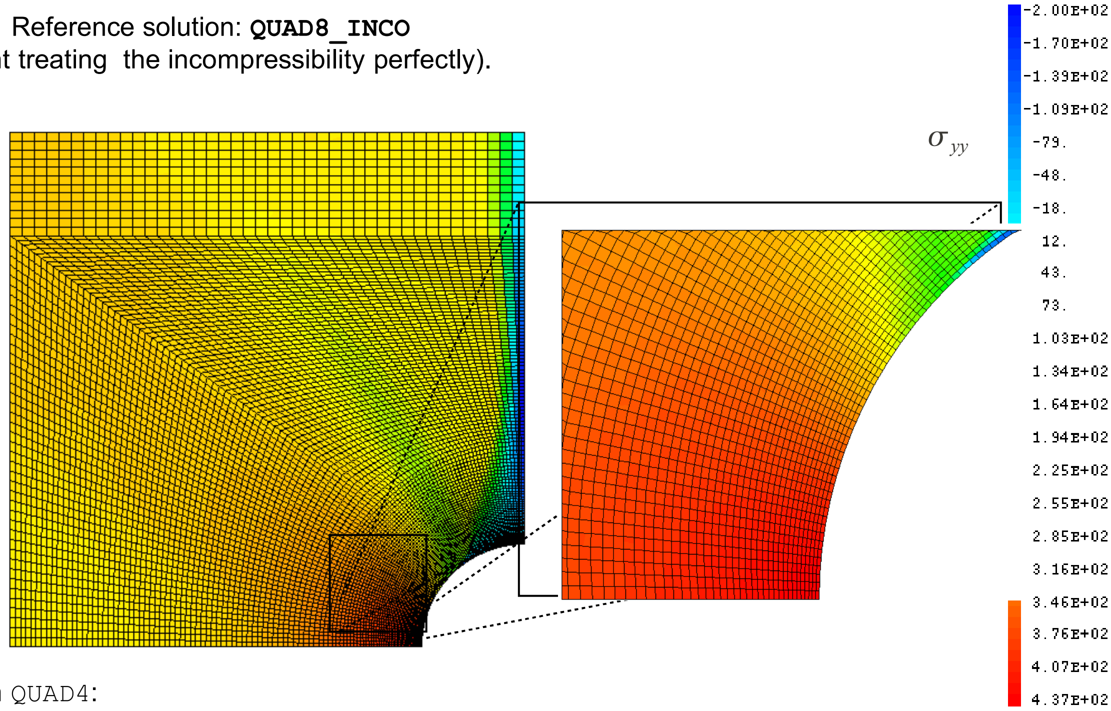
Von-put, plasticity with linear isotropic hardening.



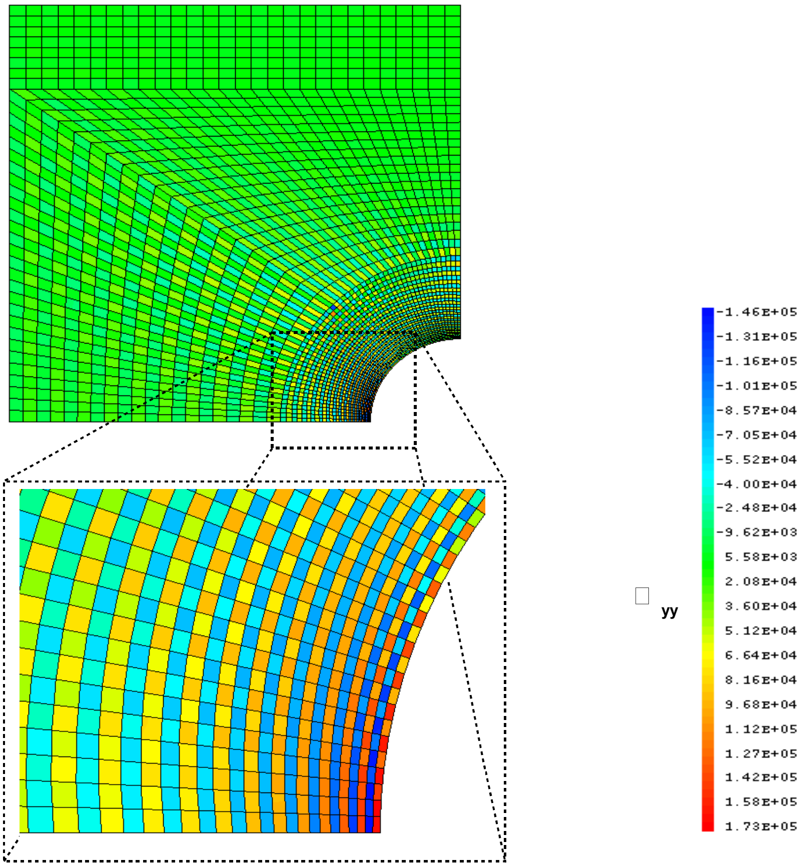
there 1.2-a

For the display of the results, each QUAD4 was cut out in 4 zones containing each one a Gauss point. It is the value of the stress in this point which we display.

Reference solution: **QUAD8_INCO**
(element treating the incompressibility perfectly).



Solution `QUAD4`:



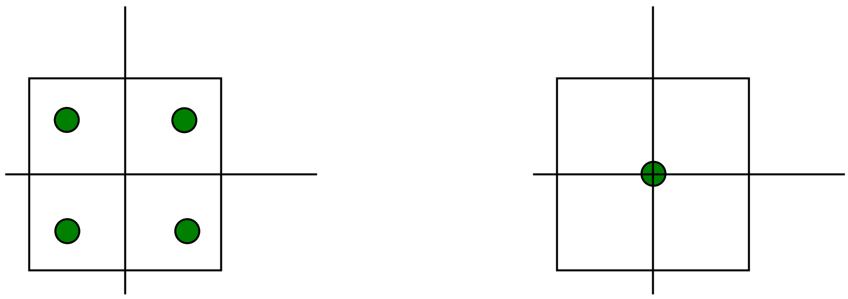
the results show that `the QUAD4` converges only at the cost of strong oscillations of stresses within each element. If these oscillations make it possible the element to put in agreement its nodal displacements and its plastic strain at constant volume, they make the results unrealistic.

2 Under integration of the QUAD4

Despite everything the potential of the isoparametric quadrilateral element for computation by finite elements, the application of a classical bilinear formulation to define its field of displacement, leads to poor results. This is explained by various reasons:

- it has an excessive stiffness ("lock"), during a request of which the cross-bending part is important.
- the classical bilinear formulation of the field of displacement is very sensitive to the distortion of the mesh and presents severe "a locking" when one applies it to an incompressible material.

A solution with these problems of numerical blocking consists in calculating the stiffness matrix via a reduced integration. The principle of this method is to consider the diagram of numerical integration during less points of integrations than one usually should not any to evaluate the stiffness matrixes exact of the element. On the basis of isoparametric element QUAD4 [R3.03.02], one modifies the number of points of integration as well as the weight and the coordinated of the latter, to create the under-integrated element which we will name in this document: QUAS4



QUAD4 QUAS4

QUAD4	QUAS4
Number of points of integration: 4	Number of points of integration: 1
Weight of each point: 1	Weight of point: 4
Coordinates: $(x, y)=(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}})$	Coordinates: $(x, y)=(0,0)$

2.1 Formulation

In the center, the operator discretized gradient used to compute: the stiffness matrix is following form:

$$B=B_c=\begin{bmatrix} b^{t}{}_x & 0 \\ 0 & b^{t}{}_y \\ b^{t}{}_y & b^{t}{}_x \end{bmatrix}$$ **éq 2.1-1**

With

$$b_x^t = N,x\,(0) = \frac{\partial N}{\partial x}|\xi = \eta = 0$$
$$b_y^t = N,y\,(0) = \frac{\partial N}{\partial y}|\xi = \eta = 0$$
éq 2.1-2

and N represents (N_1, N_2, N_3, N_4), the vector of the shape functions. Let us recall that the vectors b represent the simplest shape of the operator discretized gradient under-integrated introduced by Hallquist [bib1] and who is based on the evaluating of derivatives of the isoparametric shape functions at the origin of the reference frame (ξ, η)

That is to say:

$$b_x^t = \frac{1}{2A}\left[(y_2 - y_4),(y_3 - y_1),(y_4 - y_2),(y_1 - y_3)\right] = \text{Constant sur l'élément}$$
$$b_y^t = \frac{1}{2A}\left[(x_4 - x_2),(x_1 - x_3),(x_2 - x_4),(x_3 - x_1)\right] = \text{Constant sur l'élément}$$
éq 2.1-3

$A = (1/2).((x2 - x4).(y3 - y1) + (x3 - x1) * (y4 - y2))$: Area of the element

the stiffness matrix is written:

$$K_e = K_c = A\ .\ B_{c^T}\ .\ C\ .\ B_c$$
éq 2.1-4

C being:

- either the elastic matrix of behavior for computations in elasticity
- or the tangent matrix for the plastic designs. Let us note that during such computations, it is the integration of the constitutive law at the Gauss point (in the center in our case) which determines the value of the coefficients of C .

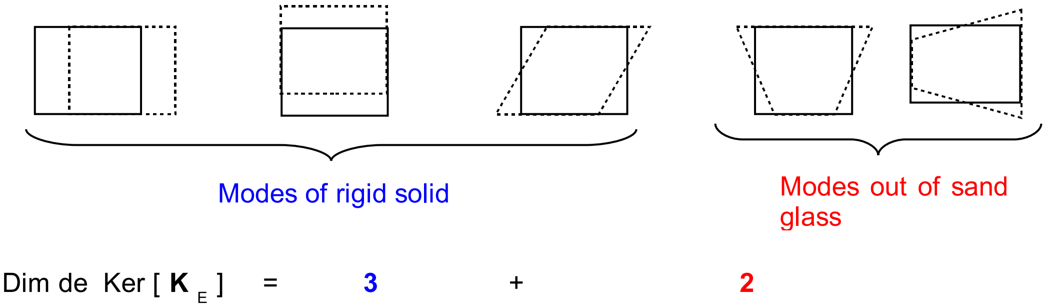
Finally, the internal forces are written:

$$F_{\text{int}} = K_e\ .\ U = B_{c^T}\ .\ \sigma_c$$
éq 2.1-5

U : **Vector of nodal displacements.**

2.2 Failure of computation in Code_Aster: parasitic modes

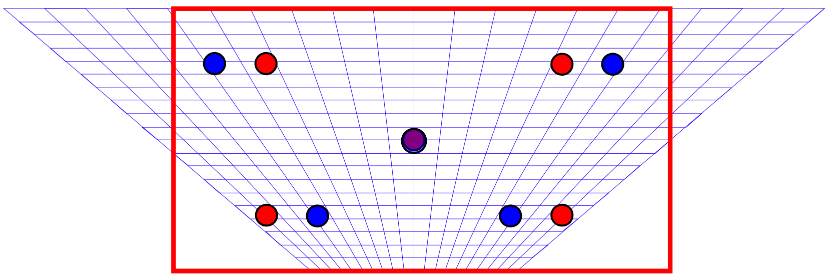
the computations launched on the case test n°1 with such an element fail. The stage of the computation which consists in reversing K_e to determine displacement nodal cannot be crossed. Indeed, in the center, the stiffness matrix is singular. By displaying the core of K_e, we note that its dimension is not any more three but five. Two vectors were added to the core and returned the inversion of the impossible stiffness matrix. The appearance of these two additional vectors in the core of K_e is directly related to the fact that we choose the center like only point of integration. In other words, there exist two fields of nodal displacements other than the fields of displacement corresponding to the rigid solid motion cancelling the internal forces. These modes represent the modes out of sand glass of the QUAD4 [Figure 2.2-a]. Thereafter, one of the stages of stabilization will consist in enriching the operator gradient discretized in order to make K_e invertible.



Appears 2.2-a

2.3 graphic Interpretation of the problem involved under integration

the problem of the under-integration of the QUAD4 is related to its modes of nodal displacements out of sand glass (request in bending). [Figure 2.3-a] shows us that, on such modes, the coordinates of the center remain unchanged. If this is in agreement with the theory of the beams (i.e in pure bending, $\varepsilon_{xx} = 0$ on neutral fiber), the classical theory of the finite elements does not make it possible to differentiate deformed state and not deformed from an element in such a case. Therefore these modes are also called modes with energy null, which on the level of Code_Aster *, make* computations unrealizable. Appear



2.3-a This

problem has for origin a value of the nonsignificant strain in the center of the strain of the QUAD4 . The next chapter will describe the approach which will enable us to calculate the strain of the QUAD4 whatever the modes of displacement of these nodes. Stabilization

3 of the QUAD4 at a Gauss point

the stabilization of the QUAS `4` takes place in two stages: To enrich

- the operator discretized gradient and B_c to thus allow to calculate strain energy related to the modes of displacements out of sand glass (hourglass *modes)*; To interpolate
- a strain field/stresses allowing to give an account of the strain/the stresses on the group of the element while integrating the QUAD4 `into` the center (assumed *strain stabilization).* Variational principle

3.1 of the problem That

Ci is drawn from the weak form of the variational principle of Hu-Washizu: éq

$$\delta\pi\left(\upsilon,\dot{\bar{\varepsilon}},\bar{\sigma}\right)=\int_{\Omega_e}\delta\dot{\bar{\varepsilon}}^T.\ \sigma\ d\Omega+\delta\int_{\Omega_e}\bar{\sigma}^T\ .\ \left(\nabla_s\upsilon-\dot{\bar{\varepsilon}}\right)\ d\Omega-\delta\dot{d}^T\ .\ f^{ext}=0$$

3.1-1 With

$$\dot{\bar{\varepsilon}}\left(x,t\right)=B\ \left(x\right).\ \dot{d}\ \left(t\right)$$

stabilization "Assumed strain" leans on the fact that the applied stress is selected orthogonal with the difference between the symmetric part of the gradient velocity and strain rate. Consequently, that enables us to write: And

$$\delta\dot{d}^T\ .\ \int_{\Omega_e}\mathbf{B^T}\ .\ \sigma\ d\Omega-\delta\dot{d}^T\ .\ f^{ext}=0$$

thus that: Enrichment

$$f^{\text{int}}=\int_{\Omega_e}B^T\ .\ \sigma\ \left(\dot{\bar{\varepsilon}}\right)d\Omega$$

3.2 of the operator gradient discretized To enrich

the operator discretized gradient consists in B_c making a new operator of it by B adding a third component. to him who γ, like and b_x, b_y is a vector of. $\Re^4$ However, as the initial operator correctly B_c calculates the gradient of the linear fields of displacement, the new component formulates γ must be orthogonal with the latter. The stages of the computation of this enriched operator are detailed in the paragraph [§1.2] of the joined document entitled: "Bibliographical Report". Note

: In this report, the new operator connects B the tensor strain rate and $\dot{\varepsilon}$ the vector nodal velocities. $\dot{d}_i$ This formulation enables us in the continuation of the report to reason in term of displacement increments, and consequently to deal with incrémentaux problems, carried out on several time step. For the elastic designs (solutions obtained in one time step) we formulate the problem starting from nodal displacements: the operator then B connects tensor strains and ε nodal displacements. u_i

The new operator B leans on the statement of the field of displacement of the QUAD4 `writes` by Belytschko and Bachrach [bib2]: éq

$$u_i = \left(\Delta^T + x.b_x^T + y.b_y^T + h_\gamma^T \right) u_i$$ **3.2-1 And**

is written: éq

$$B = B_c + B_n = \begin{bmatrix} b_{x^t} + h_{,x}\,\gamma^t & 0 \\ 0 & b_{y^t} + h_{,y}\,\gamma^t \\ b_{y^t} + h_{,y}\,\gamma^t & b_{x^t} + h_{,x}\,\gamma^t \end{bmatrix}$$ **3.2-2 With**

: are

$$\Delta = 1/4 \left[t - (t^T.x)\ b_x - (t^T.y)\ b_y \right]\ ,\ \gamma = 1/4 \left[h - (h^T.x)\ b_x - (h^T.y)\ b_y \right]$$
$$b_x^t = \frac{1}{2\mathrm{A}} \left[(y_2 - y_4),\ (y_3 - y_1),\ (y_4 - y_2),\ (y_1 - y_3) \right]$$
$$b_y^t = \frac{1}{2\mathrm{A}} \left[(x_4 - x_2),\ (x_1 - x_3),\ (x_2 - x_4),\ (x_3 - x_1) \right]$$
$$h = \xi \eta$$

ξ, η the coordinates of reference. are u_i the nodal displacement vectors and are h the values taken by the function with h the four nodes. Let us note

$$d^T = [u_x, u_y]$$

that γ ***can*** be expressed directly according to the nodal coordinates: éq

$$\gamma = \frac{1}{4} \begin{bmatrix} x_2(y_3 - y_4) + x_3(y_4 - y_2) + x_4(y_2 - y_3) \\ x_3(y_1 - y_4) + x_4(y_3 - y_1) + x_1(y_4 - y_3) \\ x_4(y_1 - y_2) + x_1(y_2 - y_4) + x_2(y_4 - y_1) \\ x_1(y_3 - y_2) + x_2(y_1 - y_3) + x_3(y_2 - y_1) \end{bmatrix}$$ **3.2-3**

the shape [éq 3.2-2] of the operator is B equivalent to that of the QUAD4 . `However` this particular writing of the operator makes it possible to differentiate the terms and to integrate into the center terms of stabilization. It is only while intervening on the value of these terms of stabilization that we will be able to improve the performances of the element. Interpolation

3.3 of the strain field:

The general shape of a field "assumed strain" within a QUAD4 is given by Belytschko and Bindeman [bib3]. It is following form: éq

$$\varepsilon_{assumed\ \text{strain}}=\begin{bmatrix}\varepsilon_{x(centre)}+q_x e_1 h_{,x}+q_y e_2 h_{,y}\\ \varepsilon_{y(centre)}+q_x e_2 h_{,x}+q_y e_1 h_{,y}\\ 2\varepsilon_{xy(centre)}+q_x e_3 h_{,y}+q_y e_3 h_{,x}\end{bmatrix}=\begin{bmatrix}\varepsilon_{x(centre)}+\varepsilon_{x(stab)}\\ \varepsilon_{y(centre)}+\varepsilon_{y(stab)}\\ 2\varepsilon_{xy(centre)}+2\varepsilon_{xy(stab)}\end{bmatrix}$$ **3.3-1 With**

: and

$$q_x=\gamma .\ u_x$$
$$q_y=\gamma .\ u_y$$

which e_1, e_2, e_3 varies according to the consideration physics of each author [Table 3.3-1]. Each triplet of values characterizes an element and causes a particular interpolation of the strain: Element

QUAD4	*e1*	*e2*	*e3*
1	0	1	ASMD
1	2 -1	/2 1	ASBQI
1	0	$-\nu$	ASOI
1	-1	0	ASOI
(1/2) 1	2 -1	/2 0	Table

3.3-1 Thus

we can deduce the statement from the operator discretized gradient rising from the supposed strain field () of $\varepsilon_{(stab)}$ the element. We note this new operator. B_n éq

$$\text{QUAD4}:\quad B_n=\begin{bmatrix}h_{,x}\gamma^t & 0\\ 0 & h_{,y}\gamma^t\\ h_{,y}\gamma^t & h_{,x}\gamma^t\end{bmatrix}$$

$$\text{ASBQI}:\quad B_n=\begin{bmatrix}h_{,x}\ \gamma^t & -\bar{\nu}\ h_{,y}\ \gamma^t\\ -\bar{\nu}\ h_{,x}\ \gamma^t & h_{,y}\ \gamma^t\\ 0 & 0\end{bmatrix}$$ **3.3-2 This**

$$\text{ASOI}(1/2):\quad B_n=\begin{bmatrix}\frac{1}{2}h_{,x}\gamma^t & -\frac{1}{2}h_{,y}\gamma^t\\ -\frac{1}{2}h_{,x}\gamma^t & \frac{1}{2}h_{,y}\gamma^t\\ 0 & 0\end{bmatrix}$$

writing enables us to test the various elements easily.
The stiffness matrix is written then in the following way: éq

$$K_e = K_c + K^{stab}$$ **3.3-3 With**

: éq

$$K_c = \int_{\Omega_e} B_{c^T} C B_c d\Omega = A \ . \ B_c \ . \ C \ . \ B_c$$ **3.3-4 And**

éq

$$K_{stab} = \int_{\Omega_e} B_{c^T} C B_n \ d\ \Omega + \int_{\Omega_e} B_{n^T} \mathbf{C} \, \mathbf{B}_{\boldsymbol{c}} \ d\ \Omega + \int_{\Omega_e} B_{n^T} \mathbf{C} \, \mathbf{B}_{\boldsymbol{n}} \ d\ \Omega$$
$$= \sum_{i=1}^{4} \mathbf{JAC}(i).\left(B_c^T . C . B_n(i) + B_n^T(i) . C . B_c + B_n^T(i) . C . B_n(i)\right)$$ **3.3-5 Finally**

we calculate the internal forces in the following way: éq

$$F_{\text{int}} = B \ . \ \left(\ C \ . \ \left(\ \varepsilon_{(centre)} \ + \ \varepsilon_{(stab)} \right) \right)$$
$$= \sum_{i=1}^{4} \mathbf{JAC}(i).\left(\quad \left(B_c + B_n(\ i\ )\right) \quad . \quad \left(C.\left(\varepsilon_c + \quad \varepsilon_{stab}(\ i\ )\right)\right) \quad \right)$$ **3.3-6 Notes**

: Although
the computation of Gauss points K_{stab} requires a sum on the four, the integration of the constitutive law which determines the value of the terms of, C is carried out in the center. In addition, the equations [éq 3.3-4] and [éq 3.3-6] shows us that computations remain relatively bulky. A solution with this problem (not yet established in Code_Aster *) consists in* carrying out computations while being placed in a reference turning with the element (cf [§3.4] of the bibliographical report). This has as an advantage:

- •to remove the computation of the cross terms: ; $B_c \ . \ C \ . \ B_n$ et $B_n \ . \ C \ . \ B_c$
- •better a processing of blocking in transverse shears;
- •a writing of the constitutive law adapted better to the problems including of the non- geometrical linearities. Integration

4 of the element in Code_Aster

element QUAS 4, has two families of Gauss points. The first family consists of a point being in the center and whose weight is worth four. Elementary computations are done naturally with the first family. The calculated options are identical to that calculated by the QUAD4 . One thus calculates in the center of the element the stresses and local variables, as well as the tangent behavior. In the frame of a post processing, each element presents constant fields.

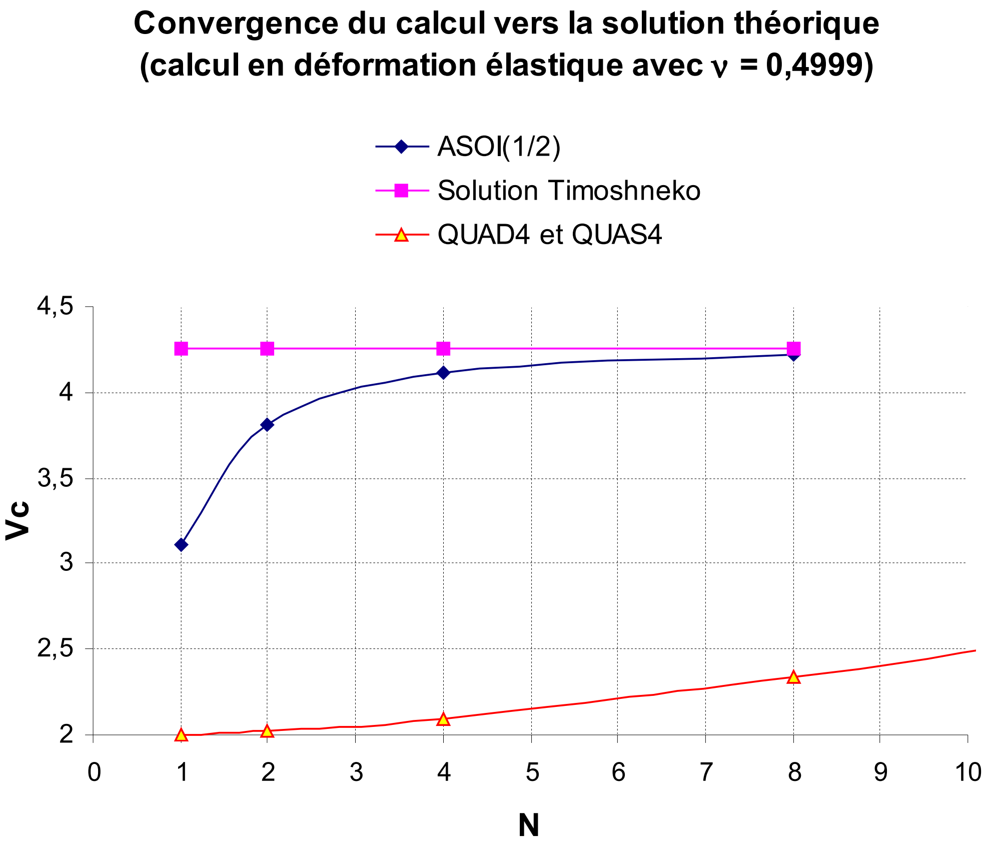
The second family is identical to that of the QUAD4 classic and comprises 4 Gauss points. She is used to compute: the matrix of stabilization. To store the forces of stabilization, one adds to the stress field as in the Gauss point 6 components. This

element is activated by choosing modelizations "C_PLAN_SI " or "D_PLAN_SI " in AFFE_MODELE for meshes QUAD4 . Only computations in small strains are possible. There remains an important limitation: in version 7.2, two types of stabilization are programmed: ASBQI and ASOI (1/2), but are not accessible by a key word in the command file. It is necessary to activate them to modify parameter PROJ in routine NMAS 2D. Put

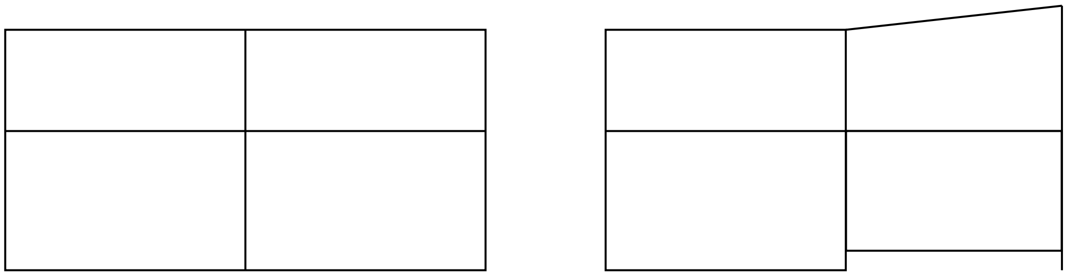
5 in obviousness of the contribution of element QUAS 4 In order to

evaluate the contribution of element QUAS 4, we used it to carry out computations of the cases tests number one and two (cf [§3]). Case

5.1 test n°1 (SSLP106) Among


Convergence du calcul vers la solution théorique
(calcul en déformation élastique avec ν = 0,4999)
ASOI(1/2)
Solution Timoshneko
QUAD4 et QUAS4
Vc
N
4,5
4
3,5
3
2,5
2
0
1
2
3
4
5
6
7
8
9
10

the elements appearing in [Table 3.3-1], only element ASOI (1/2) could be established successfully. Moreover computations converge only for the case test of the straight beam, modelled by a regular mesh. Same the computations carried out on a voluntarily deformed mesh diverge [Figure 5.1-a]. Regular



mesh ¥
Convergence **of** computation irregular

Mesh □
Divergence **of** computation Appears

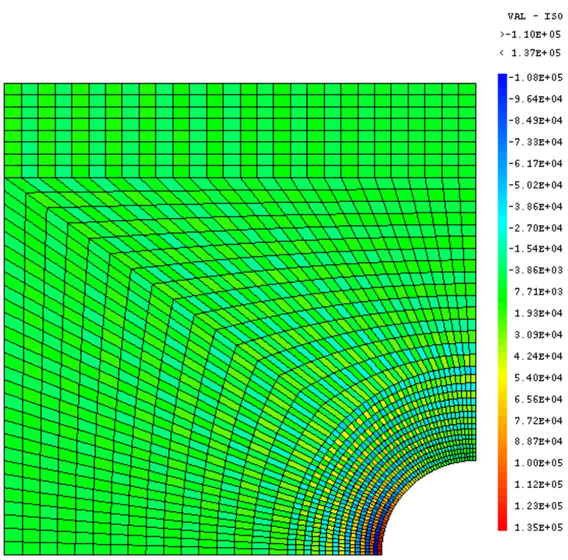
5.1-a Concerning

the computations carried out on a regular mesh, we note a clear improvement of the results. The theoretical solution is reached with a network of sixty-four elements. This reasonably enables us to say that the blocking of element QUAD4 disappeared. This test corresponds to test SSLP `106 of` the base of tests of Code_Aster . *Case*

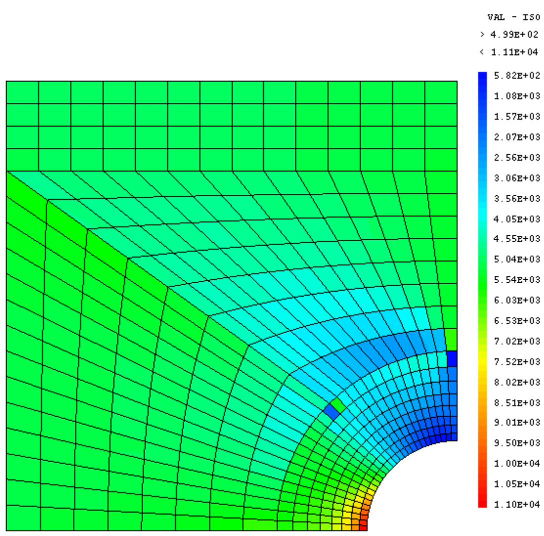
5.2 test n°2 (SSNP123) Recall

of problem: With a mesh of the type QUAD4 , computation reveals important oscillations of stresses on the mesh. We will compare the exits results of mesh QUAD4 `with` those of mesh QUAS `4:` `Computation`

in elasticity in the plane: isovaleurs of QUAD4 σ_{yy}

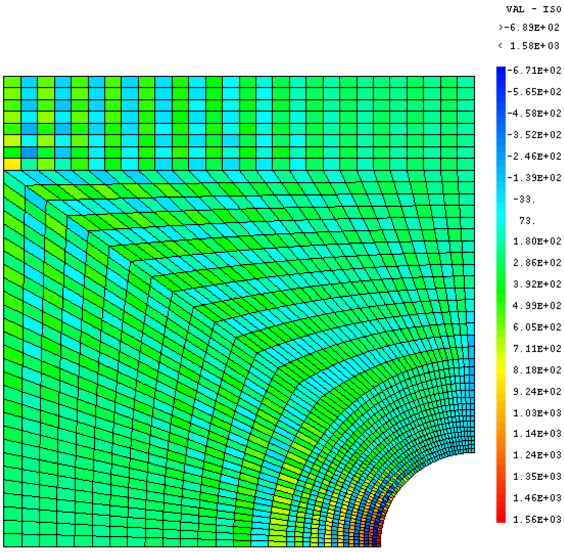


- **QUAS**
- $\nu = 0,4999$

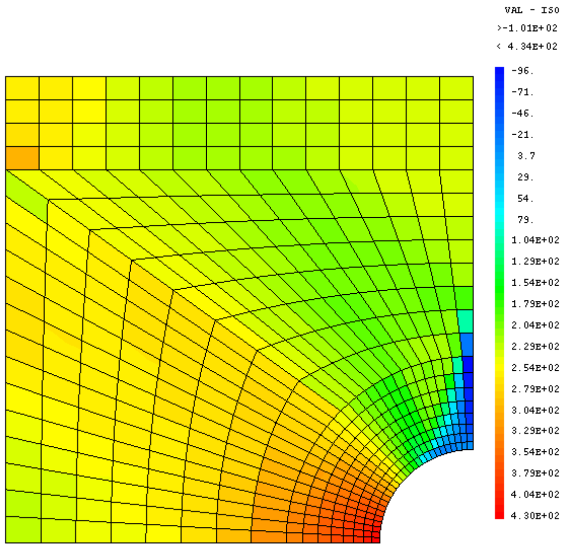


- **4 plastic**
- $\nu = 0,4999$

Designs in the plane: isovaleurs of QUAD4 σ_{yy}



- **QUAS**
- $\nu=0,3$



- **4formule**
- $\nu=0,3$

computing time indicated is an average carried out on three launching. For
the visualization of the results, each QUAD4 was divided into four zones containing each one a Gauss point. The QUAS 4 as for them are not redivisés.

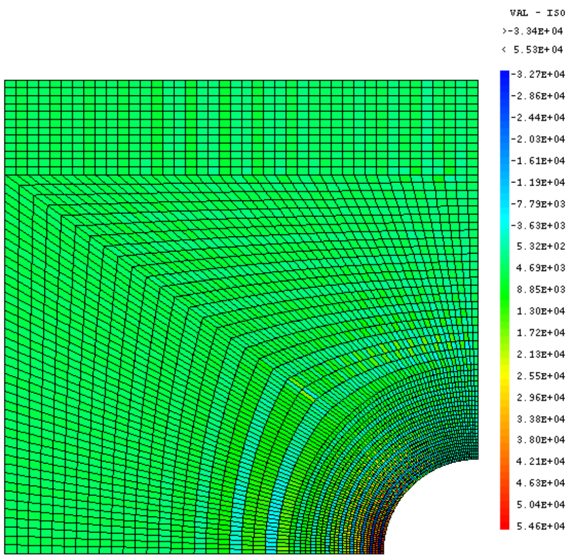
The tests carried out are voluntarily severe (near ν to 0,5). Indeed the goal here is to reach the limits of element QUAD4 in order to note the performances of the new element. Comments

5.3 We

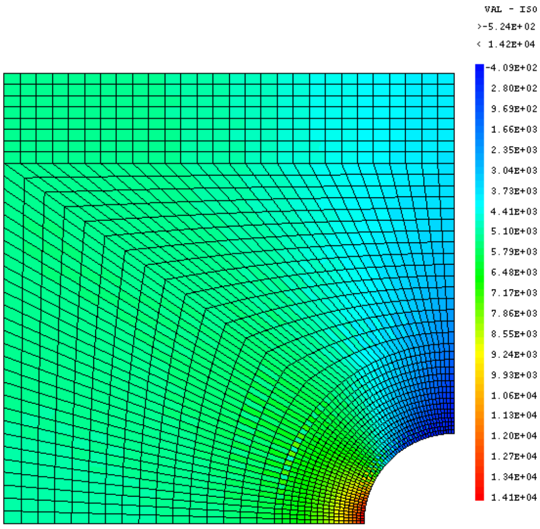
note through this series of computations that the results got with elements QUAD4 comprise strong oscillations of stresses. The results resulting from computations leaning on the QUAS 4 do not have almost any more oscillations. When they appear, they are very localised. That it is for the QUAD4 or the QUAS 4, displacement calculated with the node are A identical. Indeed, the operator gradient discretized of the QUAS 4 is deduced from the strain field of the QUAD4 (cf [éq 3.2-2]). Consequently, that it is with a grid with one or the other of these elements, the structure has the same stiffness. Taking into account

the severity of the test and in order to give us an account of the quality of the results provided by the QUAS 4, we carried out computations leaning on the meshes quadratic ones such as QUAD 8, the QUAS 8 (QUAD 8 pennies integrated with 4 Gauss points) or the QUAD 8_INCO, element treating the incompressibility perfectly. The comparison between elements with quadratic interpolation and elements with linear interpolation has little meaning. Such tests were carried out in order to have a reference solution and they enabled us to make a purely qualitative assessment on element QUAS 4. Results

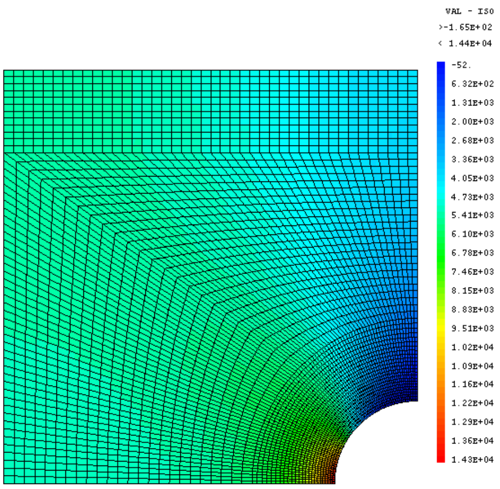
of reference: computations in elasticity in the plane with quadratic elements QUAS



- **8 Period**
- $\nu=0,4999$
- $dy(A)=6,0\,E-2$
- **of computation: Lasted** $5,6\,s$

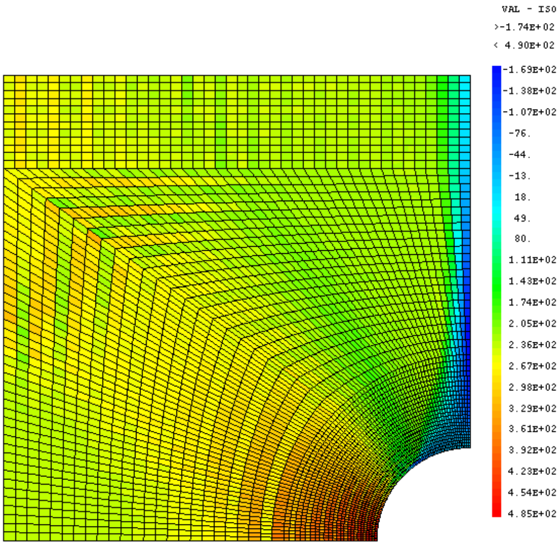


- **8 Period**
- $\nu=0,4999$
- $dy(A)=6,01\,E-2$
- **of computation: QUADS** $3,9\,s$

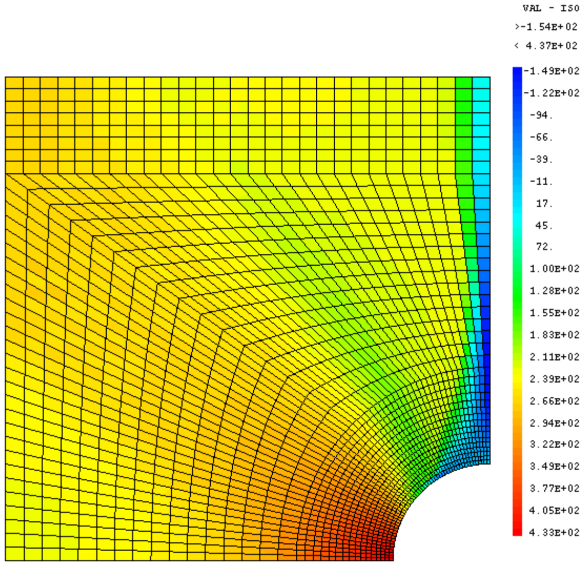


- **QUAD 8_INCO**
- $\nu=0,4999$
- $dy(A)=6,01\,E-2$
- **of computation: Plastic** $5,5\,s$

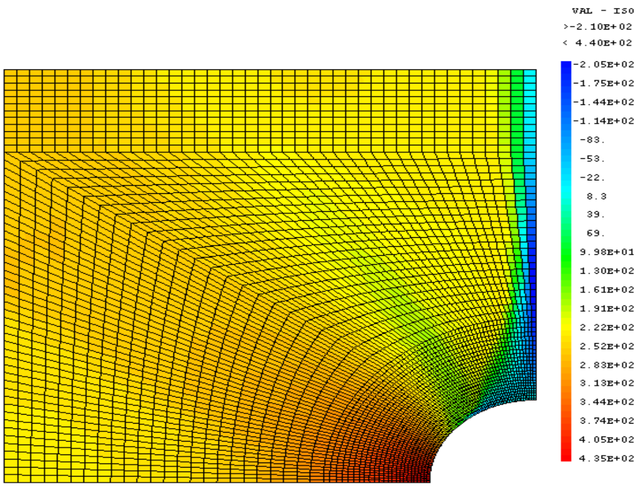
designs with quadratic elements Lasted



- **`8 Period`**
- $\nu=0,3$
- $dy(A)=1,01E-1$
- **of computation: QUAS** $17s$



- **`8 Period`**
- $\nu=0,3$
- $dy(A)=1,01E-1$
- **of computation:** $13,1s$



- **`QUAD 8_INCO`**
- $\nu=0,4999$
- $dy(A)=1,01E-1$
- **of computation: QUADS** $34,4s$

A refinement of the mesh for each computation at summer carried out. This refinement consists in doubling the number of meshes present on each stops notch. We summarize the results in the following tables: Elastic design

strain plane = 0,4999 Soil	ref.: Depl. Node A 6,01	E-02 Number of meshes

504	2016		8064		computing	
	Times Depl	Node A computing	Time Depl	Node A computing	Time Depl	Node A QUADS
8_INCO 5,5	6,01	E-02 18	6,01	E-02 QUADS		
8.5,6	6,00	E-02 15,5	6,01	E-02 QUADS		
8 IF 3,9	6,01	E-02 10,2	6,01	E-02 QUADS		
4.3,3	2,70	E-02 8,4	3,60	E-02 QUAS		
4 2,72	2,70	E-02 5,77	3,60	E-02 17,3	4,77	E-02 Table

5.3-1: Computations in elasticity Computation $\nu=0,4999$

plastic strain plane = 0,3 Soil	ref.: Depl. Node A 1,01	E-01 Number of meshes

504	2016		8064		computing	
	Times Depl	Node A computing	Time Depl	Node A computing	Time Depl	Node A QUADS
8 17	1,01	E-01 75,1	1,01	E-01 QUADS		
8 IF 13,1	1,01	E-01 62,8	1,01	E-01 QUADS		
4.6,6	9,32	E-02 22	1,00	E-01 QUAS		
4.6,5	9,18	E-02 17,33	1,00	E-01 133,2	1,01	E-01 Table

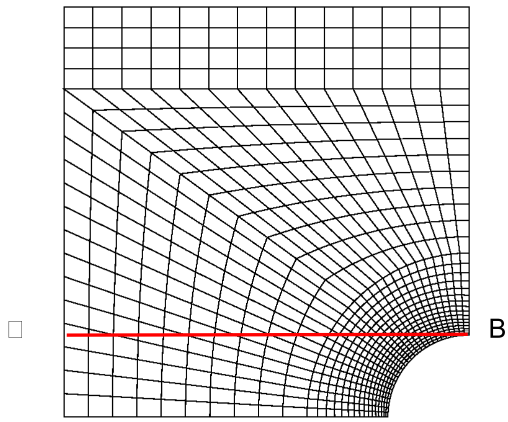
5.3-2: Plastic designs Computation $\nu=0,3$

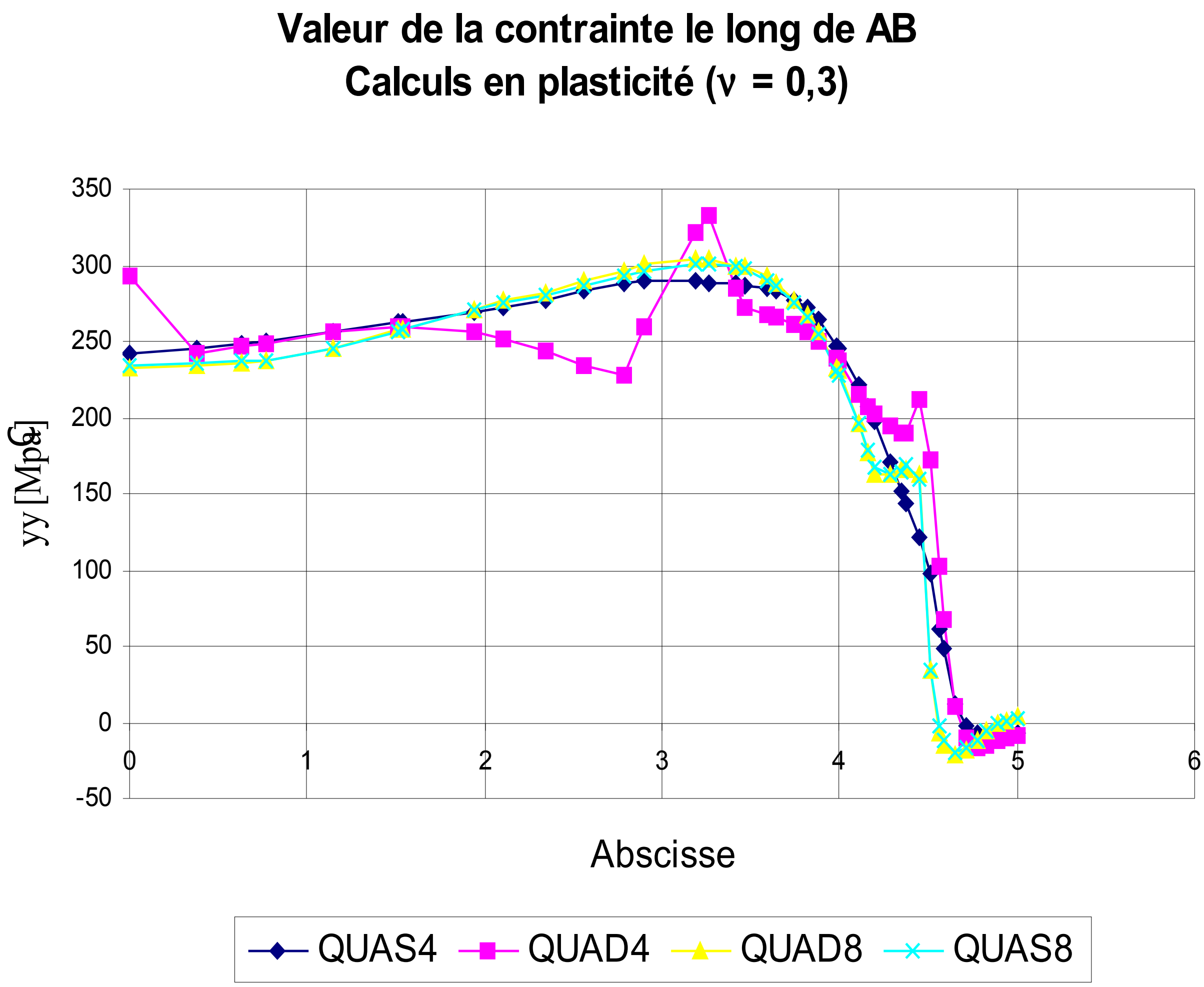
plastic strain plane = 0,4999 Soil	ref.: Depl. Node A 1,01	E-01 Number of meshes

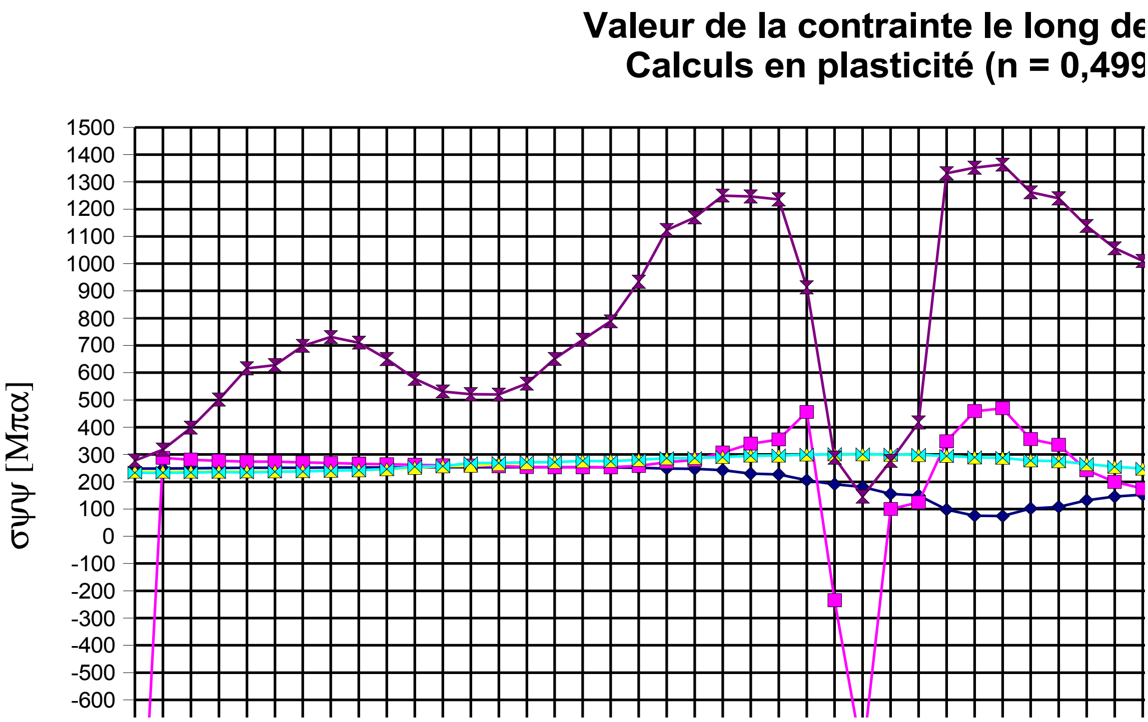
504	2016		8064		computing	
	Times Depl	Node A computing	Time Depl	Node A computing	Time Depl	Node A QUADS
8_INCO 34,4	1,01	E-01 240,8	1,01	E-01 QUADS		
8 21,7	8,84	E-02 88,17	1,01	E-01 QUADS		
8 IF 20,6	1,01	E-01 87,83	1,01	E-01 QUADS		
4.5,1	2,06	E-02 13,07	2,25	E-02 QUAS		
4 4,28	2,06	E-02 9,87	2,25	E-02 43,75	2,79	E-02 Table

5.3-3: Plastic designs In addition $\nu=0,4999$

, we traced the value of the stress along σ_{yy} a segment crossing AB the notch for a basic mesh with 504 meshes. A







on qualitative the Aspects

results: In spite of

- the wealth of the field of displacement of the mesh QUAD8, the majority of the results have oscillations . In spite of
- under integration of the QUAS8, the oscillations appear on meshes QUAS8 for certain plastic designs . Whatever the
- element used (except for the QUAD8_INCO), under integration remains essential, particularly for the materials of which approaches ν 0,5. These
- computations lead us to establish the following report: the QUAS4 remains stable with respect to the oscillations whatever the parameters of computation used. Quantitative

aspect: We

always note that the value of the displacement of the node A resulting from a mesh QUAS4 remains between 3 and 5 times lower than the reference solution (slow convergence). This value remains nevertheless identical to that calculated with a mesh QUAD4.

The profiles traced along the notch enable us to affirm that the values of stresses calculated with a mesh QUAS4 are of much better quality than those calculated with a mesh QUAD4. By taking the computing times of the QUAD4, mesh QUAS4 allows a substantial reduction of the period of computation: Time-saver

	(time *of ref. = time QUAD4) Mesh*		
504		2016	Elasticity
26%.35%.18%	$\nu=0,3$		
	$\nu=0,4999$	31%	Plasticity
16%.21%.18%	$\nu=0,3$		
	$\nu=0,4999$	24%	Let us notice

on the way that the time-savers seem to increase with the number of meshes. In plasticity in particular, this time-saver depends mostly amongst iterations of Newton necessary to convergence on computation within each time step. Conclusions

6 In

elasticity as in plasticity, that it is for materials of which is worth ν or $0,3$, $0,5$ new element QUAS4 remains stable and the always realistic results, without any oscillations of stresses. This, we noted it, is far from being the case for the QUAD4. The stability of this new element vis-a-vis cases tests as severe as those presented in this ratio is comparable to that of quadratic element QUAD8 under integrated. On the other hand

, this element has the convergence of a linear element in terms of many degrees of freedom. It is thus necessary to net with a sufficient smoothness to collect the gradients of stresses of the sought solution. This refinement necessary must be put out of balance with the time-saver induced by under - integration. On

the treated examples, the QUAS4 allowed a time-saver of significant computation of about 20% on average for elastic and elastoplastic constitutive laws. Let us note that these models are relatively inexpensive to integrate. Time-savers much more important are expected for models more difficult to integrate. Bibliography

7 Olympic Games

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8 of the versions of the document Version

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