Finite elements of mixed interface for models of cohesive zone (xxx_INTERFACE and xxx_INTERFACE_S)

Summarized:

This document presents a finite element of mixed interface for models of cohesive zone. The key keys corresponding for the modelizations in *Code_Aster* are PLAN_INTERFACE, AXIS_INTERFACE, 3D_INTERFACE, PLAN_INTERFACE_S, AXIS_INTERFACE_S and 3D_INTERFACE_S.

Contents

1 Introduction

This model of cohesive zone gives an account of the phenomena of starting of crack, propagation (in a given direction) and of final fracture. The mixed element of interface is introduced to discretize the way of cracking of which the degrees of freedom are the displacement of the two lips of crack, as well as the density of the cohesive forces. The construction of the model of finite element and the integration of constitutive law `CZM_TAC_MIX` are detailed. This document is largely inspired by the article Lorentz 5, the interested reader can refer to it for more details.

2 Construction of the finite element of mixed interface

2.1 Principle of minimization of energy in fracture mechanics

In their approach of the fracture mechanics, Frankfurt and Marigo 5 describe the state of a structure Ω by a field of displacement u which can admit discontinuities $\delta=[[u]]$ through surfaces $\Gamma(u)$. For a loading given displacement is characterized by a condition of optimality: it corresponds at least local of defined potential energy as the sum of elastic strain energy, cohesive energy and the work of the external forces:

$$E_{\text{pot}}(u)=E_{\text{el}}(u)+E_{\text{fr}}([[u]])-W_{\text{ext}}(u)$$
$$E_{\text{el}}(u)=\int_{\Omega/\Gamma(u)}\Phi(\varepsilon(u))\,d\Omega \;\;;\;\; E_{\text{fr}}(\delta)=\int_{\Gamma(u)}\Pi(\delta)\,dS$$

where ε indicates the tensor of the strains, Φ the density of strain energy (voluminal) and Π the density of cohesive energy (surface). In this formulation, the mechanisms of fracture are regarded as reversible. The irreversibility will be introduced into the following part, from local variables. The characteristics of the problem are deduced from minimization, namely:

- the forecast of the way of cracking, since one takes into account all possible discontinuous displacements;
- a criterion in stress for the crack initiation (see 5).
- the cohesive model connects the discontinuity of displacement δ to the vector of tension t from derivative (generalized) of Π ;
- the conditions of contact, are managed via the cohesive energy to which one adds a function of indicatrix prohibiting the interpenetration of the lips.

However, to take account of all possible discontinuous displacements involves numerical difficulties related to the discretization of functional space $BD(\Omega)$. In particular, to authorize discontinuities through each one of the finite elements led to a dependence of the results to the mesh. Alternative works are completed, based on the regularization of discontinuities 5 or the adaptation of mesh 5. The latter are however restricted with energies of surface of the type Griffith (without cohesive forces) and lead to important numerical difficulties. This is why we introduce a simplification into our model by considering that surfaces potential discontinuities of displacement Γ are applied *a priori* and that they do not depend any more u .

2.2 Search for point saddles by a method of decomposition – coordination

In spite of the postulate on the direction of cracking, the minimization of energy is not obvious taking into account the non-derivability of the energy of surface. One bases oneself here on the technique of decomposition – coordination introduced into 5, which condenses the non-derivability on a local level (Gauss points).

2.2.1 Augmented Lagrangian

the relation between the field of discontinuity δ and the field of displacement u is introduced explicitly into minimization, total energy E thus depends explicitly on u and on δ :

$$E(u,\delta)=\int_{\Omega\setminus\Gamma}\Phi(\varepsilon(u))\,d\Omega-W_{ext}(u)+\int_{\Gamma}\Pi(\delta)\,d\Gamma$$

The minimization of potential energy is then equivalent to the problem of minimization under stress according to (displacement pertaining to all kinematically admissible displacements):

$$\min_{\substack{u,\delta\\ [[u]]=\delta}} E(u,\delta)$$

The linear stress $[[u]]=\delta$ is treated by dualisation : a solution (u,δ) of corresponds to a point saddles (u,δ,λ) Lagrangian following, where λ corresponds to the field of the Lagrange multipliers:

$$\boldsymbol{L}(u,\delta,\lambda)\underset{déf}{=}E(u,\delta)+\int_{\Gamma}\lambda\cdot([[u]]-\delta)\,d\Gamma$$

Lastly, to gain in coercity compared to δ , which will prove to be necessary thereafter, one adds a quadratic term of penalization not having no influence on the solution since it is equal to zero when the stress in checked. The Augmented Lagrangian one $\boldsymbol{L}_r$, with the coefficient of penalization r is written then:

$$\boldsymbol{L}_r(u,\delta,\lambda)\underset{déf.}{=}E(u,\delta)+\int_{\Gamma}\lambda\cdot([[u]]-\delta)\,d\Gamma+\int_{\Gamma}\frac{r}{2}([[u]]-\delta)^2\,d\Gamma$$

2.2.2 Characterization of the point saddles

the conditions of optimality of order 1 for the Augmented Lagrangian one, are written in the following way:

$$\forall\ \delta\delta\quad \int_{\Gamma}\left[t-\lambda-r\,([[u]]-\delta)\right]\cdot\delta\delta=0\quad\text{avec}\quad t\in\partial\Pi(\delta)$$

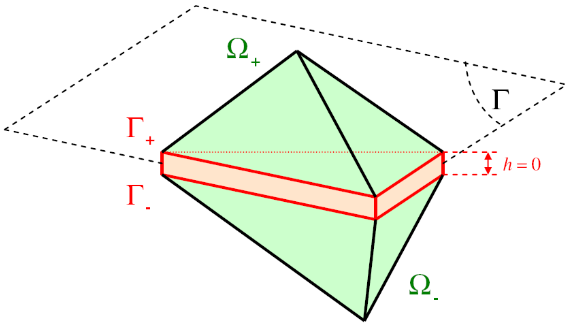
$$\forall\ \delta u\quad \int_{\Omega\setminus\Gamma}\sigma:\varepsilon(\delta u)+\int_{\Gamma}\left[\lambda+r([[u]]-\delta)\right]\cdot[[\delta u]]=W_{\text{ext}}(\delta u)\quad\text{avec}\quad \sigma=\frac{\partial\Phi}{\partial\varepsilon}(\varepsilon)$$

$$\forall\ \delta\lambda\quad \int_{\Gamma}([[u]]-\delta)\cdot\delta\lambda=0$$

The meaning of the under-differential $\partial\Pi$ is given in part 3. A this stage, one is satisfied to say that $t\in\partial\Pi(\delta)$ means that t and δ are connected by the cohesive model. The equation thus gives an interpretation of the multiplier of Lagrange λ : except the term of penalization, it is the cohesive forces. The equation expresses the equilibrium of the forces in the volume and along the surface of discontinuity Γ . Lastly, the stress between the field of displacement and its discontinuities imposes.

2.3 Discretization of the finite element

Since the trajectory of crack Γ is a priori *defined* and thanks to the assumption of the small disturbances, the spatial discretization of the system - can lean on a finite element of interface. Let us suppose that the subdomains Ω_- and Ω_+ (parts of Ω respectively in lower part and with the top of Γ) are discretized by tetrahedrons or hexahedrons so that the nodes on each side of Γ coïncident1Cela[1]. In this case, the degenerated prisms or hexahedrons can be used to discretize Γ and connect the two lips Γ_- and Γ_+ potential cohesive crack (see Appear 2.3-a).



Appear 2.3-a : Discretization by finite element of interface.

That is to say a mesh characterized by the parameter h (for example, maximum size of the finite elements). In the field Ω one adopts a quadratic interpolation with of the finite elements of Lagrange classics (P2-continuous). The discretized space of **the fields of displacements** U_h is written:

$$U_h=\{u\ ;\ \forall\ x\in\Omega\ \ u(x)=[N(x)]\{U\}\}$$

where $\{U\}$ indicates the vector of nodal displacements and $[N]$ the matrix of the quadratic shape functions. The trace of the displacement interpolated on Γ_- and Γ_+ is also quadratic per pieces and the discontinuity of displacement is expressed in the following way:

$$\forall\ x\in\Gamma\ \ u_{|\Gamma_-}(x)=[N_-(x)]\{U\}\ \ ;\ \ u_{|\Gamma_+}(x)=[N_+(x)]\{U\}$$
$$\forall\ x\in\Gamma\ \ [[u(x)]]=[D(x)]\{U\}\quad \text{avec}\ \ [D(x)]\underset{déf.}{=}[N_+(x)]-[N_-(x)]$$

where $[N_-]$ and $[N_+]$ correspond respectively to the trace of $[N]$ on Γ_- and Γ_+ where $[D]$ the matrix of the functions of the quadratic forms indicates which interpolates the jump of displacement. Let us note that it can be useful to introduce a rotation into $[D]$ in order to obtaining the components of $[[u]]$ in a coordinate system local and thus distinguishing the normal and tangential parts.

The field of the Lagrange multipliers λ is interpolated on Γ by linear shape functions per pieces (P1-continuous), leading to space discretized of the Lagrange multipliers $\boldsymbol{L}_h$ according to:

$$\boldsymbol{L}_h=\{\lambda\ ;\ \forall\ x\in\Gamma\ \ \lambda(x)=[L(x)]\{\Lambda\}\}$$

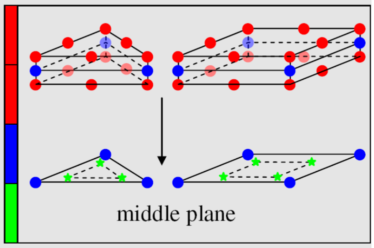
where $\{\Lambda\}$ corresponds to the nodal unknowns of the multiplier of Lagrange and $[L]$ indicates the matrix of the linear shape functions on Γ . In this way, the stress $[[u]]=\delta$ imposed by is carried out only with the weak meaning.

1 does not pose a priori a problem with the algorithms of mesh, at least for cracks having a simple form, for example plane

To finish, the discretization of **the field of discontinuity** $\delta \in D_h$ is based on the collocation points on Γ , of coordinates x_g . These points correspond to Gauss points, of weights ω_g , used to compute: the integrals. Here a standard modelization, and a modelization known as under-integrated are distinguished.

- In the standard modelization, Gauss points are 3 (segments), 6 (triangles) or 9 (quadrilaterals) per element. This corresponds to an interpolation P2-discontinuous of δ . This choice has the advantage of limiting the risk of appearance of null pivots in the tangent matrix, but it does not check condition LBB within the limit of a parameter of penalization r infinite.
- In the under-integrated modelization, Gauss points are 2 (segments), 3 (triangles) or 4 (quadrilaterals) per element. This corresponds to an interpolation P1-discontinuous of δ . This choice checks best condition LBB within the limit $r\rightarrow\infty$, but it is likely to reveal null pivots in the tangent matrix. These null pivots appear when the stiffness of the elements in contact with the interface is not enough to ensure coercity of the formulation, in particular when the interface is on a symmetry plane, or between a volume and a structural element (bar, grid or membrane).

The spatial discretization, the notations as well as the diagrams of the finite elements for the under-integrated modelization are summarized on Appear 2.3-b.

Field	Shape functions	Unknowns	Interpolation	Finite element
displacement $\mathbf{u} \in \mathcal{U}_h$	cont-P2 $[\mathbf{N}(\mathbf{x})]$	nodal $\{\mathbf{U}\}$	$\mathbf{u}(\mathbf{x}) = [\mathbf{N}(\mathbf{x})]\{\mathbf{U}\}$	
displ. discontinuity $[\![\mathbf{u}]\!]$	cont-P2 $[\mathbf{D}(\mathbf{s})]$	nodal $\{\mathbf{U}\}$	$[\![\mathbf{u}]\!](\mathbf{s}) = [\mathbf{D}(\mathbf{s})]\{\mathbf{U}\}$	
Lagrange mult. $\lambda \in \mathfrak{L}_h$	cont-P1 $[\mathbf{L}(\mathbf{s})]$	nodal $\{\Lambda\}$	$\lambda(\mathbf{s}) = [\mathbf{L}(\mathbf{s})]\{\Lambda\}$	
displ. discontinuity $\delta \in \mathcal{D}_h$	disc-P1	Gauss points δ_g	$\delta(\mathbf{s}_g) = \delta_g$	

Appear 2.3-b : Spatial discretization

the field δ disappears from the total formulation thanks to static condensation. Indeed, with the adopted discretization, the resolution of come down to satisfy the cohesive model in each collocation point:

$$t_g=\lambda_g+r\left([\![u_g]\!]-\delta_g\right)\in\partial\Pi(\delta_g) \quad \text{avec} \quad \begin{cases}[\![u_g]\!]=[D_g]\{U\} & ; \; [D_g]=[D(x_g)] \\ \mu_g=[L_g]\{\Lambda\} & ; \; [L_g]=[L(x_g)]\end{cases}$$

The integration of the constitutive relation (i.e the resolution of, cf left following), makes it possible to calculate function δ_g of and $\{U\}$ which $\{\Lambda\}$ one notes: δ

$$t_g=\lambda_g+r\left([\![u_g]\!]-\delta_g\right)\in\partial\Pi(\delta_g) \quad\Leftrightarrow\quad \delta_g=\hat{\delta}\left([\![u_g]\!],\lambda_g\right)=\delta(U,\Lambda)$$

The parameter of penalization makes it possible r to ensure the unicity of whatever and $\{U\}$ (cf $\{\Lambda\}$ left following). That constitutes a requirement to guarantee robustness of the model. There

is then the nonlinear system whose unknowns are nodal nodal displacements $\{U\}$ and the multiplying of Lagrange: $\{\Lambda\}$

$$\int_{\Omega\setminus\Gamma}[\nabla N]^T:\boldsymbol{\sigma}(U)+\sum_g \omega_g[D_g]^T\cdot\left([L_g]\{\Lambda\}+r[D_g]\{U\}-r\,\delta(U,\Lambda)\right) = \{F_{\text{ext}}\}$$

$$\sum_g \omega_g \left[L_g\right]^T \cdot \left(\left[D_g\right]\left\{U\right\} - \delta\left(U, \Lambda\right)\right) = 0$$

The integral of volume and the nodal vector of the external forces are calculated in a usual way. Lastly, the system is solved simultaneously compared to and $\{U\}$ by means of $\{\Lambda\}$ a method of Newton (generalized), where the tangent operator is symmetric since - corresponds to a search for point saddles. Integration

3 of cohesive model CZM_TAC_MIX

the cohesive behavior is determined by the density of cohesive energy. $\Pi(\delta)$ Although the variational formulation presented in the preceding part is independent of the choice of cohesive energy, we are interested now in a particular cohesive model which one details specificities. These main features are: conditions

- of contact (not interpenetration of the lips); perfect
- initial dependancy (not of regularization of the energy of surface); coupling
- between the modes of fracture (tension and shears); total
- fracture, (null cohesive forces beyond of a certain threshold of damage); irreversibility
- of the fracture. That

means in particular that there is no final friction, nor distinction between the mechanisms of fracture in tension and shears. Note:

> *The key word Code_Aster corresponding to this cohesive model is CZM_TAC_MIX . For more details on this model (local variables, parameters of entry, coherent tangent matrix) and on other cohesive models, one can refer to 5 5 7.02.11). Preliminary*

3.1 notations Because

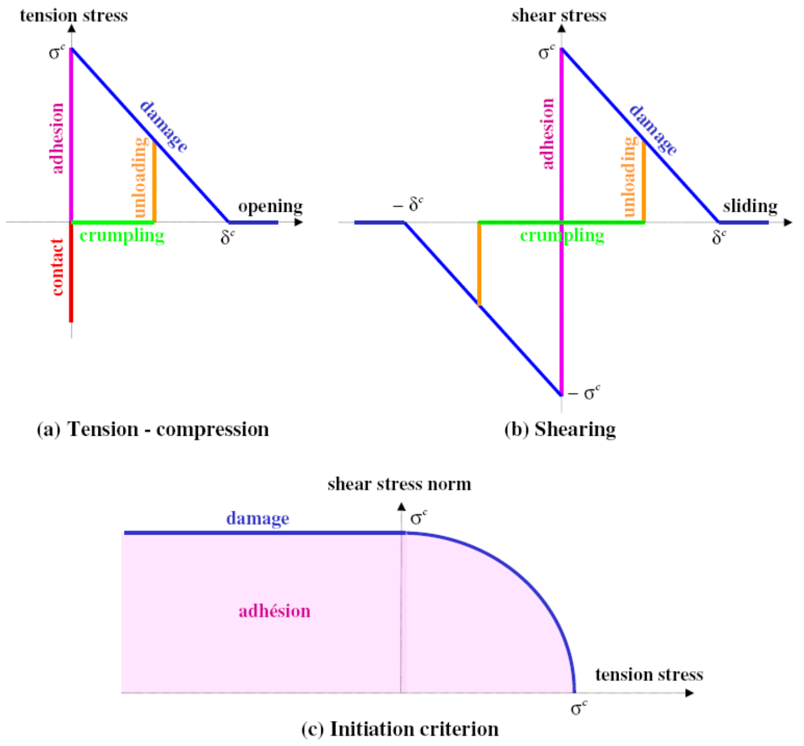
of the stress of noninterpenetration of the lips of crack, the normal direction n for the surface of potential discontinuity (opening Γ /compression) must be distinguished from the directions in the plane (sliding, shears). With this intention, the following notations are introduced, where any v vector quantity represents: and

$$v_n = v \cdot n \;\; ; \;\; v_{//} = v - v_n n \;\; ; \;\; \langle v \rangle_+ = \langle v_n \rangle \, n + v_{//} \;\; ; \;\; \|v\|_+ = \left(\langle v \rangle_+ \cdot \langle v \rangle_+\right)^{1/2}$$

where $\langle . \rangle$ the positive part of a scalar indicates. Definition

3.2 of the cohesive model

the cohesive model suggested by Heel and Curnier 5 5 the five points mentioned previously and returns in the frame of the energy formulation. Responses of the model subjected to a pure tension and pure shears, as well as the criterion of starting in stress are detailed Appears 3.2 3.2-a Figure



3.2 3.2-a Responses of the model Heel Curnier CZM_TAC_MIX in pure traction and compression (A), pure shears (b) and criterion of starting in stress (c)

corresponding cohesive energy is defined as follows: First of all

, discontinuities of displacement in opening and shears are gathered in a single scalar which δ_{eq} measures the amplitude of discontinuity, thus answering item 3 3 Then

$$\delta_{eq}=N(\delta)=\sqrt{\delta\cdot\delta}$$

, one takes account of irreversibility 3 3 a new scalar local variable which κ measures the maximum level of loading running:

$$\kappa(t)=\max_{t'<t}\ \delta_{eq}(t')$$

Cohesive energy depends on the local variable and κ the jump of equivalent displacement. δ_{eq} The conditions of contact 3 3 are managed by an indicating function which imposes a positive normal discontinuity: $\delta_n\geq 0$

$$\Pi(\delta,\kappa)=I_{\mathbb{R}^+}(\delta_n)+\psi(\max(\delta_{eq},\kappa))$$

The function characterizes ψ in particular the reaction to a request in mode I pure. According to 5 5 must ψ be an increasing and differentiable function, where $\sigma^c=\psi'(0)$ the critical stress defines, parameter of the criterion of starting 3 3 on the Figure 3.2 3.2-a In addition, the stability of the process of fracture requires that is ψ concave 5 5 Lastly, the final fracture 3 3 occurs as soon as reached ψ its higher limit for G^c a finished value, $\delta_{eq}=\delta^c$ where is G^c the energy of fracture and δ^c the critical opening beyond which the cohesive forces are cancelled. Thus

, one chooses the following function ψ which corresponds to the responses represented on the Figure 3.2 3.2-a with

$$\psi(\delta_{eq})=\begin{cases} G^c \dfrac{\delta_{eq}}{\delta^c}\left(2-\dfrac{\delta_{eq}}{\delta^c}\right) & \text{if } \delta_{eq}\leq\delta^c \\ G^c & \text{if } \delta_{eq}\geq\delta^c \end{cases}$$

the relation between materials parameters; A

$$G^c=\frac{1}{2}\sigma^c\ \delta^c$$

this stage, the energy and the cohesive model which in drift are entirely defined. However, one can explicitly express the relation between the discontinuity of displacement and δ the vector forced, t in a way condensed in following differential inclusion, where is $\partial\Pi$ the under-differential (see Π Clarke 5 5 : For

$$t\in\partial\Pi(\delta)$$

a value given of, κ one can interpret the under-differential like $\partial\Pi(\delta)$ the cone formed by all the slopes of directional derivatives of in Π for δ all the acceptable directions. Mathematically, that is formulated in the following way: where

$$\partial\Pi(\delta)=\left\{t\in R^3\ ;\ \forall\ v\in R^3\ \ t\cdot v\leq\Pi^{\circ}(\delta,v)\right\}$$

is $\Pi^{\circ}(\delta,v)$ the directional derivative from Π ratio with in δ direction: v This

$$\Pi^{\circ}(\delta,v)=\limsup_{\substack{\rho\to0^+\\ d\to\delta}}\frac{\Pi(d+\rho\,v)-\Pi(d)}{\rho}$$

definition coincides with the gradient of everywhere Π where this one is differentiable. According to the definition, the points deserving a special attention are, $\delta=0$ and $\delta_n=0$. $\delta_{eq}=\kappa$ One thus deduces from the four following cases (magazines color on the Figure 3.2 3.2-a : Not

1) (and $\kappa=0$) $\delta=0$: perfect dependancy, i.e criterion of starting (connects pink) Field
$$\partial\Pi(\delta)=\left\{t\in R^3\ ;\ \|t\|_+\leq\sigma^c\right\}$$

2) where: return to zero $\delta_{eq}<\kappa$ with stress null (connects green) Very
$$\partial\Pi(\delta)=\left\{t_n\,n\ ;\ t_n\leq0\ \text{ et }\ \delta_n\geq0\ \text{ et }\ t_n\ \delta_n=0\right\}$$

3) cone: discharge $\delta_{eq}=\kappa>0$ vertical from the stress (connects orange) Field
$$\partial\Pi(\delta)=\left\{t_n n+\rho\ \delta\ ;\ 0\leq\rho\leq\frac{\psi'(\kappa)}{\kappa}\ \text{ et }\ t_n\leq0\ \text{ et }\ \delta_n\geq0\ \text{ et }\ t_n\ \delta_n=0\right\}$$

4) where: damage $\delta_{eq}>\kappa$ (connects blue) Note:
$$\partial\Pi(\delta)=\left\{t_n n+\psi'(\delta_{eq})\frac{\delta}{\delta_{eq}}\ ;\ t_n\leq0\ \text{ et }\ \delta_n\geq0\ \text{ et }\ t_n\ \delta_n=0\right\}$$

There exists

- *a field for the cohesive force, where discontinuity is null: it is the criterion of starting represented Figure 3.2 3.2-a to nonthe derivability in. Π $\delta=0$ The form of the field depends on the statement on. δ_{eq} The condition*
- *of Kuhn and Tucker, which appears in - makes it possible to describe the conditions of contact (red Figure 3.2-a 3.2 3.2-a is negative t_n , and corresponds to a compression whose value is not defined by the cohesive model. There null*
- *exists a jump between the return to zero with stress (connects green) and the damage (connects blue), the cohesive force is thus not continuous compared to the jump of displacement. In*
- *the case of a mode I pure or a mode of pure shears, the responses of the cohesive model are those represented on the Figure 3.2 3.2-a The maximum values in tension and shears are equal because of the choice of the norm. Numerical integration*

3.3 According to

, the numerical integration of the cohesive model comes down to calculating for δ_g values given of and of $[[u_g]]$, and λ_g this for each Gauss point (one omits now the index). In fact g , is a characterization of the following minimum: Moreover

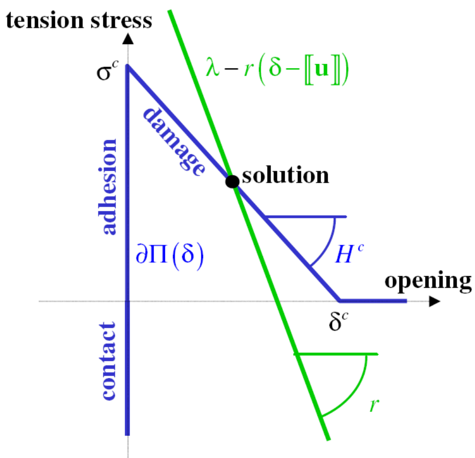
$$\min_{\delta\in\mathbb{R}^3}\left[\lambda\cdot([[u]]-\delta)+\frac{r}{2}([[u]]-\delta)^2+\Pi(\delta,\kappa)\right] \quad\Leftrightarrow\quad \lambda+r[[u]]-r\,\delta\in\partial\Pi(\delta,\kappa)$$

, the evolution of the (voir) local variable κ must be taken into account. A temporal discretization is then necessary, consider a series of times corresponding $t^0<t^1<\ldots<t^n$ quantities and the, λ^n and $[[u^n]]$. δ^n During the resolution κ^n , the iterative process is divided into two parts: The second

$$\delta^n=\underset{\delta\in\mathbb{R}^3}{\operatorname{argmin}}\left[\lambda^n\cdot([[u^n]]-\delta)+\frac{r}{2}([[u^n]]-\delta)^2+\Pi(\delta,\kappa^{n-1})\right]$$

$$\kappa^n=\max(\kappa^{n-1},\delta^n_{eq})$$

part is commonplace, minimization thus amounts solving the first with known $\kappa=\kappa^{n-1}$ parameter. A graphic interpretation of differential inclusion is provided on the Figure 3.3-a 3.3-a I pure without discharge): the solution corresponds to the intersection of the linear function (with $\delta\rightarrow\lambda+r[[u]]-r\,\delta$ a negative slope given by the coefficient of penalty) with r the graph. Appear $\partial\Pi(\delta,\kappa^{n-1})$



3.3-a 3.3-a of the integration of the behavior. So that

integration is robust, it is necessary that the function between hooks in is strictly convex compared to (i.e that δ the minimum is single). While introducing, the lenitive H_c modulus of the cohesive model, one displays a sufficient condition to satisfy this condition (see 5 for more 5 details): Note:

$$r > \max_{x \geq 0} |\psi''(x)| = \frac{\sigma^c}{\delta^c} = H^c$$

is

$r = 100 \times H^c$ *a value recommended to the user of Code_Aster. One supposes*

now that the condition is met , moreover the function between hooks in is semi - continuous in a lower position, which guarantees the existence and the unicity of the minimum. Let us detail

the numerical integration of the model now, i.e. the resolution of according to . Taking into account $\tau = \lambda + r[[u]]$ the existence and unicity of the solution, it is interesting to use differential inclusion and the writing of the under-differential provided by to. The four cases thus are found: : perfect

- si $\kappa = 0$ et $\|\tau\|_+ \leq \sigma^c = \psi'(0)$ dependancy: return to zero

$$\delta = 0$$

- si $\kappa > 0$ et $\|\tau\|_+ < r\kappa$ with stress null: discharge

$$\delta = \frac{\langle \tau \rangle_+}{r}$$

- si $\kappa > 0$ et $r\kappa \leq \|\tau\|_+ \leq r\kappa + \psi'(\kappa)$ vertical: damage

$$\delta = \kappa \frac{\langle \tau \rangle_+}{\|\tau\|_+}$$

- si $r\kappa + \psi'(\kappa) < \|\tau\|_+$ Note

$$\delta = \delta_{eq} \frac{\langle \tau \rangle_+}{\|\tau\|_+} \;\; ; \;\; \delta_{eq} \text{ solution de } \psi'(\delta_{eq}) + r\delta_{eq} = \|\tau\|_+$$

: the distinction

or not N $\kappa = 0$ *"is not necessary since seems a typical case of. To conclude*

, the various modes of the cohesive model correspond to the solutions provided δ by -. These last are provided according to. No $\tau = \lambda + r[[u]]$ numerical method N" is necessary, indeed, the only equation to be solved (in) is linear per pieces. Conclusion

4 a model

finite element of interface, is proposed in order to model the cohesive crack evolution along a preset way. This last is compatible with the use of classical voluminal finite elements. Its unknowns are nodal displacements on the lips of cohesive crack as well as the nodal Lagrange multipliers, corresponding to the surface density of the cohesive forces. The Lagrangian one of the problem is increased, that makes it possible to ensure a condition of convexity (via the parameter of penalization) which guarantees the unicity of the solution during the local integration of the model. This approach

presents however the limits and the following disadvantages: The potential

- trajectories of crack must be applied a priori. Additional
- degrees of freedom, correspond to the cohesive forces, are introduced. However their number remains relatively low since they are restricted with the potential trajectories of crack. The introduction
- of Lagrange multipliers leads to a mixed formulation: the resolution of the problem thus returns to a search for point saddles and more to that of a minimum as it was the case in the initial energy formulation 5. It is 5necessary
- to increase the Lagrangian one to have a property of local convexity. That implies the introduction of a parameter of penalization, without influence on the continuous problem, but which could affect the results of the discretized problem. However, the numerical examples in 5 show 5 that this sensitivity remains low and disappears with refinement from the mesh. The local
- integration of the cohesive model rests on the computation of discontinuities of displacement starting from the cohesive forces. An opposite approach is in general adopted in the literature for the elements of interface. However

a certain number of interesting properties emerge: No regularization

- of the cohesive model is necessary, in particular with regard to the initial dependancy or the condition of contact. The choice
- of a quadratic discretization for displacements and linear for the Lagrange multipliers makes it possible to satisfy condition LBB. The latter makes it possible to ensure the convergence of the solution with the refinement of the mesh in terms of displacements and cohesive forces (see numerical example in 5). The rate 5
- of convergence which one would obtain without interface is not disturbed by the presence of the elements of interface (see 5). The search 5
- for point saddles leads to a symmetric tangent matrix. This model
- of interface is compatible with the usual algorithms of resolution of Code_Aster *such as* the method Newton, the linear search or the control of the loading. This is illustrated by computations 2D and 3D in 5, thus 5showing the robustness and the reliability of such a model. Bibliography

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6 of the versions of the document Index Doc.

Version	**Aster Author (S**	**) or contributor (S), organization Description**	**of the modifications A 9.4 J.**
Laverne		EDF/R & D /AMA E.Lorentz EDF/R & D /SINETICS initial Text	11.3 Mr.
	David	EDF/R & D /AMA Modification	concerning the number of points used for the discretization of the jump